Module 2:

ELASTIC PROPERTIES OF MATERIALS

Elasticity: It is that property of materials by virtue of which it completely regains its original shape & size after the removal of deforming force.

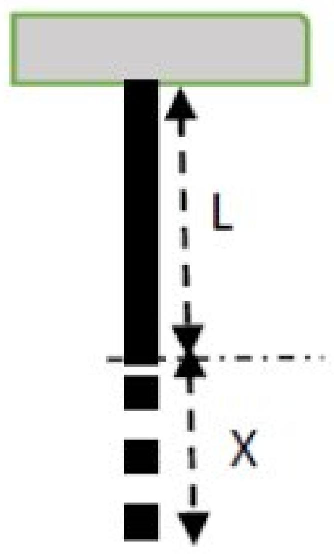
Stress and Strain:

Stress: The stress is given by the ratio of the applied force to the area of its application

Strain: The deformation produced by the external force, accompanies a change in dimensions. '*The ratio of the change in dimensions to original dimensions is called strain.*'

TYPES OF STRESS AND STAIN.

a) **Tensile stress or linear stress or longitudinal stress:** *It is the stretching force acting per unit area of the body along its length.*



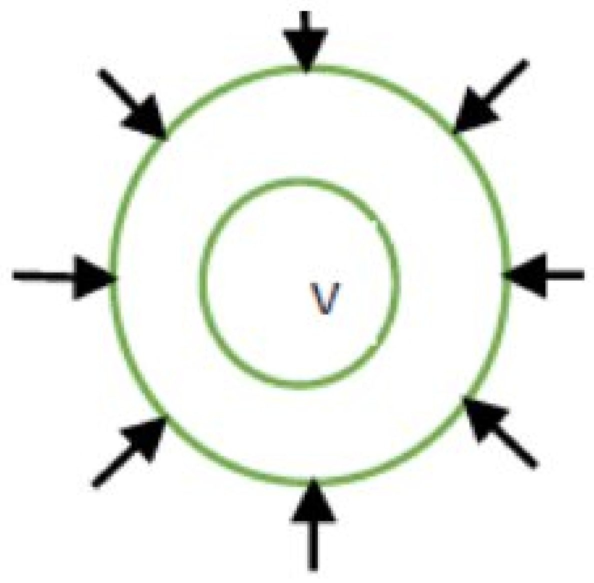
Let us consider the wire fitted to the roof of length 'L' and it has a cross-section area A is elongated to the new length by the action of a force.

$$Longitudinal\ stress\ = \frac{F}{A}$$

Longitudinal strain: '*It is the ratio of change in length to original length by the action of the force.*'

$$\boldsymbol{longitudinal\ strain = \frac{change\ in\ length}{original\ length} = \frac{x}{L}}$$

Compressive stress or Volume stress (Pressure):



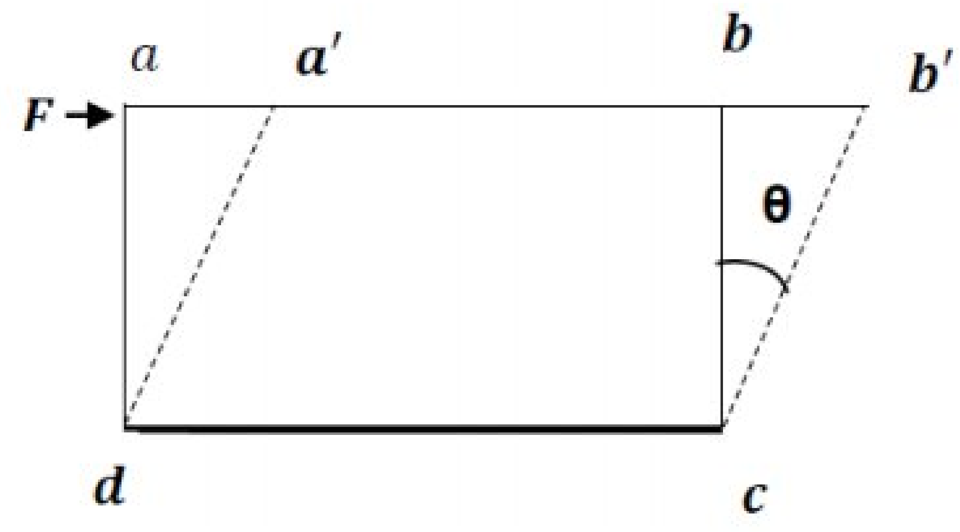
It is the uniform pressure (Force per unit area) acting normally all over the body. When the Deforming force is applied normally and uniformly to the entire surface of a body' it produces a volume strain (without changing its shape in case of solid bodies). The applied force per unit area gives normal stress or pressure. If F is the force applied uniformly and normally on a surface area.

$$Compressive\ stress\ or\ pressure = F/A$$

If a uniform force is applied all over the surface of a body (which is having a suitable shape for such an application) the body (Fig. 2) undergoes a change in its volume (however, the shape is retained in case of solid bodies). If v is the change in volume and V is the original volume of the body then.

$$\text{Volume strain} = \frac{\text{Change in volume}}{\text{original volume}}$$

Shear Stress or Tangential Stress:



It is the force acting tangentially per unit area on the surface of a body. If a force is applied tangentially to a free portion of the body with another part being fixed, its layers slide one over the other; the body experiences a turning effect and changes its shape. This is called shearing and the angle through which the turning takes place is called shearing angle.

$$\text{Hence, } tangential\ stress\ = \frac{F}{A}$$

Shear Strain

In the case of shearing, the shearing angle itself is a measure the ratio of change in dimensions to original dimensions.

$$Shearing\ strain\ (\theta) = \frac{x}{L}$$

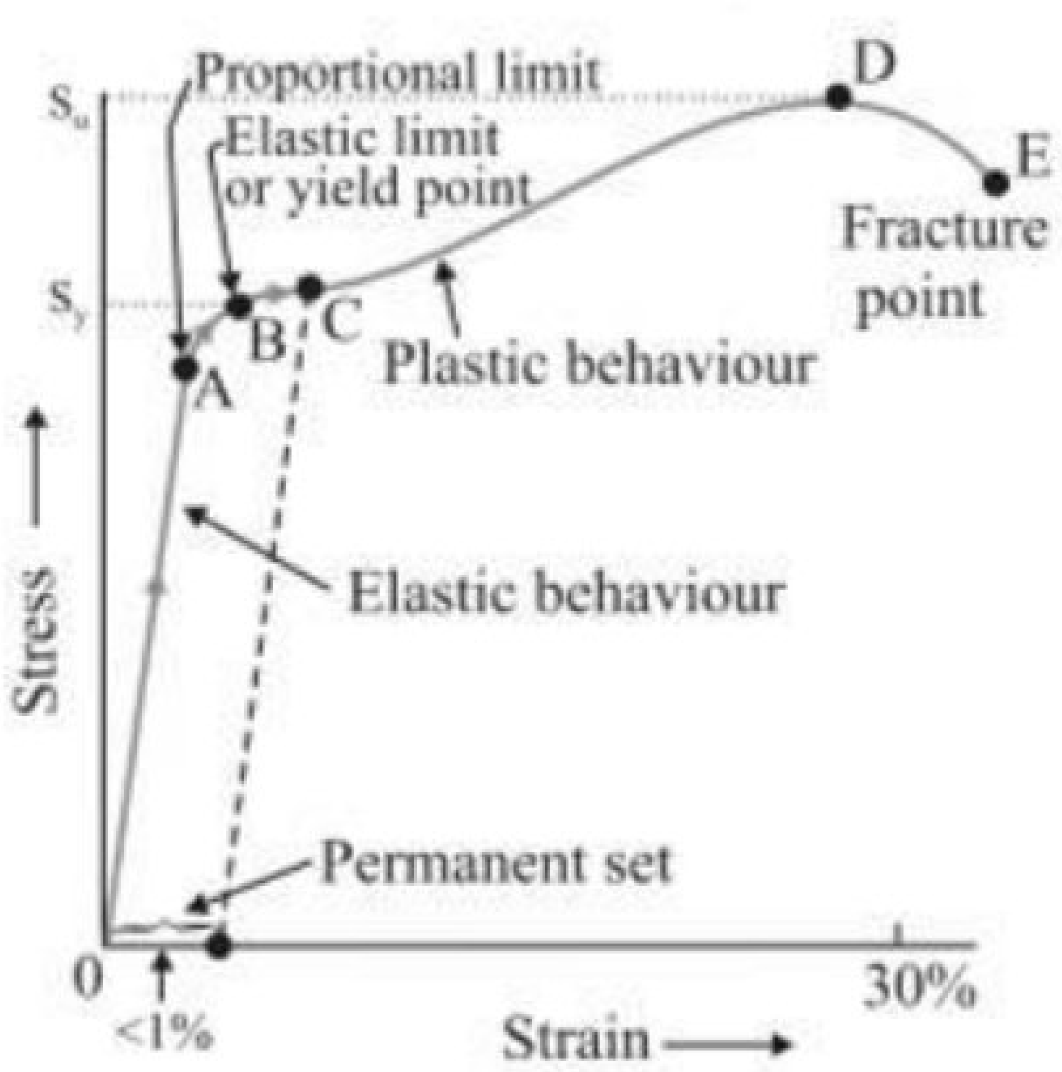
Hooke's law

Stress is directly proportional to strain within the elastic limit

$$Stress\ \propto strain$$

$$\frac{Stress}{strain} = Constant\ (elastic\ modulus)$$

Stress-Strain Diagram:



OA=Proportionality Range.
A=Proportionality Limit.
B=Yield Point.
C=Plastic Limit.
D = Ultimate Strength.
E= Fracture Point.

From stress and strain fig.

- We can see that in the region between 'O' and 'A', the curve is *linear. Hence, **Hooke's Law** obeys in this region*. In the region from 'A' to 'B', the stress and strain are not proportional. However, if we remove the load, the body returns to its original dimension.
- The point 'B' in the curve is the ***Yield Point*** or the ***elastic limit,*** and the corresponding stress is the ***Yield Strength*** (Sy) of the material.
- Further, stress is increased, exceeding the Yield Strength the strain increases rapidly even for a small change in the stress. This is shown in the region from 'B' to 'D' in the curve.
- If the load is removed at, say a point 'C' between 'B' and 'D,' the body does not regain its original dimension. Hence, even when the stress is zero, the strain is not zero, and the deformation is called ***plastic deformation***. This is a permanent set.
- Further, the point 'D' is the ultimate tensile strength (Su) of the material. Hence, if any additional strain is produced beyond this point, a ***fracture*** can occur (point 'E').
- If, he ***ultimate strength*** and fracture points are close to each other (points 'D' and 'E'), then the material is brittle.
- The ultimate strength and fracture points are far apart (points 'D' and 'E'), then the material is ductile.

STRAIN HARDENING STRAIN SOFTENING

Strain Hardening: ***Strain or stress hardening*** is the strengthening of a metal by applying the compressive pressure till its plastic deformation. Which results the material become stronger by increase its yield point this effect are is strain or stress hardening. It is an important industrial process that is used to harden metals or alloys which do not respond to heat treatment. It is also 'work hardening or cold working.

Strain softening

Cretan material subjected to compressive stress whose yield strength is reduced upon stress strain softening Soil, concrete usually shows this behaviour.

Elastic modulus

Young's modulus: it is defined as the ratio of longitudinal stress to longitudinal strain.

$$Young'smodulus = \frac{longitudinal\ stress}{longitudinal\ strain} = \frac{F/A}{x/L} = \frac{FL}{xA}$$

Rigidity modulus: it is defined as the ratio of Shearing stress to Shearing strain.

$$Rigidity\ modulus = \frac{\text{Shearing}\ stress}{\text{Shearing}\ strain} = \frac{F/A}{x/L} = \frac{FL}{xA}$$

Bulk modulus: is defined as the ratio of volume stress to volumetric strain.

$$Bulk\ modulus = \frac{\text{Volume}\ stress}{\text{Volume}\ strain} = \frac{F/A}{v/V} = \frac{PV}{v}$$

Longitudinal strain coefficient (α): it is defined as **a** longitudinal strain produced per unit stress

Longitudinal strain=x/L

Unit Stress (T) the order of stress required to produce unit stress (x/L)

$$Lanitudinal\ stain\ coefficient = \frac{x/L}{T} = \frac{x}{LT}$$

Lateral deformation and lateral strain:

Whenever the elastic body subjected to the longitudinal stress, the deformation not only that takes place in terms of its length (longitudinal strain) but also thickness will also experiences change called lateral deformation. The deformation of the body perpendicular to the direction of force called lateral strain.

$$\text{Lateral strain} = \frac{Change\ in\ diameter}{\text{original diameter}} = \frac{d}{D}$$

Lateral strain coefficient (β):

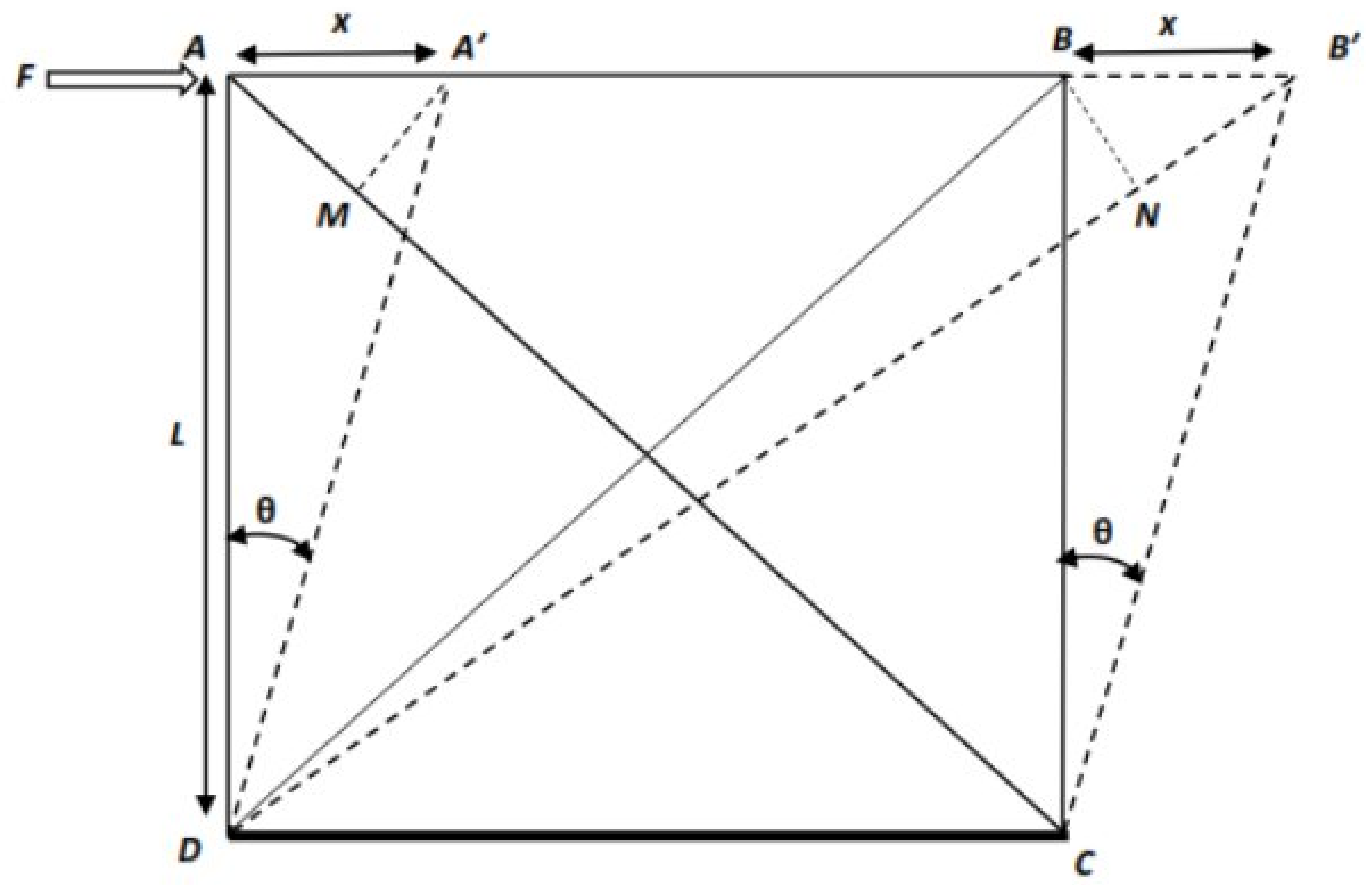
It is defined as the lateral strain produced per unit stress (T) called lateral strain

$$\beta = \frac{d/D}{T} = \frac{d}{DT}$$

Poisson's ratio: Within the elastic limit, Poisson ratio is defined as the ratio of lateral strain to the longitudinal strain $\sigma = \frac{\beta}{\alpha}$

The theoretical limit of Poisson's ratio lies between **−1 to 0.5** because. A. shear modulus and bulk's modulus should be positive.

Relation between Elongation stain, shearing strain and compression strain



The diagonal element AC contracted to $A'C$ similarly, the diagonal part BD is elongated to $B'D$

But, $AA' = BB' = x$, $AB = BC = CD = AD = L$ AC=BD=DN

The elongation and compression strain is

$$Elongation\ stain = \frac{B'N}{BN} = \frac{B'N}{BD} - - - - - (1)$$

$$Compression\ stain = \frac{AM}{AC} - - - - - - - - (2)$$

The value of diagonal length is determined by using Pythagoras's theorem (ABC)

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = L^2 + L^2 = 2L$$

$$AC = \sqrt{2}L - - - - - - -(3)$$

After the stress, the two isosceles right-angle triangles is formed $(\angle AMA')$ & $(\angle BNB')$

$$MA \quad = AA' \cos 45 = x \cdot (\frac{1}{\sqrt{2}}) - - - (4)$$

$$NB' = BB' \cos 45 = x \cdot \left(\frac{1}{\sqrt{2}}\right) - - - - - -(5)$$

Elongation strain is obtained by Substitute (3) and (5) in (1)

$$Elongation\ stain = \frac{B'D}{BD} = \frac{x \cdot \left(\frac{1}{\sqrt{2}}\right)}{\sqrt{2}L}$$

$$\because\ Tangential\ strain\ (\theta) = x/L$$

$$Elongation\ stain = {}^{x}/_{2L} = {}^{\theta}/_{2} - - - - - -(6)$$

Similarly, compression strain is obtained by substitute (3) and (4) in (2)

$$Compression\ stain = \frac{x \cdot (\frac{1}{\sqrt{2}})}{\sqrt{2}L}$$

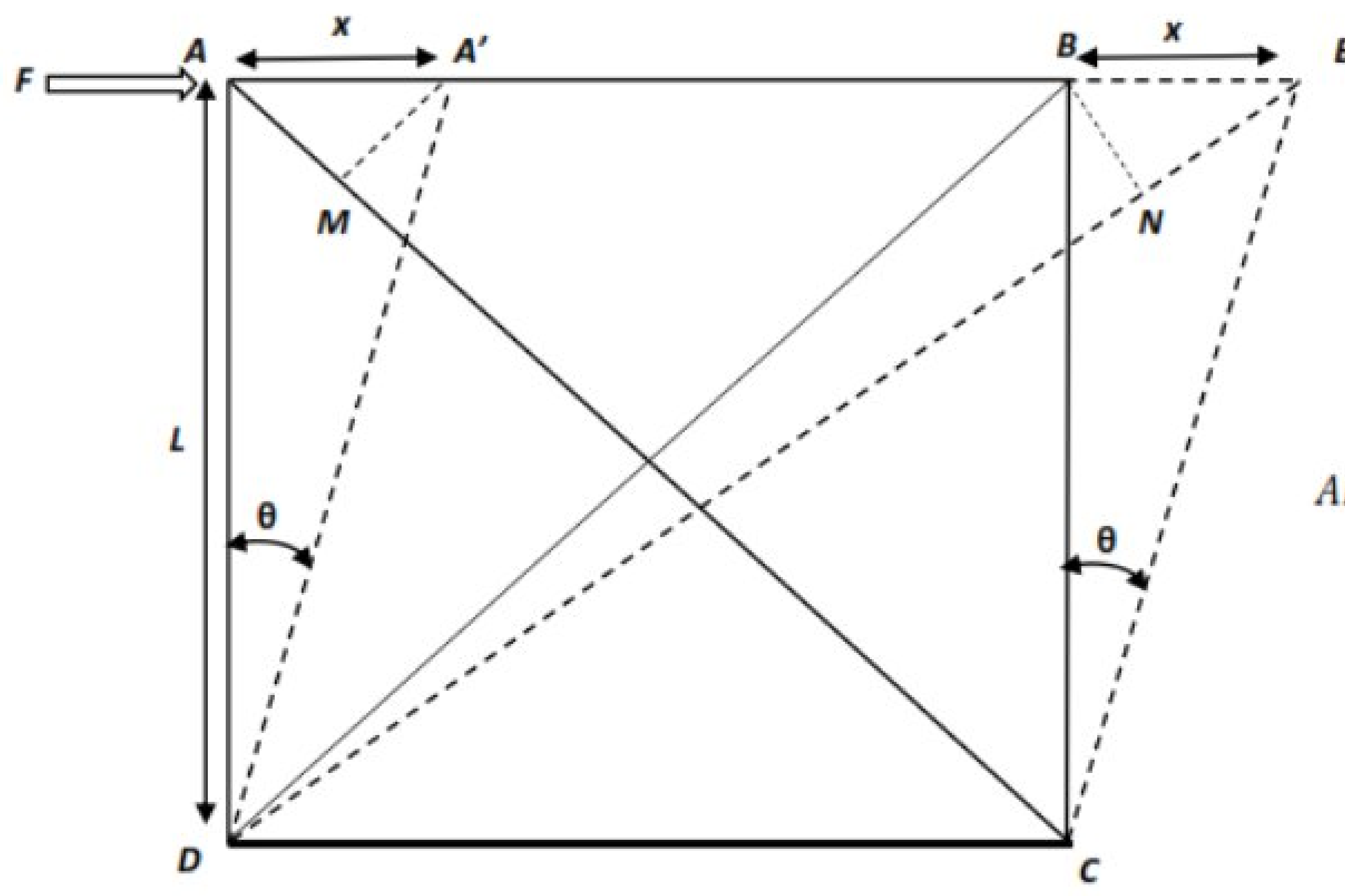
$$Compression\ stain = {}^{x}/_{2L} = {}^{\theta}/_{2} - - - - - -(7)$$

Add (6) and (7)$\Longrightarrow$

$$\therefore \qquad {}^{\theta}/_{2} + {}^{\theta}/_{2} = \theta$$

Therefore, the shearing strain is the sum of elongation strain and compressive strain

The relation between young's modulus, rigidity modulus and Poisson ratio



$$AA' = BB' = x$$

$$AB = BC = CD = AD = L$$

AC=BD=DN

The diagonal element BD is elongated to $B'D$

The strain produced along the diagonal is equal to $T(\alpha + \beta) ------(1)$

Along the DB is

$$stain = \frac{B'N}{BD} ----- (2)$$

From (1) and (2)

$$T(\alpha + \beta) = \frac{B'N}{BD} ----(3)$$

The value of diagonal length is determined by using Pythagoras's theorem (ABC)

$$DB^2 = BC^2 + DC^2$$

$$DB^2 = L^2 + L^2 = 2L$$

$$DB = \sqrt{2}L ------(4)$$

After the stress, the two isosceles right-angle triangles is formed $(\angle AMA')$ & $(\angle BNB')$

$$NB' = BB' \cos 45 = x \cdot \left(\frac{1}{\sqrt{2}}\right) ------(5)$$

Substitute (4) and (5) in (3)

$$T(\alpha + \beta) = \frac{x \cdot \left(\frac{1}{\sqrt{2}}\right)}{\sqrt{2}L}$$

$$T(\alpha + \beta) = \frac{\theta}{2}$$

$$\frac{T}{\theta} = \frac{1}{2(\alpha+\beta)}$$

$$\eta = \frac{1}{2(\alpha + \beta)}$$

$$\eta = \frac{1}{2\alpha\left(1 + \frac{\beta}{\alpha}\right)}$$

$$\eta = \frac{1/\alpha}{2\left(1 + \frac{\beta}{\alpha}\right)}$$

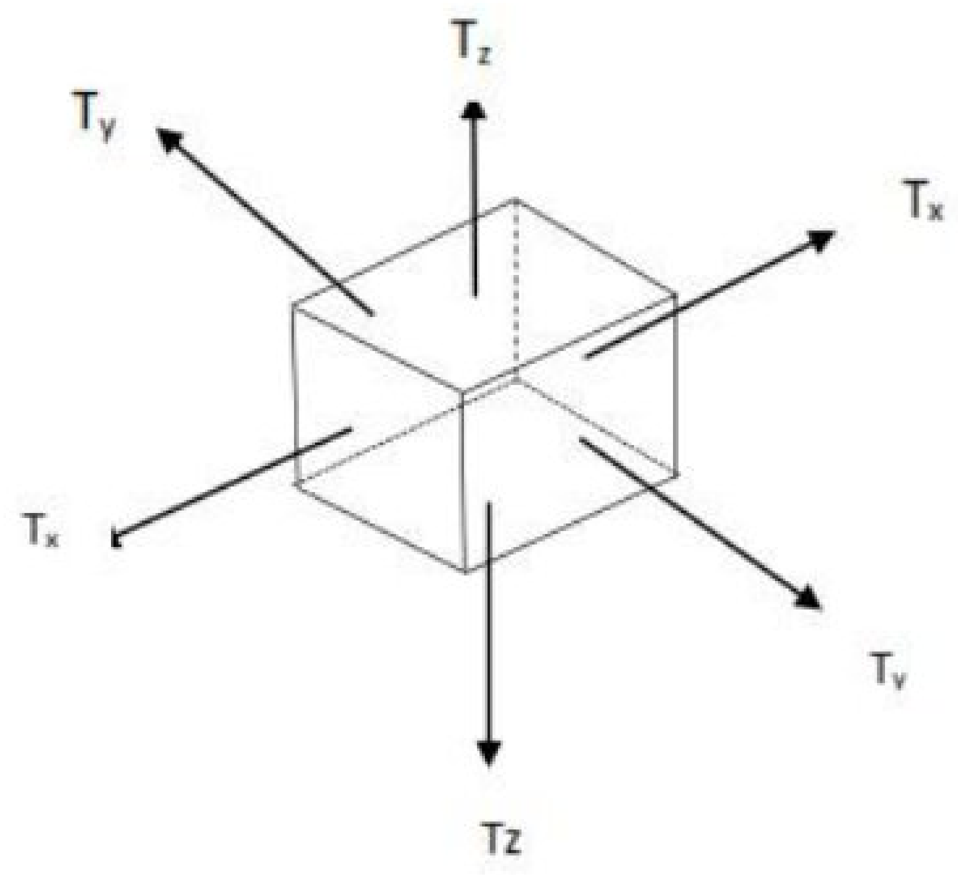
$$\boldsymbol{\eta = \frac{Y}{2(1+\sigma)}} \quad or \quad \boldsymbol{Y = 2\eta(1+\sigma)}$$

Relation between K Y and σ

Consider a cube of unit volume, T_x, T_y, T_z are the stress acting along the respective axis.

$'\alpha'$ Elongation per unit stress along the Force.

$'\beta'$ Compression per unit stress perpendicular to the force.



If the elongation stress produces the increase in the length along the x-axis then αT_x is the strain.
Similarly, the remaining two sides are compressed hence the compression stain is βT_y & βT_z

If the outward stress is actin along the x-axis then the length the side is

$$1 + \alpha T_x - \beta T_y - \beta T_z$$

$$1 + \alpha T_y - \beta T_z - \beta T_x$$

Similarly, along Z axis

$$1 + \alpha T_z - \beta T_x - \beta T_y$$

Volume of the cube after the stress is

$$(1 + \alpha T_x - \beta T_y - \beta T_z)(1 + \alpha T_y - \beta T_z - \beta T_x)(1 + \alpha T_z - \beta T_x - \beta T_y)$$

$$1 + \alpha(T_x + T_y + T_z) - 2\beta(T_z + T_x + T_y)$$

$$1 + (\alpha - 2\beta)(T_x + T_y + T_z)$$

$$T_x = T_y = T_z = T$$

Volume of the cube after the stress is

$$1 + (\alpha - 2\beta)3T$$

$$The\ change\ in\ the\ volume\ = volume\ after\ stress\ -\ original\ volume$$

$$1 + (\alpha - 2\beta)3T - 1$$

$$The\ change\ in\ the\ volume = (\alpha - 2\beta)3T$$

$$volume\ strain = \frac{change\ in\ volume}{original\ volume}$$

$$volume\ strain = \frac{(\alpha - 2\beta)3T}{1}$$

$$Bulk\ modulus = \frac{p}{(\alpha - 2\beta)3T}$$

$$K = \frac{T}{(\alpha - 2\beta)3T}$$

$$K = \frac{1}{(\alpha - 2\beta)3}$$

$$K = \frac{1}{3\alpha(1 - 2\frac{\beta}{\alpha})}$$

$$K = \frac{1/\alpha}{3(1 - 2\frac{\beta}{\alpha})}$$

$$K = \frac{Y}{3(1 - 2\sigma)}$$

Relation between K, n, Y

$$\boldsymbol{\eta = \frac{Y}{2(1+\sigma)} ----(1)}$$

$$\boldsymbol{K = \frac{Y}{3(1-2\sigma)} ----(2)}$$

Up on rearrange the equation (1) & (2)

$$\frac{Y}{\eta} = 2(1+\sigma)$$

$$\frac{Y}{3K} = (1-2\sigma)$$

Add

$$\frac{Y}{\eta} + \frac{Y}{3K} = 2 + 2\sigma - 1 - 2\sigma$$

$$\frac{Y}{\eta} + \frac{Y}{3K} = 3$$

$$\frac{3KY + \eta Y}{3\eta K} = 3$$

$$\boldsymbol{Y = \frac{9\eta K}{3K+\eta}}$$

Relation between K, σ, n

$$\boldsymbol{2\eta(1+\sigma) = Y --- (1)}$$

$$3K(1-2\sigma) = Y --- (2)$$

From (1) and (2)

$$\boldsymbol{2\eta(1+\sigma)} = 3K(1-2\sigma)$$

$$\boldsymbol{2\eta + 2\eta\sigma} = 3K - 6\sigma K$$

$$\boldsymbol{2\eta\sigma} + 6\sigma K = 3K - \boldsymbol{2\eta}$$

$$\boldsymbol{\sigma}(\boldsymbol{2\eta} + 6K) = 3K - \boldsymbol{2\eta}$$

$$\boldsymbol{\sigma} = \frac{3K - \boldsymbol{2\eta}}{(\boldsymbol{2\eta} + 6K)}$$

Beams:

Beams are the structural element that primarily resists loads applied laterally to the beam's axis.

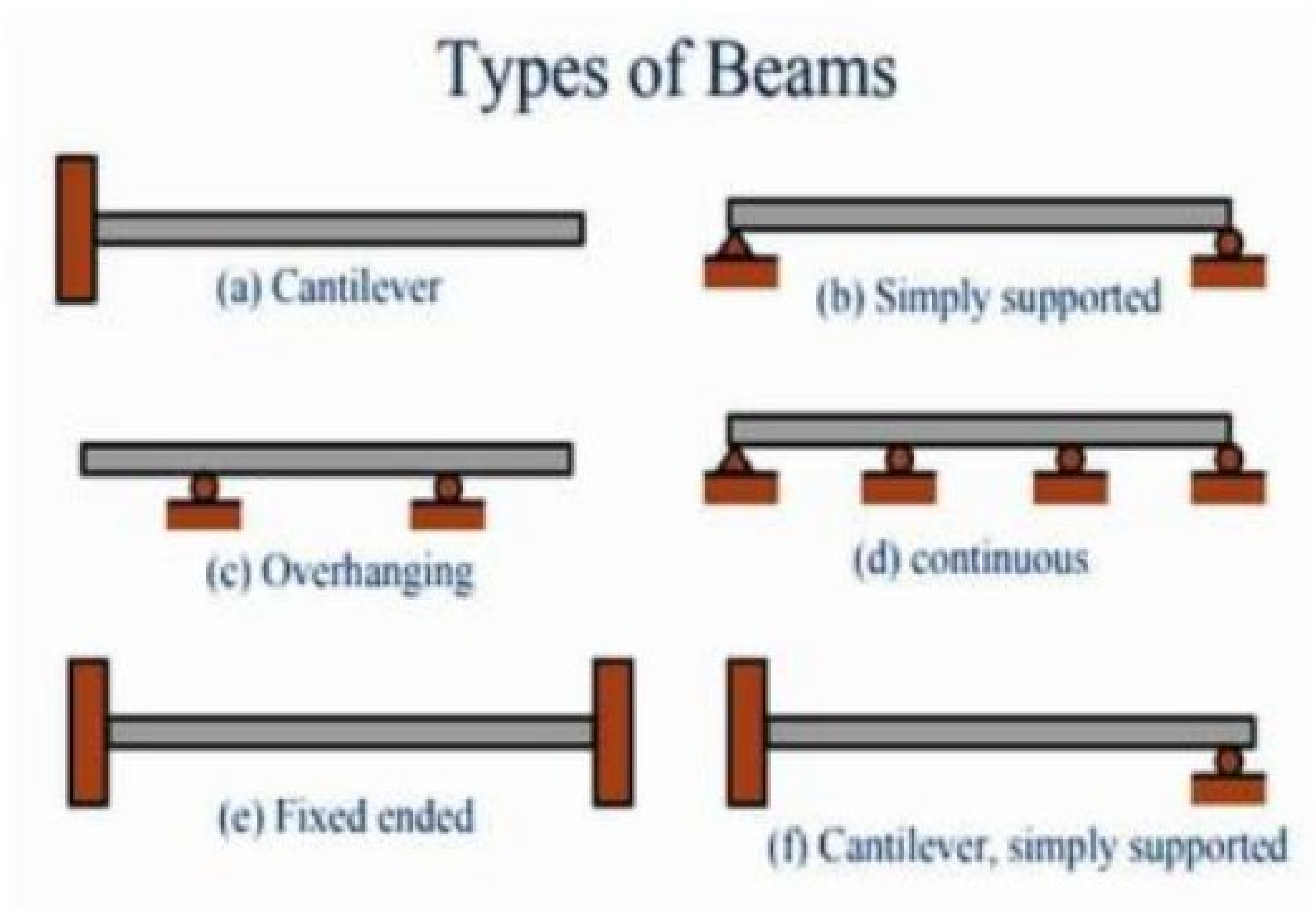
Neutral plane:

The neutral plane is the surface within the beam between these zones, where the material of the beam is not under stress, either compression or tension. Any line within the neutral plane parallel to the axis of the beam is called the deflection curve of the beam. Above and below the neutral axis the material experiences elongation and compression

Types of Beams

(a) Cantilever: A cantilever beam is fixed at one end and free at another end. It can be seen in the image Fixed.

(b) Simply supported: A supported beam is a type of beam that has pinned support at one end and roller support at the other end. Depending on the load applied, it undergoes shearing and bending. It is one of the simplest structural elements in existence.



(c) Overhanging: it is the type of beam some part of its length extended after the pin support

(d) Continuous Simple beam: A continuous beam has more than two supports distributed throughout its length. It can be seen in the image

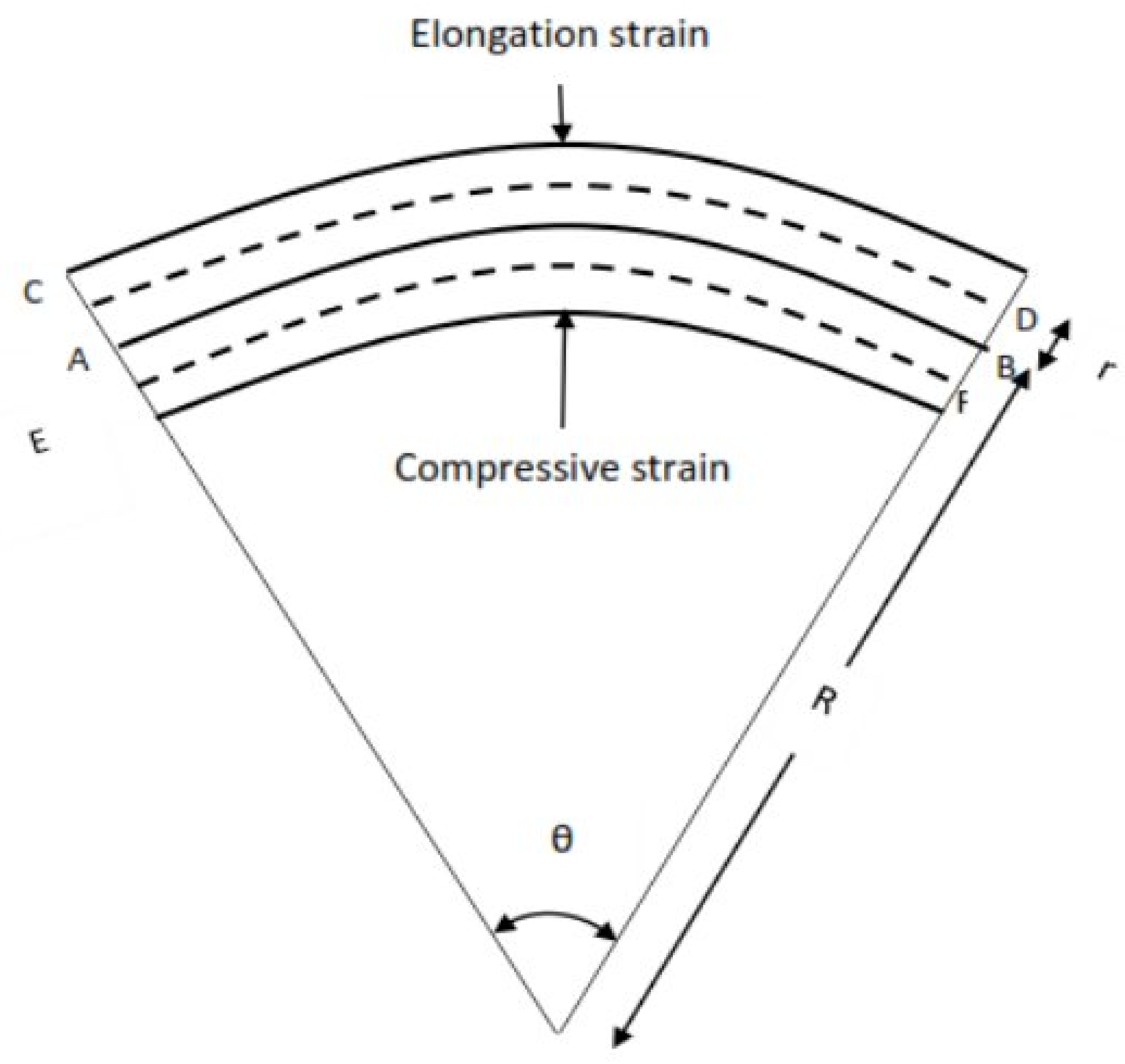
(e) Fixed ended beams: As the name suggests, a fixed beam is a type of beam whose both ends are fixed.

Applications

1. Construction of buildings, bridges.
2. Designing of Vehicle chassis frames of trucks. Etc.,

Bending moment of Beam

Consider a uniform beam made up of large number of parallel equidistant planes above and below the neutral axis after the bending it forms the arcs of a circle with radius of curvature 'R' above the neutral planes an elongation strain and below compressive strain acted.



The arc length AB is (original length) –neutral axis

$$AB = R\theta$$

The arc length CD is (elongated Length)

$$CD = (R + r)\theta$$

$\therefore Change\ in\ length\ = CD - AB$

$Change\ in\ length\ = (R + r)\theta - R\theta = r\theta$

$\because Original\ length\ = AB = R\theta$

Therefore,

$$\textbf{Linear Strain} = \frac{Change\ in\ length}{Original\ length} = \frac{r\theta}{R\theta} = \frac{r}{R}$$

$$\textbf{Youngs modulus } (\boldsymbol{Y}) = \frac{longitudinal\ stress}{longitudinal\ strain}$$

$$Longitudinal\ stress = Y \times linear\ strain$$

$$Y \times \frac{r}{R}$$

$$\because Stress = \frac{F}{a}$$

$$\frac{F}{a} = Y \times \frac{r}{R}$$

$$F = \frac{Yar}{R}$$

$$\therefore Moment\ of\ force = F \times distance\ from\ neutral\ axis$$

$$F \times r = \frac{Yar}{R} \times r$$

$$= \frac{Yar^2}{R}$$

The beam is made up of large number of planes, a moment of force for whole beam is

$$momment\ of\ force = \frac{Y}{R}\sum ar^2$$

$$moment\ of\ force = \frac{Y}{R} I_g$$

Where $I_g = \sum ar^2$ is the geometrical moment of inertia or radius of gyration.

$$\textbf{Bending moment of rectangular beam} = \frac{Y}{R}\left(\frac{b.d^3}{12}\right)$$

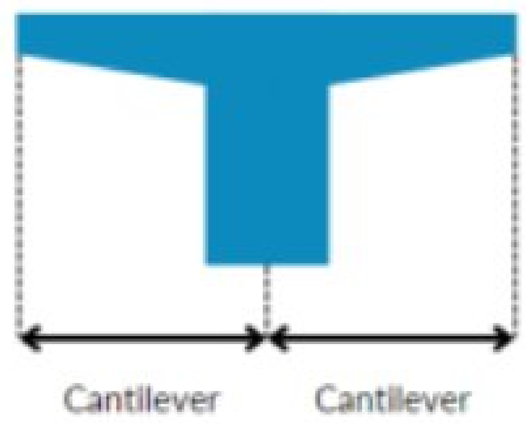
$$\textbf{Bending moment of circular beam} = \frac{\pi Y}{4R} r^4$$

Where: b =breadth, d=thickness, R =bending radius of curvature, Y =young's modulus, r=radius of circular beam

CANTILEVER: A cantilever is a rigid structural element that extends horizontally and is supported at only one end. Typically it extends from a flat vertical surface such as a wall, to which it must be firmly attached.

Fig (a) Simple cantilever

Fig(b) Bridge section



A good example of a cantilever beam is a balcony. A balcony is supported on one end only, the rest of the beam extends over open space; there is nothing supporting it on the other side. The cantilever beam is used in;

1. In Buildings.
2. Cantilever bridges.
3. Overhanging projections and elements.
4. Balconies such as in Frank Lloyd Wright's Falling Water.
5. Machinery and plants such as cranes.
6. Overhanging roofs like shelters and stadium roofs.
7. Shelving and Furniture.

I -Section girder and their Engineering Applications

I section beam it is suitable for resisting *flexural stresses*. As more area is away from neutral axis compared to rectangular or circular section *(flanges),* its moment of inertia (MOI) increases as we write

MOI=Ar^2

As moment of inertia increases, moment resisting capacity also increases which makes I section better as compared to other cross section.

This is better option for designer to save material. So to sum up I section is better than other section as it has more flexural strength for same area of other section and it is an economical section also.

Applications:

- A plate girder is a steel beam that is widely used in bridge construction. Girder is required to carry heavy loads on relatively long spans
- It is a built up I-beam section, used to carry heavy loads which cannot be carried economically by rolled I-sections. It is made by welding the steel plates in I-beam shape.
- Primarily used in bridges, plate girder is used when we need deeper sections having¬ higher stiffness to carry heavy loads.
- I-section beams are especially important in civil constructions as they substitute for numerous support structures which obviously help in saving both time and money.

- The bridge in which girders are used for supporting its deck is called the girder bridge.

Elastic Materials: Thus elasticity is defined as; it is that property of a body which it regains its original shape and size when the deforming force is removed.

For example: **nylon, latex, rubber, polyester**. This behaviour is governed by Hooke's Law, which understands the relationship between stress and strain under a Modulus of Elasticity. Elastic materials can be natural, semi-synthetic or synthetic, depending on their degree of elaboration through the hand of man.

There are no bodies in nature which are perfectly elastic. However, there is a material formed by adding 0.5% of phosphor to bronze to increase its stiffness and wear resistance. This alloy is called phosphor – bronze. It comes very close to the definition of a perfectly elastic body. Also, it is possible to make filaments of quartz by exposing quartz in the form of rods to oxyhydrogen flame. These quartz fibers can also be considered under a perfectly elastic body. However in our common use, those which are well known as elastic materials are the elastomers.

Elastomers are the polymers with high elastic nature. Natural rubber, synthetic rubber, silicone rubber etc are all elastomers. They can be extended many more times their original size. After releasing from the stretching force, they manage to return to a size almost the same as the original size (though, over number cycles, they gradually lose this ability). Yet against the common sense, technically, Rubber is considered less elastic than steel. It is because, steel returns very quickly to its original form whereas, rubber takes considerably longer time for reinstatement. On this account, steel scores over rubber as more elastic.

Fundamentals of fracture

Fracture:

Fatigue is the mechanical failure of the material due to the permanent cracks or breakage. This may be due to the overstress applied to the body exceeding to its ultimate strength.

Ductile and brittle fracture:

1. **Brittle fracture:**

   Brittle fracture is the sudden and rapid metal failure in which the material shows little or no plastic strain. This is characterized by quick failure without any warning. The generated cracks propagate rapidly and the material collapses all of a sudden. Brittle Fracture is a condition that occurs when a material is subjected to temperatures that make it less elastic, and therefore more brittle. The potential for material to become brittle depends on the type of material that is subjected to these low temperatures. Some materials, such as carbon and low alloy steels will become brittle at low temperatures and therefore susceptible to damage ranging from cracking to shattering or disintegration of equipment. When a material becomes brittle, the consequences can be very serious. If the brittle material is subjected to an impact or an equivalent shock (ex. rapid pressurization) the combination could potentially lead to a disastrous failure under certain conditions.

2. **Ductile fracture:**

   Ductile fracture is the material failure that exhibits substantial plastic deformation prior to fracture. The ductile fracture process is slow and gives enough warnings before final separation. Normally, a large amount of the plastic flow is concentrated near the fracture faces. Ductile fracture occurs over a period of time and normally occurs after yield stress, where as brittle fracture is fast and can occur at lower stress levels than a ductile fracture. That is why Ductile fracture is considered better than brittle fracture.

Fatigue failure:

Fatigue failure is when the surface of a material begins to crack or fracture, causing the part to weaken; it is due to cyclic load. Typically, the first stage of fatigue failure is crack initiation. Crack initiation occurs once applied stress exceeds tensile strength. The next stage that occurs is crack growth.

Factors affecting fatigue

- **Effect of stress concentration.**
  Stress concentration (also called a stress raiser or a stress riser) is a location in an object where the stress is significantly greater than the surrounding region. Stress concentrations occur when there are irregularities in the geometry or material of a structural component that cause an interruption to the flow of stress. This arises from such details as holes, grooves, notches and fillets. Stress concentrations may also occur from accidental damage such as nicks and scratches.
- **The influence of size factor:**
  Due to the inhomogeneity of the material structure and the existence of internal defects, the increase of the size will increase the failure probability of the material, thus reducing the fatigue limit of the material.
- **Influence of surface processing state:** There are always uneven machining marks on the machined surface, which are equivalent to tiny gaps, causing stress concentration on the material surface, thus reducing the fatigue strength of the material.
- **The impact of loading experience:** The overload damage means that the fatigue limit of the material will decrease after the material runs for a certain number of cycles under the load higher than the fatigue limit.
- **Influence of chemical composition:** The internal factors include the composition, microstructure, purity and residual stress of the material.
- **Effect of heat treatment on Microstructure:** Although the same static strength can be obtained for materials of the same composition due to different heat treatments, the fatigue strength can vary in a considerable range due to different microstructures.
- **Influence of inclusions:** The inclusion itself or the hole produced by it is equivalent to a tiny notch, which will produce stress concentration and strain concentration under the action of alternating load, and become the crack source of fatigue fracture, which has adverse effect on the fatigue performance of materials. The influence of inclusions on fatigue strength depends not only on the type, nature, shape, size, quantity and distribution of inclusions, but also on the strength level of materials and the level and state of applied stress.