MODULE - 1

OSCILLTIONS AND WAVES

Nature manifests itself in the form of matter and radiation, therefore concepts of matter (particle) and radiation (wave) are very basic and fundamental in Physics.

It is very essential to transport energy from one place to another and there are two ways transporting energy.

1] Energy can be transported in the form of kinetic energy of matter in motion. Ex: A bullet fired from a gun carries kinetic energy and transfers it to the target

2] Energy can be transported from one point to another by a wave without the actual movement of particles in the medium, i.e called as wave motion. Ex: Radiation received from Sun is wave motion.

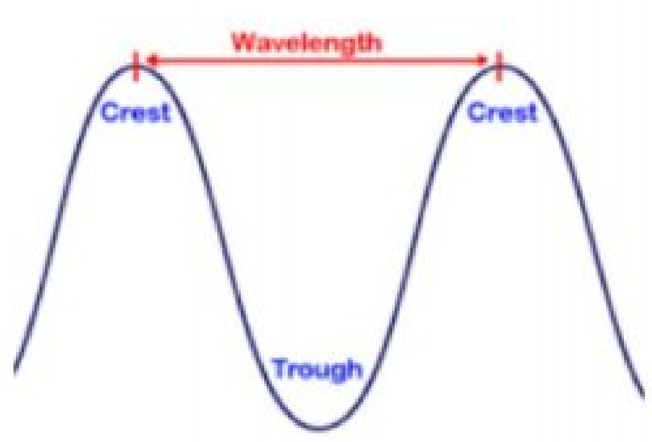
Depending on the medium of propagation, waves can be classified as

1] Mechanical waves

2] Non mechanical waves (electromagnetic waves)

A wave can be defined as " The transmission of energy from one place to another through a medium without the actual translation of the medium in the direction of the energy flow"

A **wave** is produced by the periodic disturbance/periodic oscillations at a point in the given medium.



Ex: Sound waves are **produced** because of the vibration of particles of the **medium** or the body. Sound propagates in the form of **waves** . These **waves** carry mechanical energy with them which is known as sound energy. (Ex: When you strike a bell, the metal vibrates, creating a sound wave)

Ex: Electromagnetic wave, is a form of energy emitted by moving charged particles. Electromagnetic waves consist of both electric and magnetic field waves. These waves oscillate in perpendicular planes with respect to each other, and are in phase. (Ex: oscillations of charge flowing back and forth in an electrical circuit, vibrations of electrons in an atom generating light waves etc.)

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SIMPLE HARMONIC MOTION (SHM)

[QUESTION : Define simple harmonic motion and explain the characteristics]

Definition of SHM: A simple harmonic motion is defined as "The motion in which the restoring force acting on the body is directly proportional to its displacement from the mean position"

Characteristics:

1. The motion is periodic
2. When the body displaced from its mean position, the restoring force acts on the body which tends to bring back the body to its mean position
3. The restoring force is directly proportional to the displacement of the body from its mean position.

Displacement: the distance of the particle measured along the path of the motion from its mean position at a given instant is called as displacement given as $\boldsymbol{x = A\,Sin\,(\omega t + \emptyset)}$
Where 'x' is displacement, 'A' is amplitude, 'ω' is angular frequency, 't' is time constant and '$\emptyset$' is initial phase

Velocity: velocity is defined as distance travelled by unit time, given as

$$\boldsymbol{v} = \frac{dx}{dt}$$

$$v = \frac{d}{dt}\ [A\,Sin\,(\omega t + \emptyset)]$$

$$v = \omega A\,Cos\,(\omega t + \emptyset)$$

Acceleration: acceleration is given by

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}\ \omega A\,Cos\,(\omega t + \emptyset)$$

$$a = -\,\omega^2 A\ Sin(\omega t + \emptyset)$$

Time period: time taken for one complete oscillation is called as period, which is given by $\omega = 2\pi f = \frac{2\pi}{T}$

Therefore $$\boldsymbol{T = \frac{2\pi}{\omega}}$$

Frequency: the number of oscillations made by the body in one second, given by

$$\boldsymbol{f = \frac{1}{T}}$$

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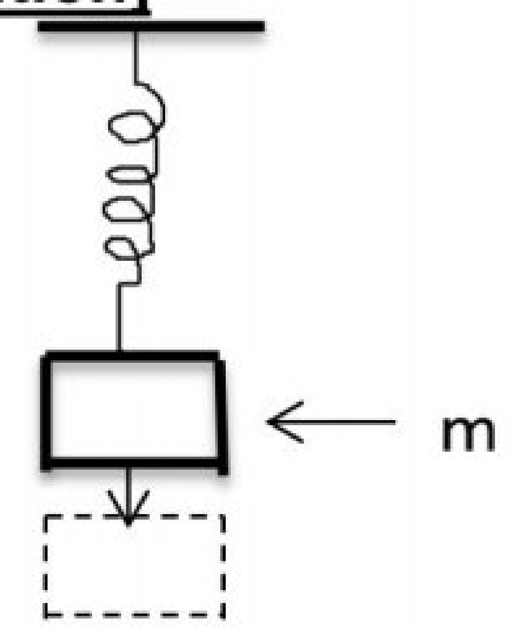
DIFFRENTIAL EQUATION OF SHM

[QUESTION: Derive expression for differential equation for SHM and mention its solution]



Consider a block of mass 'm' attached to the one end of the spring. When the mass is pulled and left to it, it oscillates along its equilibrium position. Mass is said to be performing simple harmonic motion when the restoring force (F) is proportional to the displacement (x).

i.e restoring force $\propto -x$

{Negative sign indicates the restoring force is directed opposite to the displacement}

$$F \propto -x$$

$$F = -kx \quad \text{--} \ (1)$$

Here K is the proportionality constant known as spring constant. It represents the amount of restoring force produced per unit elongation and is a relative measure of stiffness of the material.

According Newton's II law of motion, the restoring force produces acceleration given by

$$F = ma \quad \text{---------------------} \ (2)$$

where 'a' is acceleration given by $a = \frac{d^2x}{dt^2}$

from equation 1 & 2

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\boldsymbol{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$$

Putting $\frac{k}{m} = \omega^2$, the above equation becomes

$\boldsymbol{\frac{d^2x}{dt^2} + \omega^2 x = 0}$ **----------------(3) This is the general differential equation for SHM**

Here 'ω' is the angular frequency given by $\boldsymbol{\omega = \sqrt{\frac{k}{m}}}$ radian/sec

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Therefor time period 'T' is given by $$\boldsymbol{T} = \frac{2\pi}{\omega}$$

$$\mathrm{T} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

Therefore $$\mathbf{T} = 2\pi\sqrt{\frac{m}{k}} \quad \textbf{seconds}$$

The general solution for equation (3) is given by

$$\mathbf{x} = \mathbf{A}e^{i\omega t} + \mathbf{B}e^{-i\omega t}$$

Where 'A' and 'B' are constants to be determined by initial conditions

MECHANICAL SIMPLE HARMONIC OSCILLATOR:
MASS SUSPENDED TO A SPRING (VERTICAL OSCILLATIONS)

[QUESTION: Derive the expression for period of oscillation for a mass spring oscillator]

For a light spiral spring within elastic limit, the tension of the spring is proportional to the extension of the spring beyond its length, i.e it obeys Hook's law

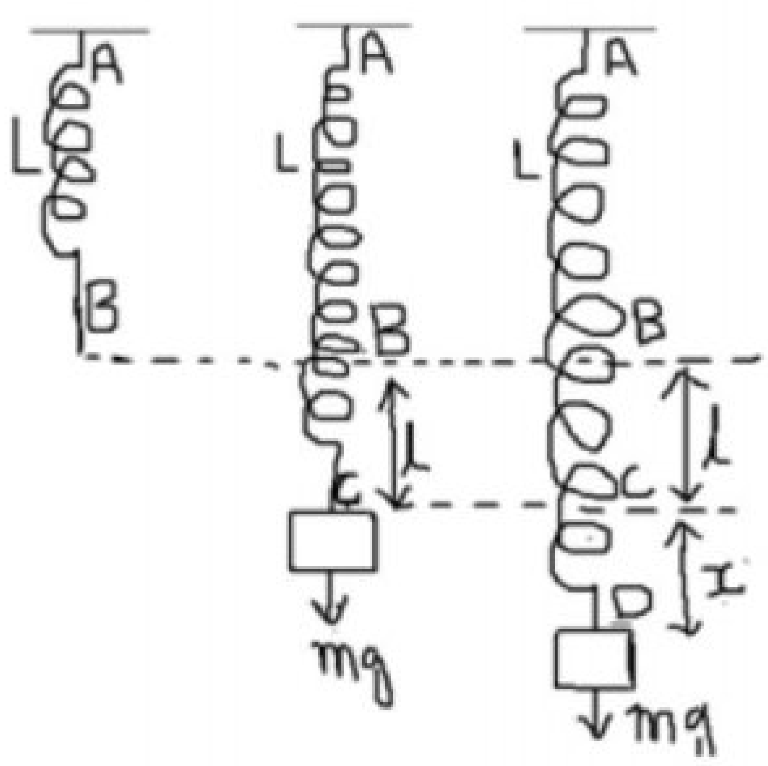

Figure shows a spring of length 'L' suspended freely from a support at the position 'A' if a mass 'm' is attached to its free end 'B' then the spring stretches downward and it length will increase say by $l = BC$.

According to Hook's law, the force exerted by the spring on the mass is directly proportional to its displacement

if 'l' is the displacement then $$F \,\alpha\, l$$
$$F = kl$$

Where 'k' is called as proportionality constant which depends on the material, size of the spring balance, the tension 'T' on the spring. 'k' is also called as spring constant or stiffness factor or force constant.

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But W.K.T $F = mg$

Therefore the tension on the spring is equal to

$T = mg = kl$ ------------------------- (1)

Now if the mass is further displaced to the position 'D' through a small distance 'x' then the tension in the spring is T' given by

$T' = k\,(l + x)$ ------------------------ (2)

Therefore the total force acting on the mass is given by

$F = (T - T')$

$F = kl - k\,(l + x)$

$F = -kx$

The time period 'T' is given by

$$T = \frac{2\pi}{\omega}$$

We know that $\omega = \sqrt{\frac{k}{m}}$

Therefore $T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$

Or $T = 2\pi\sqrt{\frac{m}{k}}$

From equation (1) we have $mg = kl$

Or $k = \frac{mg}{l}$

Therefore $T = 2\pi\sqrt{\frac{m}{\frac{mg}{l}}}$

$$\boldsymbol{T = 2\pi\sqrt{\frac{l}{g}}}$$

The time period of spring with large 'k' (i.e strong spring) will be less and is directly proportional to mass suspended.

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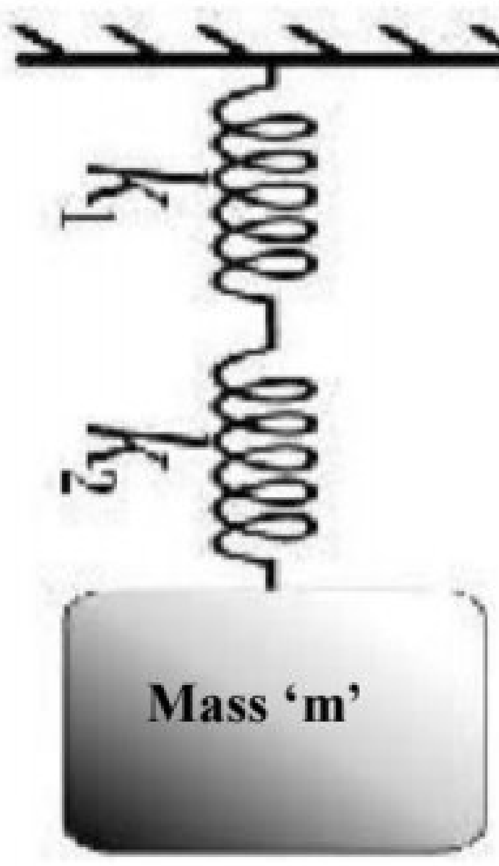
EXPRESSION FOR SPRING CONSTANT FOR SERIES COMBINATION

[QUESTION: Derive the expression for equivalent force constant for springs in a series combination. What is the expression for period of its oscillation]

Consider a load 'm' suspended through two springs $S_1 and\ S_2$ with spring constants k_1 and k_2 in series combination. Both the springs experience same pull (i.e same force) by the mass 'm'. S_1 extends by x_1 and S_2 extends by x_2. Thus the mass m comes down showing a total extension $x = x_1 + x_2$



↓

F = mg

Therefore total extension is given by

$$x = x_1 + x_2 \qquad \{\text{but W.T.T } F = -kx \ or\ x = -\frac{F}{K}\}$$

$$-\frac{F}{k} = -\frac{F}{k_1} - \frac{F}{k_2}$$

If K_s is the equivalent spring constant for the series combination of the springs, then we have

$$-\frac{F}{k_s} = -\left(\frac{F}{k_1} + \frac{F}{k_2}\right)$$

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

If there are multi spring connected in series then

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_2} + - - - -$$

If a mass 'm' is attached to the bottom of such a series combination of springs and set for oscillations, then its period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{k_s}}$$

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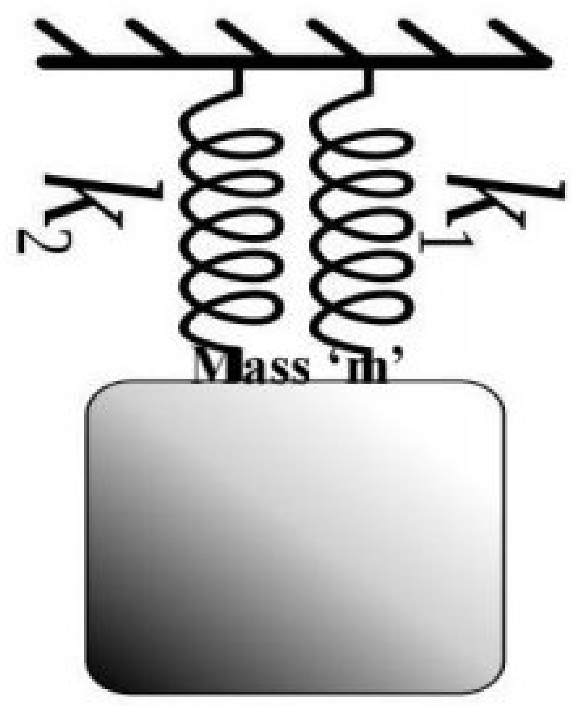
EXPRESSION OF SPRING CONSTANT FOR PARALLEL COMBINATION

[QUESTION: Derive the expression for equivalent force constant for springs in parallel combination. What is the expression for period of its oscillation]



Consider a load suspended through two springs with spring constants k_1 and k_2 in parallel combination.
If F_P is restoring force shared by each spring then

$$F_P = F_1 + F_2 \qquad \{\text{but W.T.T } F = -kx\}$$

$$-K_P\,x = -k_1x - k_2x$$

If K_P is the equivalent stiffness factor of the parallel combination of the springs, then we have

$$-K_P\,x = -x\,(k_1 - k_2)$$

$$\boldsymbol{K_P = k_1 + k_2}$$

If there are multi spring connected in parallel then

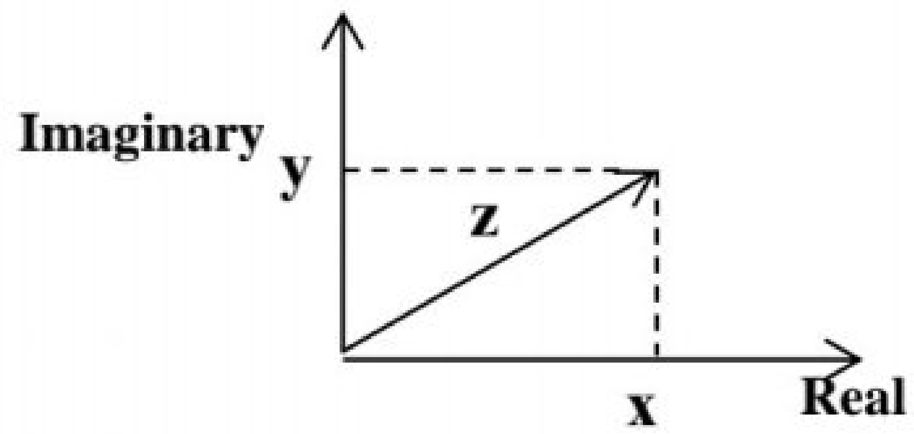
$$\boldsymbol{K_P = k_1 + k_2 + k_3 + ---}$$

If a mass 'm' is attached to the bottom of such a parallel combination of springs and set for oscillations, then its period of oscillation is given by

$$\boldsymbol{T = 2\pi\sqrt{\frac{m}{k_P}}}$$

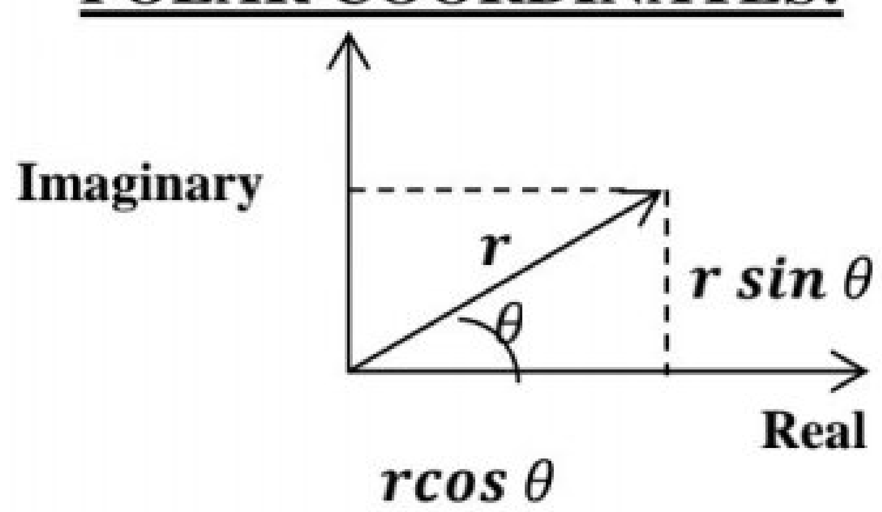
COMPLEX NOTATION

Complex numbers are a convenient tool to mathematically analyze sinusoidal functions. It can be used to represent amplitude and phase of a periodically varying function.



A complex number z has the form $\boldsymbol{Z = x + iy}$ where $i = \sqrt{-1}$ which is termed as imaginary. Here x is said to be the real part and y is imaginary.

POLAR COORDINATES:



Polar coordinates are represented by 'r' and 'θ' where 'r' is the magnitude of 'z'

Here $$Z = r\,cos\theta \; + ir\sin\theta$$
$$Z = r\,(cos\theta \; + i\sin\theta)$$

From Eulers formula, we can write $e^{i\theta} = \; cos\theta \; + i\sin\theta)$

Therefore $\boldsymbol{Z = re^{i\theta}}$

Phasors are Time Independent complex quantities used to represent periodically varying parameters.

Ex: Alternating current is represented as $I(t) = Ie^{i\omega t}$

Alternating voltage is represented as $V(t) = Ve^{i\omega t}$

Here 'I' and 'V' are phasors

A periodic force is expressed in phasor form as

$F = F_0\, Cos\; \omega t$ here 'F' is phasor

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FREE OSCILLATIONS

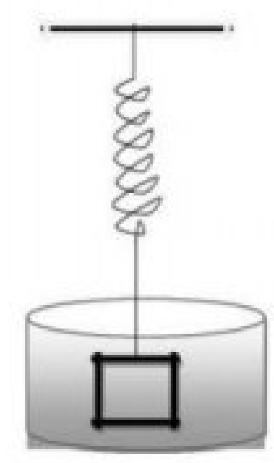
The oscillations are said to be free oscillations when there are no external forces. The object oscillates with natural frequency. If there is no resistance offered to the motion of the oscillations by any sources such as air, friction or internal forces, then the body keeps on oscillating indefinitely and such oscillations are called free oscillations.

DAMPED OSCILLATIONS

In a damped harmonic oscillator, the amplitude decreases gradually, this is called as damping. Damping may be due to friction or viscous force. Ex. A pendulum immersed in liquid (water) exhibits damped oscillations.

EXPRESSION FOR THE PERIOD AND AMPLITUDE OF DAMPED HARMONIC MOTION

[QUESTION : What are damped vibrations/oscillations? Give the theory of damped vibrations/oscillations, and find the condition of heavy, critical and light damping]



Oscillating mass in a liquid.

The above figure shows a spring with attached mass at one end is immersed in water medium. When a body oscillations in a medium, there are two forces acting on it

[1] The restoring force in opposite direction proportional to the displacement 'x' which tends to bring back the body to its initial positions and is given by

$$F \propto -x$$

$$F = -kx \text{ ----------------------------- (1)}$$

Where 'k' is proportionality constant or force constant/spring constant/stiffness factor.

[2] The resistive force proportional to the velocity but oppositely directed given by

$$F \propto -v$$

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$$F = -rv$$

Or $$F = -r\frac{dx}{dt} \quad \text{------------------------------ (2)}$$

Where 'r' is resistive force per unit velocity and is called as damping constant. Negative sign indicates that, the forces are acting opposite to the direction of the particle motion.

Therefore the net force acting on the oscillating body is given by

$$F = restoring\ force + retarding\ force$$

Therefore from equation (1) and (2)

$$F = -kx - r\frac{dx}{dt}$$

But we have force = mass x acceleration

$$F = m\,\frac{d^2x}{dt^2}$$

'm' is the mass of the oscillating body

Therefore the equation of motion of the particle is given by

$$m\,\frac{d^2x}{dt^2} = -k\,x - r\,\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}\,x - \frac{r}{m}\,\frac{dx}{dt}$$

Or $$\frac{d^2x}{dt^2} + \frac{r}{m}\,\frac{dx}{dt} + \frac{k}{m}\,x = 0$$

Or $$\frac{d^2x}{dt^2} + 2b\,\frac{dx}{dt} + \omega^2\,x = 0 \quad \text{------------------------------ (3)}$$

Where $2b = \frac{r}{m}$ and $\omega^2 = \frac{k}{m}$. The quantity '2b' gives the resistive force per unit mass per unit velocity and 'b' is called as damping coefficient.

Equation (3) is known as the differential equation of damped vibration or damped simple harmonic motions (SHM)

Finding solutions for the above equation (3)

Let $x = e^{\alpha t}$ be a trial solution of the equation

Differentiating the above equation with respect to 't'

$$\frac{dx}{dt} = e^{\alpha t}\alpha \quad \text{---------------------- (*)}$$

$$\text{Or} \quad \frac{dx}{dt} = \alpha x \quad \text{-------------- (4)} \quad \{\text{becaz } x = e^{\alpha t}\}$$

Differentiating equ (*) again w.r.t 't'

$$\frac{d^2x}{dt^2} = \alpha\,(e^{\alpha t}\,\alpha)$$

$$\frac{d^2x}{dt^2} = \alpha^2 e^{\alpha t}$$

$$\frac{d^2x}{dt^2} = \alpha^2 x \quad \text{-------- (5)} \quad \{\text{becaz } x = e^{\alpha t}\}$$

substituting equation (4) and (5) in equ (3)

We have $\alpha^2 x + 2b\alpha x + \omega^2 x = 0$

Or $(\alpha^2 + 2b\alpha + \omega^2) = 0$ (because $x \neq 0$)

The standard solution for the above quadratic equation is given by

$$\alpha = -b \pm \sqrt{b^2 - \omega^2}$$

Thus the general solution of the equation (3) is given by

$$x = A_1 e^{\left(-b + \sqrt{b^2 - \omega^2}\right)t} + A_2 e^{\left(-b - \sqrt{b^2 - \omega^2}\right)t}$$

$$\boldsymbol{x = e^{-bt}\left[A_1 e^{\left(\sqrt{b^2 - \omega^2}\right)t} + A_2 e^{\left(-\sqrt{b^2 - \omega^2}\right)t}\right]} \quad \text{------------------ (6)}$$

Where A_1 and A_2 are two arbitrary constants to be determined.

Depending upon the relative values 'b^2' and 'ω^2' three different cases of damping arises. They are as follows.

CASE 1: HEAVY DAMPING (OR) OVER DAMPING

Here $b^2 > \omega^2$

This indicates the exponential decay of displacement with respect to time, i.e here the

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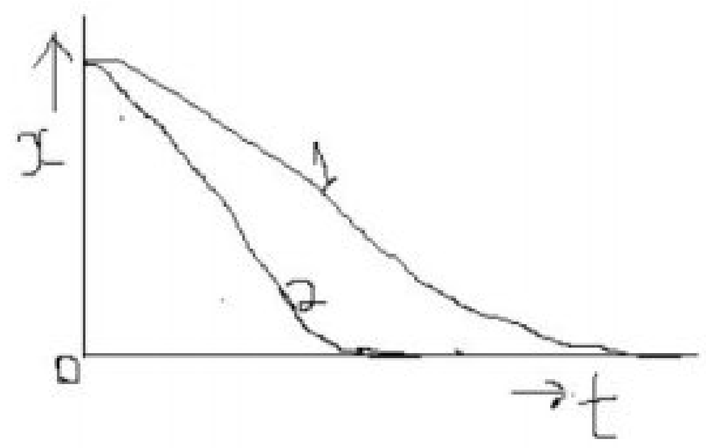
body after passing through its maximum displacement, simple comes back to equilibrium position and rest there. It ceases to vibrate anymore. This type of motion is called as heavy damped or over damped motion.

Ex: pendulum moving in a thick oil

CASE 2: CRITICAL DAMPING

Here $b^2 = \omega^2$, here the damping term effect is balanced by that of the stiffness term. Here, the oscillations decreases slowly in the beginning and then rapidly to approach the value of zero to attain the equilibrium position. such motion is called as **critical damped motion**

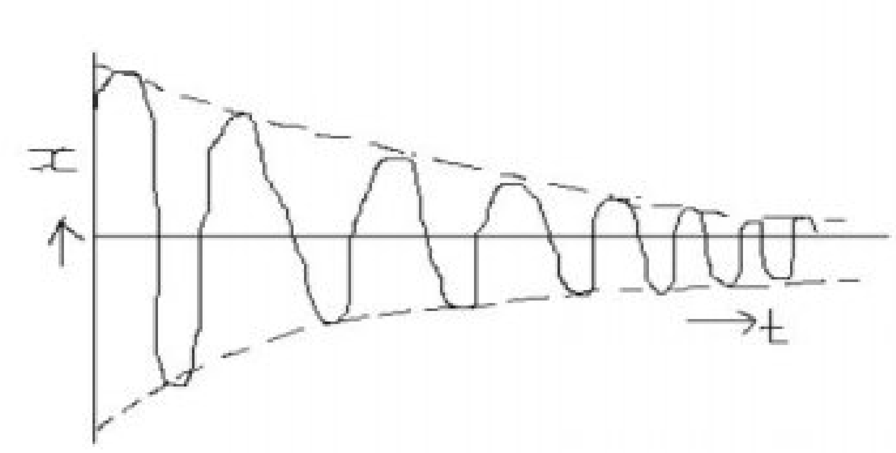
Ex: Voltmeter, ammeter exhibits this type of motion in which pointer moves to correct position and comes to rest without any oscillation.



1 → heavy damped curve , 2 → critical damped curve

CASE 3: LIGHT DAMPING (OR) UNDER DAMPING

Here $b^2 < \omega^2$, here the restoring force and resistive forces acting on the body are such that, the body oscillates with diminishing amplitude as the time propagates and finally comes to ta halt at the equilibrium position as shown in figure



Ex: Motion of pendulum in air, motion of the coil of ballistic galvanometer, electrical oscillations of LCR circuit.

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QUALITY FACTOR (Q)

The amplitude of damped oscillatory motion decays with time because of the dissipation of energy. Quality factor is defined as the $'2\pi'$ times the ratio of the energy of the oscillator to the energy lost per cycle. Hence quality factor is given by

$$Q = 2\pi \frac{eneryg\ of\ the\ oscillator}{energy\ lost\ per\ period}$$

$$Q = 2\pi \frac{E}{PT} \text{ ------------------------- (1)}$$

Where 'P' is power dissipated and 'T' is time period

Here power dissipated is given by $P = \frac{E}{\tau}$ Where 'τ' is called as relaxation time

So equation (1) can be written as $Q = 2\pi \frac{E}{\frac{E}{\tau}T}$

$$Q = \frac{2\pi\tau}{T}$$

$$\boldsymbol{Q = \omega\tau} \qquad (\text{because } \omega = \frac{2\pi}{T})$$

It is clear that higher the value of 'Q' higher will be the relaxation time 'τ', i.e lower damping.

FORCED OSCILLATIONS AND RESONANCE

When a body vibrates with a frequency other than its natural frequency under the action of an external periodic force are called as **forced oscillations**.

Ex: Vibrations of a tuning fork when exposed to an external periodic force, vibrations of a bridge under the influence of marching soldiers.

EXPRESSION FOR AMPLITUDE AND PHASE OF FORCED VIBRATIONS

[QUESTION : What are forced oscillations ? Obtain and expression for amplitude and phase of the body undergoing forced vibrations]

When a body is forced to vibrate/oscillate by applying an external periodic force, the forces acted upon it are given by

[1] The restoring force proportional to the displacement 'x' which tends to bring back

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the body to its initial positions and is given by

$$F = -kx \text{ ----------------------------- (1)}$$

Where 'k' is proportionality constant or force constant/spring constant/stiffness factor.

[2] The retarding force or resistive force proportional to the velocity but oppositely directed given by

$$F = -rv$$

Or $$F = -r\frac{dx}{dt} \text{ ----------------------------- (2)}$$

Where 'r' is resistive force per unit velocity and is called as damping constant. Negative sign indicates that, the forces are acting opposite to the direction of the particle motion.

[3] External periodic force acting on the body is given by

$$F\,sin\,(pt) \text{ ----------------------------- (3)}$$

Where 'p' is the angular frequency of the external periodic force.

Therefore the net force acting on the vibrating body is given by

$$\boldsymbol{F = restoring\ force + retarding\ force + external\ periodic\ force}$$

Therefore from equation (1) and (2) and (3) we have

$$F = -kx - r\frac{dx}{dt} + F\,sin\,(pt) \text{ ---------------- (*)}$$

But we have force = mass x acceleration

$$F = m\,\frac{d^2x}{dt^2}$$

'm' is the mass of the oscillating body

Therefore the equation (*) of motion of the body is given by

$$m\,\frac{d^2x}{dt^2} = -kx - r\frac{dx}{dt} + F\,sin\,(pt)$$

Or $$\frac{d^2x}{dt^2} + \frac{r}{m}\,\frac{dx}{dt} + \frac{k}{m}\,x = \frac{F}{m}\,sin\,(pt)$$

$$\frac{d^2x}{dt^2} + 2b\,\frac{dx}{dt} + \omega^2x = \frac{F}{m}\,sin\,(pt) \text{ ------------------ (4)}$$

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Where $\frac{r}{m} = 2b$ and $\frac{k}{m} = \omega^2$

The solution for the above differential equation (4) can be written as

$$x = a\sin\,(pt - \alpha) \quad \text{------------------ (5)}$$

Here 'a' and 'α' are unknowns to be found

Differentiating the above equation (5) with respect to 't' we have

$$\frac{dx}{dt} = ap\cos\,(pt - \alpha) \quad \text{------------------ (6)}$$

Differentiating again we get

$$\frac{d^2x}{dt^2} = ap\,[-\sin\,(pt - \alpha)\,.\,p]$$

$$\frac{d^2x}{dt^2} = -a\,p^2\sin\,(pt - \alpha) \quad \text{------------------ (7)}$$

Substituting equation (5), (6) and (7) in equation (4), we get

$$-a\,p^2\sin(pt - \alpha) + 2bap\cos(pt - \alpha) + \omega^2 a\sin(pt - \alpha) = \frac{F}{m}\ sin\,(pt)$$

$$-a\,p^2\sin(pt - \alpha) + 2bap\cos(pt - \alpha) + \omega^2 a\sin(pt - \alpha) = \frac{F}{m}\ sin\,[(pt - \alpha) + \alpha]$$

{Here $sin\,[(pt - \alpha) + \alpha]$ cab be considered as sin (A+B) = sinA cosB + cosA sinB}

$$-a\,p^2\sin(pt - \alpha) + \omega^2 a\sin(pt - \alpha) + 2bap\cos(pt - \alpha) = \frac{F}{m}[sin\,(pt - \alpha)\cos\alpha + cos\,(pt - \alpha)\sin\alpha]$$

$$-a\,p^2\sin(pt - \alpha) + \omega^2 a\sin(pt - \alpha) + 2bap\cos(pt - \alpha) = \left(\frac{F}{m}\right)\ sin\,(pt - \alpha)\cos\alpha + \left(\frac{F}{m}\right)\ cos\,(pt - \alpha)\sin\alpha$$

By equating coefficients of $sin\,(pt - \alpha)$ on both sides of the equation, we get,

$$-a\,p^2 + \omega^2 a = \frac{F}{m}\cos\alpha$$

Or $$\omega^2 a - a\,p^2 = \frac{F}{m}\cos\alpha$$

$$a\,(\omega^2 - p^2) = \frac{F}{m}\cos\alpha \quad \text{------------------ (8)}$$

Similarly by equating coefficients of $cos\,(pt - \alpha)$ on both sides of the equation, we get,

$$2bap = \frac{F}{m}\sin\alpha \quad \text{------------------ (9)}$$

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TO DETERMINE AMPLITUDE OF FORCED VIBRATIONS

Squaring and adding equation (8) and (9)

We get

$$[a\,(\omega^2 - p^2)]^2 + (2bap)^2 = \left(\frac{F}{m}\right)^2 + [cos^2\,\alpha + sin^2\,\alpha]$$

$$a^2\,[(\omega^2 - p^2)^2 + 4b^2p^2] = \left(\frac{F}{m}\right)^2 \qquad \{\boldsymbol{becaz\ cos^2\,\alpha + sin^2\,\alpha = 1}\}$$

$$\boldsymbol{a = \frac{\left(\frac{F}{m}\right)}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}}} \qquad \text{------------------ (10)}$$

The above equation represents the amplitude of forced vibrations

Substituting equation (10) in eq. (5), we have

$$x = \frac{\left(\frac{F}{m}\right)}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}}\ \sin\,(pt - \alpha)$$

TO DETERMINE PHASE OF FORCED VIBRATIONS

Dividing equation (9) by (8), we get

$$\frac{\frac{F}{m}\ \sin\alpha}{\frac{F}{m}\ \cos\alpha} = \frac{2bap}{a\,(\omega^2 - p^2)}$$

$$\boldsymbol{\tan\alpha = \frac{2bp}{(\omega^2 - p^2)}} \qquad \text{------------------ (11)}$$

The above equation represents the phase of forced vibrations

DEPENDENCE OF AMPLITUDE AND PHASE ON THE FREQUENCY OF THE APPLIED FORCE

[QUESTION: Discuss the dependence of amplitude and phase of a forced vibrations on the frequency of the applied external force]

Here p is the frequency of the oscillating body due to applied external force. As p can be varied, we can have three different cases

Case 1: $p \ll \omega$

Therefore $\omega^2 - p^2 \approx \omega^2$ and as p^2 is very small $\sqrt{4b^2p^2} \approx 2bp \approx 0$

Therefore amplitude from equation (10) becomes

$$\boldsymbol{a} = \frac{F/m}{\omega^2}$$

For $\boldsymbol{P} \ll \boldsymbol{\omega}$, $\boldsymbol{\alpha = 0}$

Since $\alpha = 0$ displacement and force will be in same phase

<u>Case 2</u>: $\boldsymbol{p = \omega}$

Therefore $\omega^2 = p^2$

Therefore amplitude from equation (10) becomes

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2}}$$

$$\text{Or} \quad \boldsymbol{a} = \frac{\left(\frac{F}{m}\right)}{2bp}$$

For $\boldsymbol{p = \omega}$, Phase $\boldsymbol{\alpha} = \frac{\pi}{2}$ This indicates that, the displacement ha a phase lag of $\frac{\pi}{2}$ with respect to the phase of the applied force.

<u>Case 3</u>: $\boldsymbol{p \gg \omega}$

Therefore $(\omega^2 - p^2)^2 \approx (p^2)^2$

Therefore amplitude from equation (10) becomes

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2 + [(p^2)^2]}}$$

Here as p keeps increasing, the damping factor b becomes very small

Therefore $4b^2p^2 \ll p^4$

Therefore amplitude becomes

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{p^4}}$$

$$a = \frac{\left(\frac{F}{m}\right)}{p^2}$$

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Here the phase $\alpha = -\pi$, Here as p becomes larger, the displacement develops a phase lag that approaches the value of π with respect to the phase of the applied force.

RESONANCE

[QUESTION: Define resonance with few examples ? discuss the condition for resonance and hence write a short note on significance /sharpness of resonance]

The phenomenon of making a body oscillate with its natural frequency under the influence of the external frequency of the periodic force is called as resonance.

Examples of resonance :

- Tuning of musical instruments
- Helmholtz resonator
- The vibrations caused by an exited tuning fork in an another identical tuning fork.
- Tuning of a radio transistor

The frequency at which the amplitude of the forced oscillations becomes maximum is the condition for resonance.

For forced vibrations, we know that the amplitude is given by

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}}$$

At resonance $\boldsymbol{p = \omega}$

Therefore $\omega^2 - p^2 = 0$

Therefore amplitude from equation (10) becomes

$$a_{max} = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2}}$$

Or $$a_{max} = \frac{\left(\frac{F}{m}\right)}{\boldsymbol{2bp}}$$

Therefore the sharpness of resonance depends inversely on '$\boldsymbol{b}$' where '$\boldsymbol{b}$' is the damping factor.

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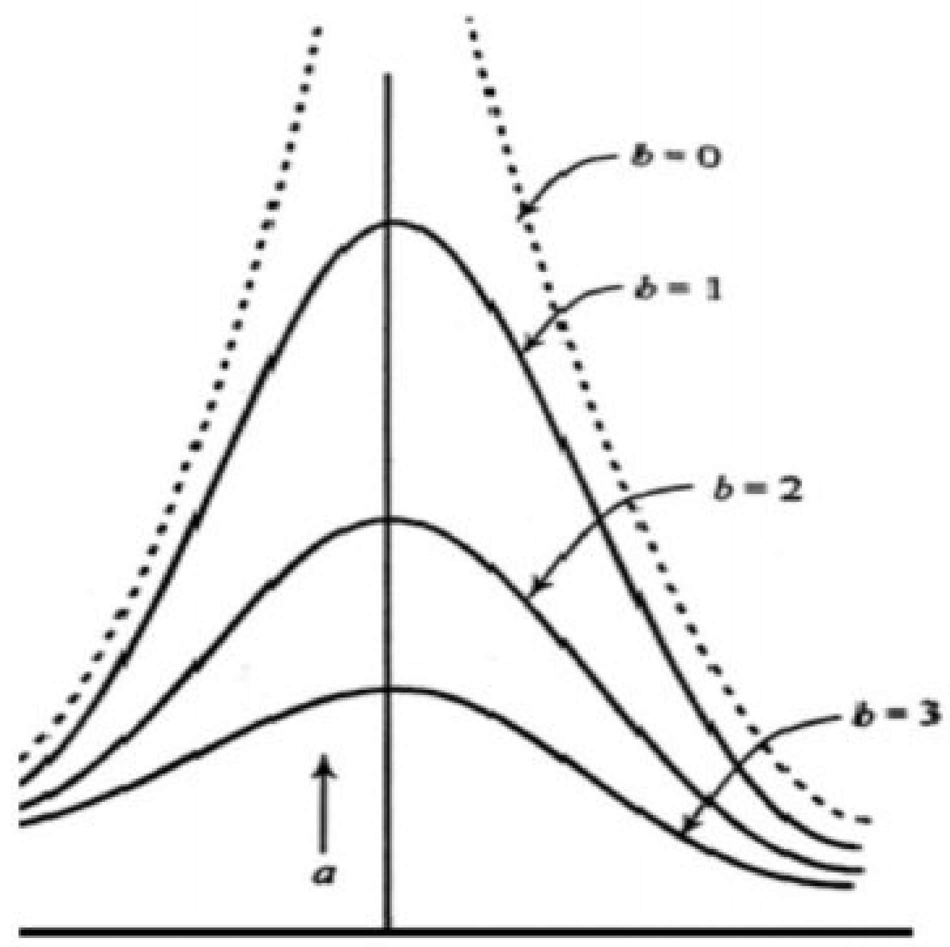
Significance and effect of damping on the amplitude of forced oscillations

Figure shows the response of the amplitude to the changes in damping. Here we can note that the curves are flat for larger values of b (i.e b=3, b=2) and hence resonance is also flat. The curves for smaller values of b (i.e at b = 1) exhibits the pronounced and a sharp peak hence it is referred as sharp resonance. But the curve which is in dotted line shows that the value of amplitude is infinity at b =0, which is a special case which never exits in reality.



Sharpness of resonance

$$P = \omega$$

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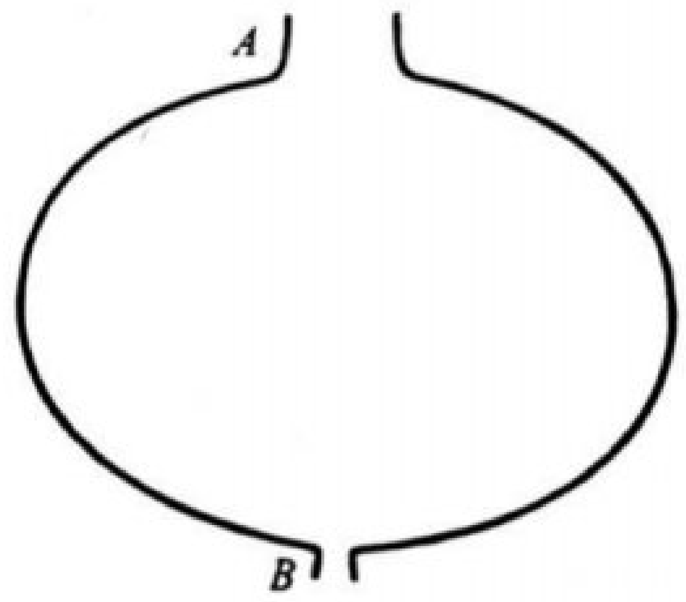
HELMHOLTZ RESONATOR (EXAMPLE OF MECHANICAL RESONANCE)

[QUESTION: Write a short note on Helmholtz resonator]



HELMHOLTZ RESONATOR

Helmholtz resonator is a cavity in which, a stream of air is set to vibrate by flowing in and out of it through an entry point at 'A'. It is used to analyse the quality of musical notes. It is a hollow cavity with two narrow opened ends at the point 'A' and ''B. The end 'B' is held near the ear with end 'A' open for the entry of the air carrying musical note. The air streams in and out of 'A' vibrates through 'A'. The air enclosed in such a hollow cavity will have definite value for its natural frequency 'ω'. When the frequency of the incoming air is same as the frequency of the air inside the cavity, amplitude becomes maximum and the sound is heard, thus the resonating cavity resonates for the note of that particular frequency.

It can be shown that, the square of the natural frequency of oscillation is inversely proportional to the volume of the resonating cavity 'V'

$$\text{i.e} \quad \omega^2 \propto \frac{1}{V}$$

$$\omega^2 = \frac{K}{V}$$

Where 'K' is proportionality constant

$$\boldsymbol{\omega^2 V = K}$$

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SHOCK WAVES

[QUESTION: Define mach number and distinguish between acoustic, ultrasonic, subsonic and supersonic waves]

Mach Number : it is defined as the ratio of the speed of the object to the speed of sound in the given medium

Mach Number $= \dfrac{\textit{Speed of the object}}{\textit{Speed of sound in the medium}}$

Mach Number $= \dfrac{v}{a}$

DISTINCTION BETWEEN

Acoustic, ultrasonic, subsonic, transonic, supersonic and hypersonic waves

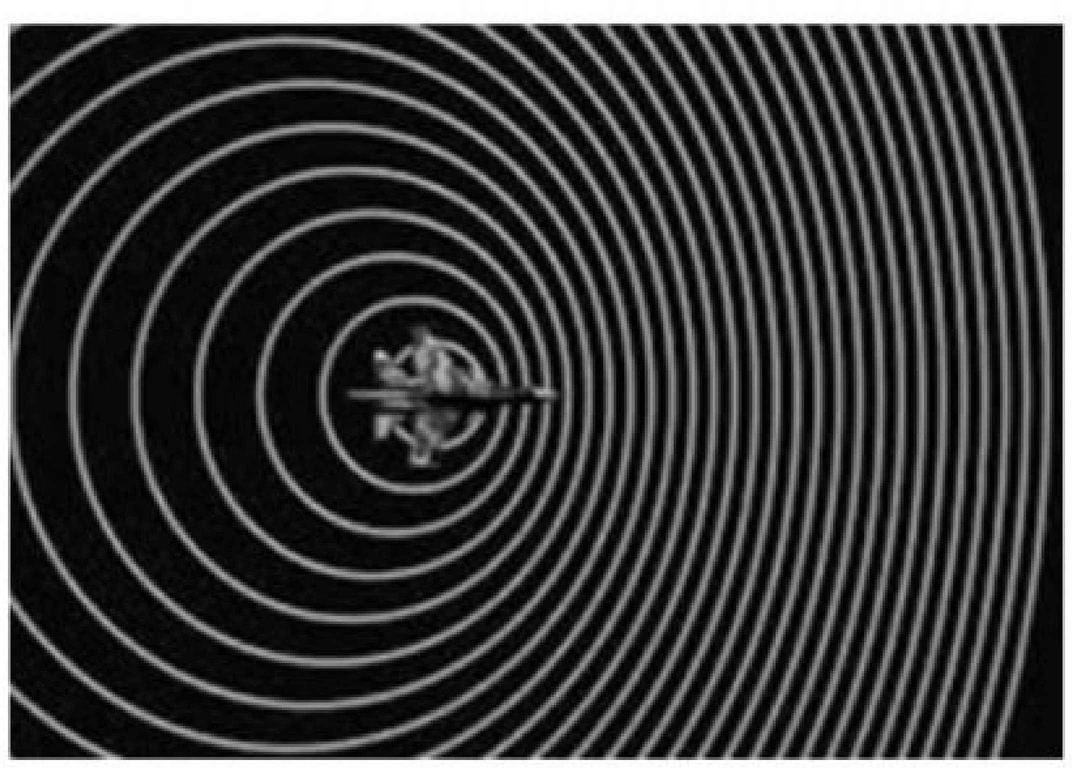
Acoustic waves: They are simply sound waves move with a speed of 333 m/s, frequency within 20 KHz and have small amplitude

Ultrasonic Waves : They are pressure waves moves with the velocity of sound, but their frequency is > 20 KHz.

Subsonic waves : (Mechanical waves) For a moving object, if its speed is less than the speed of sound, then they have subsonic waves.

All subsonic waves have Mach Number < 1

Egs : Almost all vehicles like motor cars, trains, flights, flying birds moves with subsonic speeds



As shown in figure, for a body moving with subsonic speed, the sound emitted by it moves ahead of the body as it is faster than the body.

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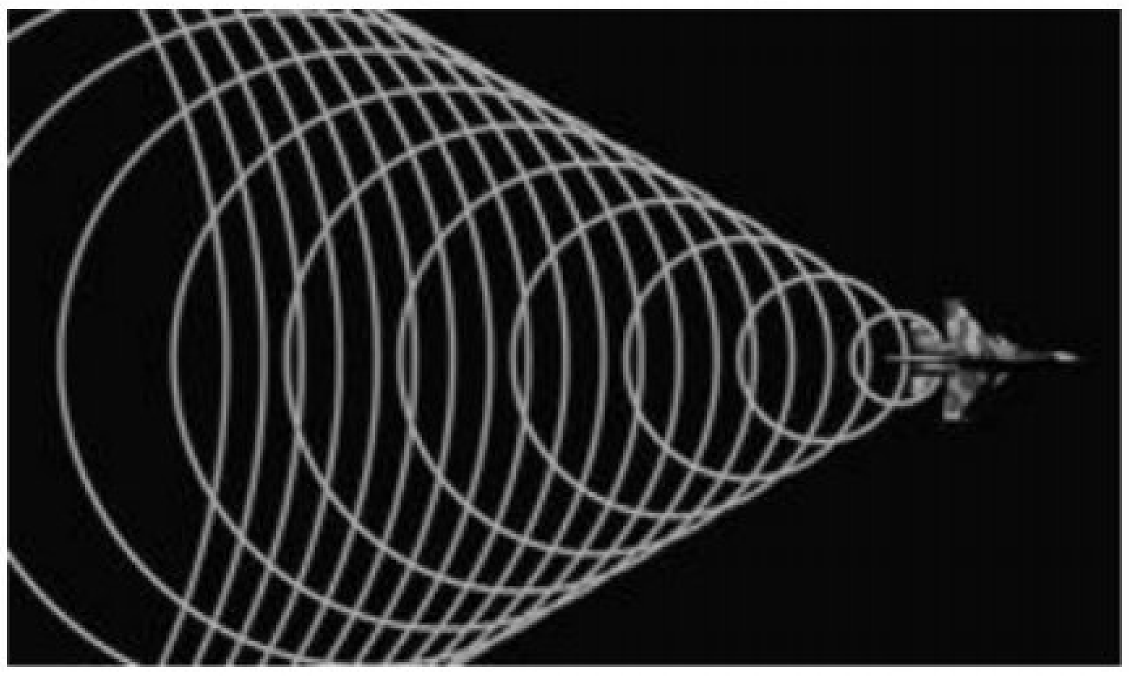
Supersonic Shock Waves : For a moving object, if its speed is greater than the speed of the sound, then they are called as supersonic shock waves. Amplitude of supersonic waves is high and it affects the medium in which it is travelling

The Mach Number for such objects is > 1

As shown in the figure, when the body moves with supersonic speed it moves ahead of its own sound waves as travels faster than sound waves.

Egs : fighter planes fly with the speed of supersonic waves



Transonic waves : Here the speed range of the body overlaps on the subsonic and supersonic ranges. Therefore the transonic range for speeds $0.8 < M < 1.2$, hence there will be overlapping of some of the characteristics of both the subsonic and supersonic speeds.

Hypersonic waves : a special class of waves called hypersonic waves, they travel with the speeds for which mach number > 5. An hypersonic flow is accompanied by a shock layer in its front. Here also there is overlapping area between supersonic and hypersonic flow.

DESCRIPTION OF A SHOCK WAVE AND ITS PROPERTIES

[QUESTION: what are shock waves, mention few properties of a shock wave]

Any object that propagates at supersonic speeds, it gives rise to a shock wave.

- Shock waves can be produced by a sudden dissipation of mechanical energy in a small space.
- Shock waves are characterized by sudden increase in pressure, temperature and density of the gas through which it propagates.
- In nature shock waves are produced during earth quakes and when lighting strikes

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- Strong shock waves posses higher values of mach number Egs : Nuclear Explosion
- Weak shock waves possess low values of mach number Egs : Busting of an automobile tyre.

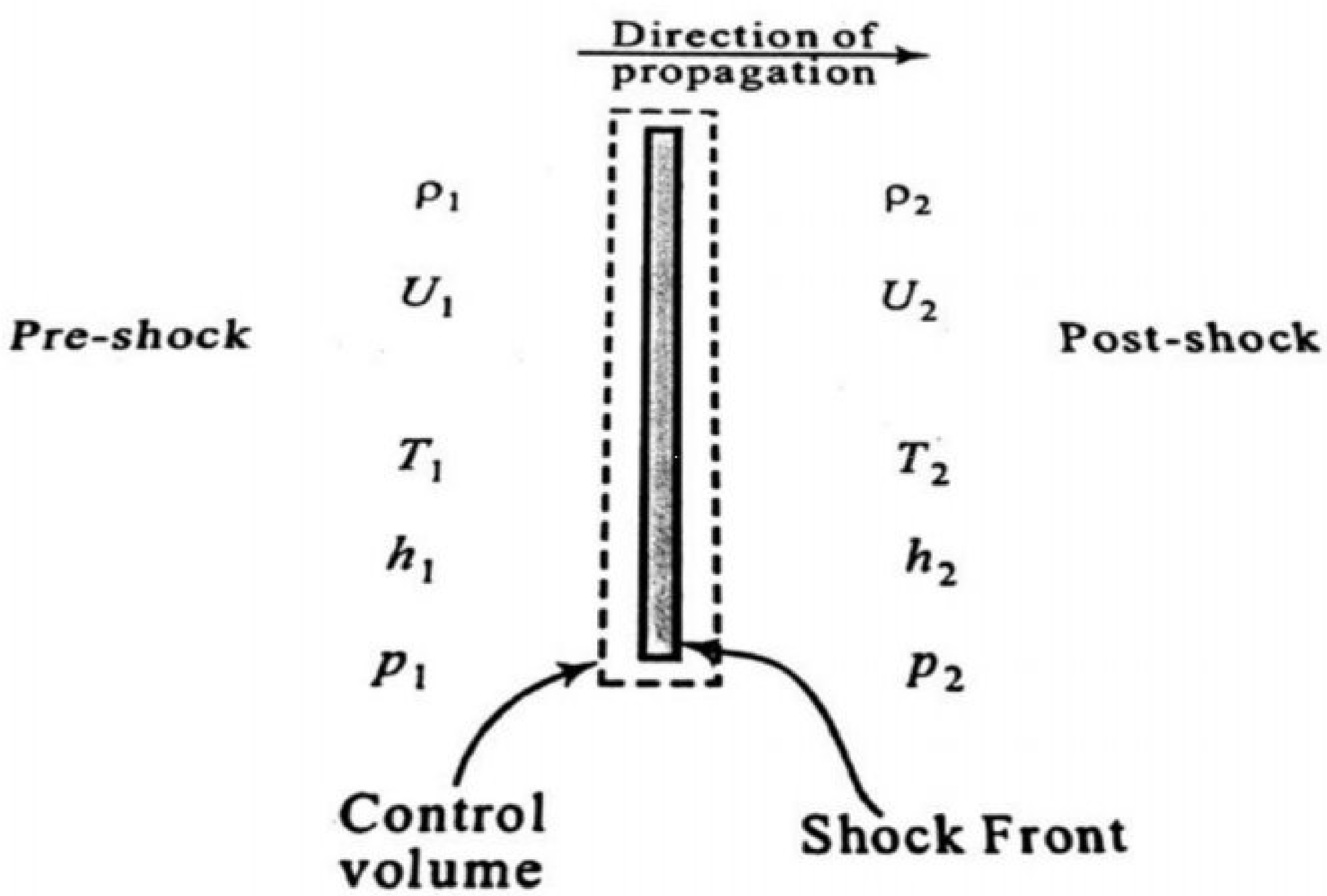
➢ They always travel in the medium with mach number exceeding 1.

➢ Shock waves obey the laws of fluid dynamics.

➢ They increase the entropy of the medium they travel

CONTROL VOLUME

[QUESTION: Explain control volume]



Shock waves are analyzed with a model called control volume. It is the one dimensional confinement in the medium with two surfaces. One on the pre-shock side and the other one on the post- shock side. Their inter separation is very small. On the pre shock side density, flow velocity , internal energy, temperature, specific enthalpy and pressure are respectively ρ_1, U_1, T_1 , h_1 and $P_1$and on the post – shock side they are ρ_2, U_2, T_2 , h_2 and P_2 respectively.

It is assumed that, within this volume, the heat energy is constant. The equations of mass, momentum and energy are the governing equations for the control volume.

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BASIC CONSERVATIONAL LAWS OF MASS, MOMENTUM AND ENERGY

[QUESTION : Explain the conservational laws of mass, momentum and energy of a shock wave]

Law of conservation of mass : it states that " the total mass of any isolated system remains unchanged and is independent of any chemical or physical changes that could occur within the system"

The law of conservation of mass is given by

$$\rho_1 U_1 = \rho_2 U_2$$

Where ρ_1 and ρ_2 are the initial and final values of density U_1 and U_2 are the velocities before and after the creation of shock wave

Law of conservation of momentum : " It states that, when two objects collide in an isolated system, the total momentum of the two object before collision = total momentum of the two objects after collision"

The law of conservation of momentum is given by

$$P_1 + \rho_1 U_1^2 = P_2 + \rho_2 U_2^2$$

Where P_1 and P_2 are the pressure before and after the creation of shock wave in shock tube.

Law of conservation of energy : "It states that, the total energy of a closed system remains constant and is independent of any changes occurring within the system"

The law of conservation of energy is given by

$$h_1 + \frac{U_1^2}{2} = h_2 + \frac{U_2^2}{2}$$

Where h_1 and h_2 are the heat of enthalpy before and after the creation of shock wave in shock tube

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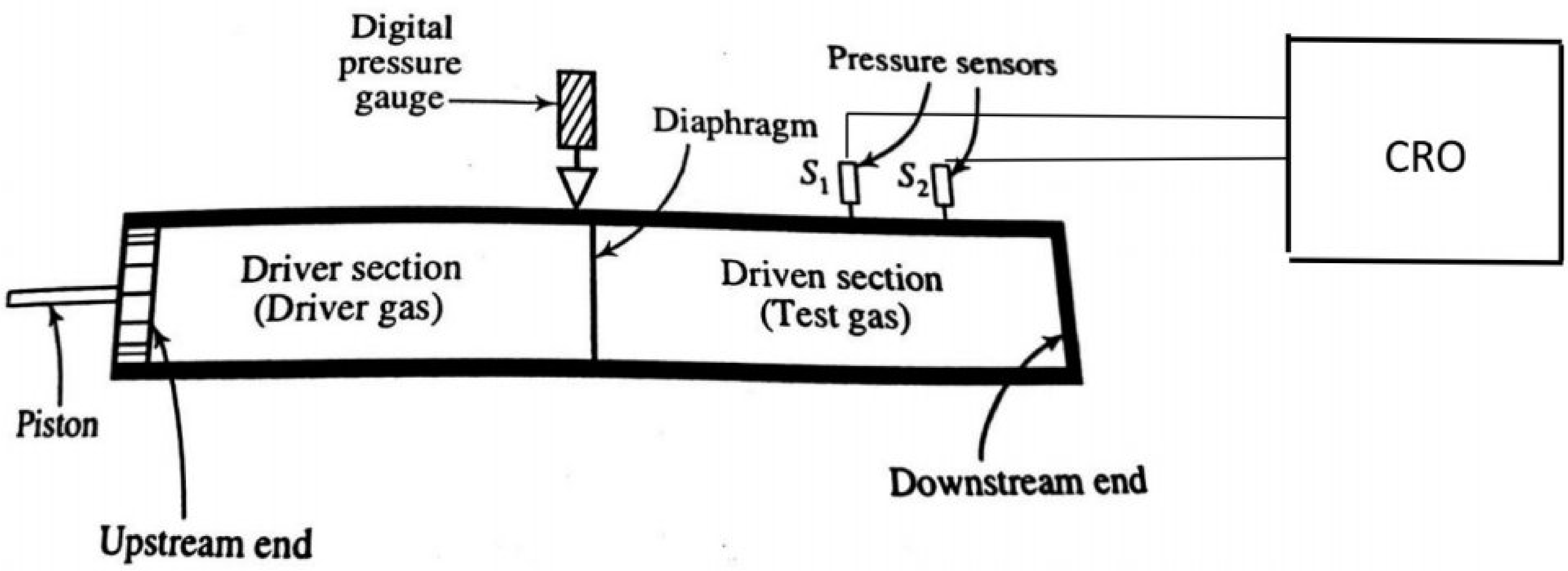
REDDY SHOCK TUBE EXPERIMENT

[QUESTION: Explain the construction and working of Reddy shock tube experiment]



Reddy shock tube consists of a cylindrical stainless steel tube of about 30mm diameter and of length 1m. The tube is divided into two sections each of length 50cm.one is called as driver tube and the other one is called as driven tube. These two tubes are separated by a 0.1mm thick diaphragm (the diaphragm may be Aluminium or paper). The Reddy tube has a piston fitted to the driver section whereas the driven section end is closed. A digital pressure guage is mounted in the driver section. Two piezoelectric sensors $S_1$and $S_2$are mounted 70mm apart towards the closed end of the shock tube. The driver section is filled with a gas (helium gas) at relatively high pressure due to the compressing action of the piston. Similarly driven section is filled with Argon gas.

WORKING : When the piston is pushed hard into the driver tube, the driver gas compresses and the diaphragm ruptures and the driver gas (helium) rushes into the driven section and pushes the driven gas (Argon) towards the downstream end. This action generates a shock wave that travels through the length of the driven section. The shock wave instantly raises the temperature and pressure of the driven gas. This propagating shock wave gets reflected from the downstream end. After reflection, the driven gas further undergoes compression boosting its temperature and pressure to still higher values. The pressure raise caused by primary shock wave and also the reflected shock wave are sensed as signals by the sensors S_1 and S_2 respectively. These signals are recorded in a digital cathode ray oscilloscope (CRO). From these recording of the CRO, the shock arrival times are found out by the associated time base calculations. Using this data, mach number, pressure and temperature can be calculated.

APPLICATIONS OF SHOCK WAVES

[QUESTION : Explain the applications of shock waves]

1] Cell information: DNA can be pushed inside the cell by passing a shock wave of appropriate strength. Also the functionality of DNA will not be affected by the impact of the shock wave.

2] Wood preservation: By using shock waves, chemical preservatives in the form of solutions could be pushed into the interiors of the wood samples such as bamboo. This process of introducing preservatives into the wood is much faster and more efficient, also saves wood from microbiological decaying.

3] Use in pencil industry: while manufacturing pencils, the wood is softened by soaking in the polymer at 70^0 C for about three hours and takes days for wood to dry. But in modern process, using shock waves the wood is placed in the liquid and a shock wave is sent through. The liquid gets into the wood almost instantaneously and don't required long time for drying. This shock wave treated wood is ready to use for further process without any delay.

4] Kidney stone treatment: Shock waves are used to shatter the kidney stones into smaller fragments after which, they are passed out of the body smoothly through urinary tracts.

5] Shock waves assisted needleless drug delivery: by using shock waves, drugs can be injected into the body without using needles. The drug is filled inside the cartridge which is kept pressed on the skin and then a shock wave is sent into the using high pressure. The drug enters the body through the pores of the skin. In this process, the patient doesn't experience any pain.

6] Treatment of dry borewells: A shock wave sent through the dry borewell clears the blockages and rejuvenates the borewell into a water source.

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	MODULE – 1 Important Questions WAVES AND OSCILLATIONS
1	Define simple harmonic oscillations . Derive the expression for differential equation for SHM and mention its solution
2	What is the expression for period of oscillation for a mass spring oscillator? Derive the expression for equivalent force constant for springs in series and parallel combination. mention the expression for period of its oscillation
3	What are damped vibrations/oscillations? Give the theory of damped vibrations/oscillations, and find the condition of heavy, critical and light damping
4	What are forced oscillations ? Obtain an expression for amplitude and phase of the body undergoing forced vibrations.
5	Discuss the dependence of amplitude and phase of a forced vibrations on the frequency of the applied external force
6	Define sharpness of resonance . Write a short note on Helmholtz resonator
	Practice all worked out problems from the Basavraju text book
	SHOCK WAVES
1	Define mach number and distinguish between acoustic, ultrasonic, subsonic and supersonic waves
2	Explain the conservational laws of mass, momentum and energy
3	Explain the construction and working of Reddy shock tube experiment]
4	Explain the applications of shock waves
	Practice all worked out problems from the Basavraju text book

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