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Module 3 : Partial Differential Equations (PDE)

Defⁿ:- An equation involving one or more partial derivatives of a function of two or more variables is called a partial differential equation.

Degree & Order :→ The Order of a PDE is the order of the highest derivative and the degree of highest order derivative after clearing the equation of fractional Power.

Linear :→ A PDE is said to be Linear if its of I degree in the dependent variable of its partial derivatives.

Ex :→ $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ order - 1 degree - 1

2) $\frac{\partial^2 z}{\partial x \partial y} = xy$ order - 2 degree - 1

3) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y} + 2z$ order - 2 degree - 1

Formation of PDE by eliminating arbitrary Constants as arbitrary functions.

Given a relation of the form $f(x, y, z, a, b) = 0$ where z is a function x, y and a, b are arbitrary constants we differentiate the given relation w.r.to x & y partially and eliminate the arbitrary constants a, b to form PDE.

$p = \frac{\partial z}{\partial x} = z_x$, $q = \frac{\partial z}{\partial y} = z_y$, $r = \frac{\partial^2 z}{\partial x^2} = z_{xx}$

$s = \frac{\partial^2 z}{\partial x \partial y} = z_{xy}$ $t = \frac{\partial^2 z}{\partial y^2} = z_{yy}$

The PDE by eliminating the arbitrary constan[ts] in the following.

1) $z = (x+a)(y+b)$

by data $z = (x+a)(y+b)$ --- ①

Differentiating partially w.r.to x & y

$$p = \frac{\partial z}{\partial x} = (y+b) \quad \text{--- ②} \qquad q = \frac{\partial z}{\partial y} = (x+a) \quad \text{--- ③}$$

using ② & ③ in ① we obtain

$$\boxed{z = pq}$$

2) Form the Partial Differential equation by eliminating the arbitrary constants a & b from $(x-a)^2 + (y-b)^2 + z^2 = c^2$

Sol^n: given

$$(x-a)^2 + (y-b)^2 + z^2 = c^2 \quad \text{--- ①}$$

Here a, b, are arbitrary constants & have to be eliminated

Diff ① w.r.to x & y partially

$2(x-a) + 2zp = 0$ & $2(y-b) + 2zq = 0$

$2(x-a) = -2zp$ & $2(y-b) = -2zq$

$(x-a) = -zp$ $\qquad$ $(y-b) = -zq$

① becomes $(-zp)^2 + (-zq)^2 + z^2 = c^2$

$$z^2(p^2+q^2+1) = c^2 \;//$$

3) If $z = xy + y\sqrt{x^2-a^2} + b$ form the associate PDE

By data $z = xy + y\sqrt{x^2-a^2} + b$

Differentiating ① w r to x & y partially

$$\frac{\partial z}{\partial x} = p = y + y \cdot \frac{1}{2\sqrt{x^2-a^2}} 2x = y + \frac{xy}{\sqrt{x^2-a^2}}$$

$$\frac{\partial z}{\partial y} = q = x + \sqrt{x^2-a^2}$$

$(q-x) = \sqrt{x^2-a^2}$ using ② in we have

$p = y + \frac{xy}{q-x}$ or $(p-y)(q-x) = xy$ or

$$pq = xp + yq \;//$$

H Form the PDE by eliminating the arbitrary constants a & b from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

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Form the PDE by eliminating the arbitrary functions in the following.

1) $z = f(x^2+y^2)$

By data $z = f(x^2+y^2)$ --- ①

Differentiating ① partially w.r.to x & y we have

$$\frac{\partial z}{\partial x} = p = f'(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = q = f'(x^2+y^2)\cdot 2y$$

$\frac{p}{q} = \frac{x}{y}$ or $py = qx$ or $py - qx = 0$.

② $z = y^2 + 2f(1/x + \log y)$

By data $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$ --- ①

Differentiating ① w.r.to x & y we get

$$\frac{\partial z}{\partial x} = p = 2f'\left(\frac{1}{x}+\log y\right)\left(-\frac{1}{x^2}\right)$$

$$\frac{\partial z}{\partial y} = q = 2y + 2f'\left(\frac{1}{x}+\log y\right)\left(\frac{1}{y}\right)$$

$px^2 = -2f'\left(\frac{1}{x}+\log y\right)$ & $(q-2y)y = 2f'\left(\frac{1}{x}+\log y\right)$

$\frac{px^2}{(q-2y)y} = -1$ or $px^2 = -qy + 2y^2$

$$px^2 + qy = 2y^2$$

3) $lx + my + nz = \phi(x^2+y^2+z^2)$

By data, $lx + my + nz = \phi(x^2+y^2+z^2)$

Diff partially w.r.to x & w.r.to y

$$l + np = \phi'(x^2+y^2+z^2)(2x+2zp)$$

$$m+nq = \phi'(x^2+y^2+z^2)(2y+2zq)$$

$$\frac{l+np}{m+nq} = \frac{x+zp}{y+zq} \quad \text{or}$$

$$(x+zp)(m+nq) = (y+zq)(l+np)$$

Simplyfying $(mz-ny)p+(nx-lz)q = ly-mx$

4] Form the PDE from
$\phi(x+y+z,\ x^2+y^2+z^2)=0.$

Sol^n → $p(y-z)+q(z-x) = (x-y)$

5] $f\left(\frac{xy}{z}, z\right) = 0.$

$$\frac{\partial u}{\partial x} \Big/ \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \Big/ \frac{\partial v}{\partial y}$$

Where $u = \frac{xy}{z}$ and $v=z$

$$\frac{\partial u}{\partial x} = y\left[\frac{z-xp}{z^2}\right] \qquad \frac{\partial v}{\partial x} = p$$

$$\frac{\partial u}{\partial y} = x\left[\frac{z-yq}{z^2}\right] \qquad \frac{\partial v}{\partial y} = q$$

from ① we have

$$\frac{y(z-xp)}{x(z-yq)} = \frac{p}{q} \quad \text{or} \quad xp = yq \,//$$

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5) $z = yf(x) + x\phi(y)$

By data $z = yf(x) + x\phi(y)$

Diff w r to. x & y partially

$$\frac{\partial z}{\partial x} = p = yf'(x) + \phi(y)$$

$$\frac{\partial z}{\partial y} = q = f(x) + x\phi'(y)$$

$$\frac{\partial^2 z}{\partial x^2} = r = yf''(x)$$

$$\frac{\partial^2 z}{\partial y^2} = t = x\phi''(y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = s = f'(x) + \phi'(y)$$

② , $\frac{p - \phi(y)}{y} = f'(x)$ ③ $\frac{q - f(x)}{x} = \phi'(y)$

⑤ $s = \frac{p - \phi(y)}{y} + \frac{q - f(x)}{x}$ or

$$s = \frac{px - x\phi(y) + qy - yf(x)}{xy}$$

$\therefore xys = px + qy - [x\phi(y) + yf(x)]$

$xys + z = px + qy$

HW

Form the PDE by eliminating the arbitrary constants / functions.

1) $f(x+y+z, x^2+y^2+z^2) = 0$.

2) $z = f(x+ct) + g(x-ct)$

Solution of PDE

Solution of non homogeneous PDE by direct integr

1) Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x+3y)$

Sol^n ⇒ We have,

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) = \cos(2x+3y)$$

Integrating w.r.to x.

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \int \cos(2x+3y)\,dx + f(y)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \int \cos(2x+3y)\,dx + f(y)$$

or

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\sin(2x+3y)}{2}$$

Integrating w.r to. x.

$$\frac{\partial z}{\partial y} = \frac{1}{2}\int \sin(2x+3y)\,dx + f(y)\int 1\,dx + g(x)$$

$$\frac{\partial z}{\partial y} = \frac{-\cos(2x+3y)}{4} + x\,f(y) + g(y)$$

Integrating w.r.to y.

$$z = \frac{-1}{4}\int \cos(2x+3y)\,dy + x\int f(y)\,dy + \int g(y)\,dy + h(x)$$

$F(y) = \int f(y)\,dy$, $G(y) = \int g(y)\,dy$

Thus

$$z = \frac{-1}{12}\sin(2x+3y) + xF(y) + G(y) + h(x)$$

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(2). Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which

$\frac{\partial z}{\partial y} = -2\sin y$ When $x = 0$ and $z = 0$ if y

is an odd multiple of $\pi/2$

Sol^n:→ find z by integration and apply the given coditions to determine the arbitrary functions occuring as constants of integration.

$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \sin x \sin y$ Integrating w.r.to x.

$\frac{\partial z}{\partial y} = \sin y \int \sin x \, dx + f(y)$

$\frac{\partial z}{\partial y} = -\sin y \cos x + f(y)$ -- (1) Integrating w.r.to y

$z = (-\cos x)(-\cos y) + F(y) + g(x)$

by data $\frac{\partial z}{\partial y} = -2\sin y$ When $x = 0$ using (1)

Integrating w.r.to y

$z = (-\cos x)(-\cos y) + F(y) + g(x)$ where

$F(y) = \int f(y)\, dy$

$z = \cos x \cos y + F(y) + g(x)$

by data $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ using this (1).

$-2\sin y = (-\sin y)\cdot 1 + f(y)$

$f(y) = -\sin y$

Hence $F(y) = \int f(y)\, dy = \int -\sin y \, dy = \cos y$

(2) $z = \cos x \cos y + \cos y + g(x)$

condition that $z = 0$ if $y = (2n+1)\pi/2$ in (3)

$0 = \cos x \cos (2n+1)\pi/2 + \cos(2n+1)\pi/2 + g(x)$

$\cos(2n+1)\pi/2 = 0$. Hence $0 = 0 + 0 + g(x)$ or $g(x) = 0$.

Thus $z = \cos x \cos y + \cos y$. or $z = \cos y(\cos x + 1)$ //

(3). Solve $\frac{\partial^2 z}{\partial x^2} = xy$ subject to the conditions that $\frac{\partial z}{\partial x} = \log(1+y)$ $x=1$ & $z=0$ when $x=0$.

Soln:-> We have $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = xy$ Integrating w.r.to x

$$\frac{\partial z}{\partial x} = \frac{x^2}{2}y + f(y) \quad \text{---} (1).$$ Again integrating w.r.to x.

$$z = \frac{x^3 y}{6} + x f(y) + g(y)$$

by data $\frac{\partial z}{\partial x} = \log(1+y)$ when $x=1$ using (1).

$\log(1+y) = \left(\frac{y}{2}\right) + f(y)$ or $f(y) = \log(1+y) - (y/2)$

$$z = (x^3 y/6) + x[\log(1+y) - (y/2)] + g(y)$$

$z=0$ When $x=0$ (3) $0 = 0 + 0 + g(y)$ or $g(y) = 0$.

$$z = \frac{x^3 y}{6} + x\left[\log(1+y) - \left(\frac{y}{2}\right)\right].$$

HW

1) Solve $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y}$ subject to the conditions $\frac{\partial z}{\partial x} = \log_e x$ when $y=1$ and $z=0$ when $x=1$

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Solution of homogeneous PDE involving derivative with respect to one independent variable only

① Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that when $y=0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$.

Solⁿ:→ Let us suppose that z is a function of y only given PDE assumes the form of ODE.

$$\frac{d^2 z}{dy^2} = z \quad \text{or} \quad \frac{d^2 z}{dy^2} - z = 0 \quad \text{or} \quad (D^2-1)z = 0$$

Where $D = \frac{d}{dy}$

Auxillary equation $m^2 - 1 = 0 \Rightarrow m = \pm 1$

$\therefore z = c_1 e^y + c_2 e^{-y}$

Solution of the PDE is got by replacing c_1 and c_2 functions of x.

Solution of the PDE : $z = f(x)e^y + g(x)e^{-y}$

When $y=0$, $z = e^x$ gives $e^x = f(x) + g(x)$ ①

when $y=0$, $\frac{\partial z}{\partial y} = e^{-x}$ from ① we get

$$\frac{\partial z}{\partial y} = f(x)e^y - g(x)e^{-y}$$

Applying the condition we get

$$e^{-x} = f(x) - g(x)$$

$f(x) + g(x) = e^x$ & $f(x) - g(x) = e^{-x}$

Adding & subtracting these equations we get.

$f(x) = \frac{e^x + e^{-x}}{2} = \cosh x$; $g(x) = \frac{e^x - e^{-x}}{2} = \sinh x$.

$$z = \cosh x\, e^y + \sinh x\, e^{-y}$$

2) Solve $\partial^2 z/\partial x^2 = a^2 z$ given that when $x=0$, $z=0$ and $\frac{\partial z}{\partial x} = a\sin y$

Soln:→ $\frac{\partial^2 z}{\partial x^2} = a^2 z$

$\frac{\partial^2 z}{\partial x^2} - a^2 z = 0$

$\frac{d^2 z}{dx^2} - a^2 z = 0$

$D^2 z - a^2 z = 0$ or $(D^2 - a^2) z = 0$.

The auxillary eqn is

$(m^2 - a^2) = 0$

$m^2 = a^2$

$m = \pm a$

$z_c = c_1 e^{ax} + c_2 e^{-ax}$

The solution of PDE is

$z = c_1 e^{ax} + c_2 e^{-ax}$

But put $c_1 = f(y)$ $c_2 = g(y)$

$\therefore z = f(y) e^{ax} + g(y) e^{-ax}$ ——— ①.

Applying the condition

When $x = 0$, $z = 0$

① becomes $0 = f(y) e^0 + g(y) e^0$

$0 = f(y) + g(y)$

$f(y) + g(y) = 0$ ——— ②

and When $x = 0$, $\frac{\partial z}{\partial x} = a\sin y$

Diff eqn ① partially w.r.to x.

$\frac{\partial z}{\partial x} = a f(y) e^{ax} - a g(y) e^{-ax}$

$a\sin y = a f(y) e^0 - a g(y) e^0$

$a\sin y = a f(y) - a g(y)$

$a\sin y = a [f(y) - g(y)]$

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$\therefore \ f(y) - g(y) = \sin y$ ---- ③

On solving ② and ③.

$f(y) + \cancel{g(y)} = 0.$

$f(y) - \cancel{g(y)} = \sin y$

$2f(y) = \sin y$

$f(y) = \frac{\sin y}{2}$ & $g(y) = -\frac{\sin y}{2}$

eqⁿ ① becomes

$z = \frac{\sin y}{2} e^{ax} - \frac{\sin y}{2} e^{-ax}$

$\boxed{z = \sin y \sinh ax}$ //

3) Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$ subject to the condition s that $z = 1$ and $\frac{\partial z}{\partial x} = y$ when $x = 0$

Solⁿ:- Let us suppose that z is a function of x only. The given PDE assumes form of an ODE $(D^2 + 3D - 4)z = 0$ Where $D = \frac{d}{dx}$

A E is $m^2 + 3m - 4 = 0$ or $(m-1)(m+4) = 0 \Rightarrow m = 1, -4$

solution of the ODE $z = c_1 e^{x} + c_2 e^{-4x}$

solution of PDE : $z = f(y)e^{x} + g(y)e^{-4x}$

from ① $\frac{\partial z}{\partial x} = f(y)e^{x} + g(y)\cdot e^{-4x}(-4)$

By data $z = 1$ and $\frac{\partial z}{\partial x} = y$ When $x = 0$ Hence

① & ② becomes

$1 = f(y) + g(y)$ and $y = f(y) - 4g(y)$

By solving these we get

$f(y) = \frac{1}{5}(4+y)$ and $g(y) = \frac{1}{5}(1-y)$

$z = \frac{1}{5}(4+y)e^{x} + \frac{1}{5}(1-y)e^{-4x}$ //

Lagrange's Linear PDE

Lagrange's linear PDE of the form $Pp+Qq=R$
where P, Q, R are all functions of x, y, z
and $p = z_x$, $q = z_y$ as usual.

Step 1 :→ Given the PDE in the form $Pp+Qq=R$
we form the equation of the form
$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ known as the Auxiliary Equation.

Step 2 :→ We can consider suitable pairs which can be put in the forms like
$f(x)dx = g(y)dy$, $g(y)dy = h(z)dz$, $f(x)dz = h(z)dz$

1) Solve $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$ or $xp+yq=z$

PDE is of the form $Pp+Qq=R$
Where $P=x$, $Q=y$, $R=z$

AE $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$

Consider $\frac{dx}{x} = \frac{dy}{y}$ Which on integration

$\log x = \log y + \log c_1$ or $x/y = c_1$.

$\frac{dy}{y} = \frac{dz}{z}$ and we can lly get $y/z = c_2$

Thus the general solution of the PDE is given by,

$\phi(x/y, y/z) = 0$ //

Solution of Lagrange's linear PDE

Lagrange's Linear PDE of the form $Pp + Qq = R$
Where P, Q, R are all functions of x, y, z
and $p = z_x$, $q = z_y$ as earlier.

Step 1 :→ Given the PDE in the form $Pp+Qq=R$ we form the equation of the form

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$ known as the Auxillary Equation

Step 2 :→ We can consider suitable Pairs which can be put in the forms like

$f(x)\,dx = g(y)\,dy$, $g(y)\,dy = h(z)\,dz$, $f(x)\,dx = h(z)\,dz$

1) Solve $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$ or $xp + yq = z$

PDE is of the form $Pp + Qq = R$
Where $P = x$, $Q = y$, $R = z$

AE $$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

Consider $\frac{dx}{x} = \frac{dy}{y}$ which on integration

$\log x = \log y + \log c_1$ or $x/y = c_1$.

$\frac{dy}{y} = \frac{dz}{z}$ and we can lly get $y/z = c_2$

Thus the general solution of the PDE is given by,

$$\phi\left(x/y,\ y/z\right) = 0$$

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Solve $x^2(y^2-z^2)p + y^2(z^2-x^2)q = z^2(x^2-y^2)$

Soln. Given,

$$x^2(y^2-z^2)P + y^2(z^2-x^2)q = z^2(x^2-y^2)$$

Where $P = \frac{\partial u}{\partial x}$ and $q = \frac{\partial u}{\partial y}$ and $u = u(x, y)$

is the unknown function.

Rewriting the equation in terms of P and q

$$F(x, y, z, P, q) = x^2(y^2-z^2)P + y^2(z^2-x^2) - z^2(x^2-y^2)$$

$$\frac{dx}{x^2(y^2-z^2)} = \frac{dy}{y^2(z^2-x^2)} = \frac{dz}{z^2(x^2-y^2)}$$

$$\frac{dx}{x^2(y^2-z^2)} = \frac{dy}{y^2(z^2-x^2)}$$

$$dx \cdot y^2(z^2-x^2) = x^2(y^2-z^2)\,dy$$

Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.

The given equation is of the form $Pp + Qq = R$

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} \quad \text{(1)}$$

using the multipliers $1/x^2$, $1/y^2$, $1/z^2$ each ratio

$$\frac{1/x^2\,dx + 1/y^2\,dy + 1/z^2\,dz}{(y-z)+(z-x)+(x-y)} = \frac{1/x^2\,dx + 1/y^2\,dy + 1/z^2\,dz}{0}$$

$$1/x^2\,dx + 1/y^2\,dy + 1/z^2\,dz = 0 \Rightarrow -1/x - 1/y - 1/z = C_1$$

or

$$1/x + 1/y + 1/z = -C_1 = K_1$$

Again using the multiplier $1/x$ $1/y$ $1/z$ (1)

$$\frac{1/x\,dx + 1/y\,dy + 1/z\,dz}{x(y-z)+y(z-x)+z(x-y)} = \frac{1/x\,dx + 1/y\,dy + dz\,1/z}{0}$$

$$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0 \quad \text{which gives } xyz = K_2$$

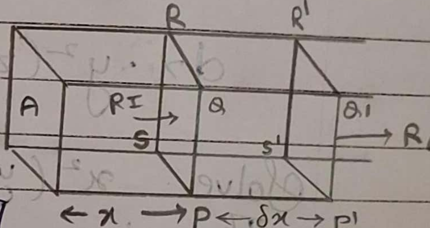
Heat And Wave Equation.

Derivation of the One dimensional heat equation

We are using the following thermodynamical Laws in respect of heat flow :

1) Heat flows from higher temp level to lower temperature level.
2) The amount of heat in a body is proportional to its mass and temp [constant of proportionality is specific heat]
3] The rate of heat flow across an area is proportional to the area and to the temp gradient $\left(\frac{\partial u}{\partial x}\right)$ normal to the area where constant of proportionality is called (K) thermal conductivity

Consider the uniform bar with



A → Cross sectional area
ρ → density [mass per unit volume]
s → Specific heat of the material
K → Thermal conductivity of material

let the 3 sides of the bar are insulated so that heat flow only in one direction [say along '+'ve direction of x-axis]

Consider an element δx between the planes PQRS : P'Q'R'S' at a distance x & $x+\delta x$ from one end.
let $\frac{\partial u}{\partial x}$ be the temp gradient at P & P'

∴ mass of the element $= A\rho\,\delta x$
The quality of heat stored in slab $= A\rho s\,\delta x\,\delta u$

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Rate of increase of heat in the slab

$$R = (A s S \delta x) \frac{\partial u}{\partial t} \quad \text{--- ①}$$

If R_I is the rate of inflow & R_o is the rate of out flow heat then

(Rate of heat flow) $R_I = -kA\left(\frac{\partial u}{\partial x}\right)_x$;

$$R_O = -kA\left(\frac{\partial u}{\partial x}\right)_{x+\delta x}$$

'$-$ve sign is due to heat flow from higher temp to lower temp.

$\therefore$ Rate of increase of heat in the slab also given by $R = R_I - R_O$

$$A s S \delta x \frac{\partial u}{\partial t} = -kA\left(\frac{\partial u}{\partial x}\right) - \left(-kA\left(\frac{\partial u}{\partial x}\right)_{x+\delta x}\right)$$

$$= kA\left[\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_x\right]$$

$$\frac{\partial u}{\partial t} = \frac{k}{sS} \times \frac{\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_x}{\delta x}$$

Now as $P' \to P$ $\quad \delta x \to 0$

$$\frac{\partial u}{\partial t} = \frac{k}{sS} \frac{\partial^2 u}{\partial x^2}$$

$$\boxed{\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}}$$

Where $c = \sqrt{k/sS}$

one dimensional Heat equation. is known as

Derivation One dimensional wave equation.

Consider a uniform flexible string stretched between the points A and B at a distance l apart.

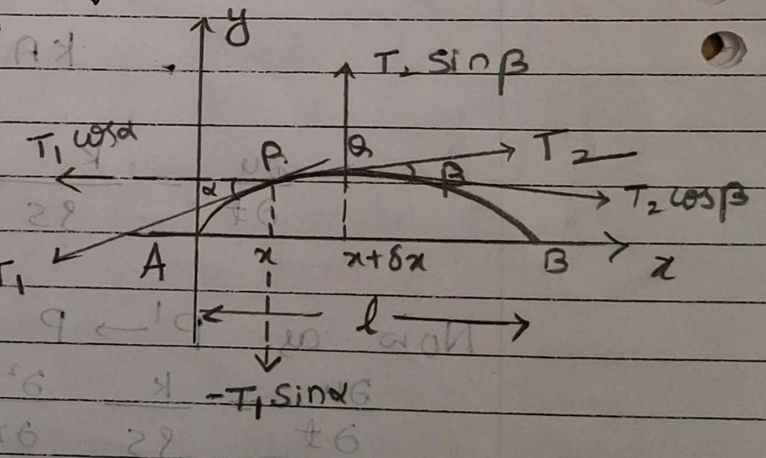
Let ρ be the mass per unit length of the string.

Here we are finding the equation of the motion of the string $y(x,t)$ at any point x and at any time $t>0$ under the following assumptions.

i) As the string is uniform tension T in the string is same throughout.

ii) The effect of gravity is neglected due to large tension in the string

iii) The motion of the string is in small transverse vibrations.



Let us consider force acting at small part of the string PQ at a distance δx apart

Since there is no motion along Horizontal, The Horizontal components are

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Same to tension in the string T.

i.e $\boxed{T_1 \cos\alpha = T_2 \cos\beta = T}$

The resultant force acting at PQ is given by

$$F = T_2 \sin\beta - T_1 \sin\alpha$$

-ve sign is due to the verticle component is acting downwords.

By Newton's 2nd Low of motion Force = mass x acceleration

$$F = ma$$

Total force $T_2 \sin\beta - T_1 \sin\alpha = (\rho\,\delta x)\dfrac{\partial^2 y}{\partial t^2}$

$$T_2 = \frac{T}{\cos\beta} \; ; \quad T_1 = \frac{T}{\cos\alpha}$$

$$\Rightarrow \frac{T\sin\beta}{\cos\beta} - \frac{T\sin\alpha}{\cos\alpha} = \rho\,\delta x\,\frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow \tan\beta - \tan\alpha = \frac{\rho}{T}\,\delta x\,\frac{\partial^2 y}{\partial t^2}$$

$$\tan\alpha = \left.\frac{\partial y}{\partial x}\right|_{P=x} \qquad \tan\beta = \left.\frac{\partial y}{\partial x}\right|_{Q=x+\delta x}$$

$$\frac{\left.\dfrac{\partial y}{\partial x}\right|_{x+\delta x} - \left.\dfrac{\partial y}{\partial x}\right|_{x}}{\delta x} = \frac{\rho}{T}$$

As point $Q \to P$ $\quad \delta x \to 0$

$\boxed{\dfrac{\partial^2 y}{\partial x^2} = c^2 \dfrac{\partial^2 y}{\partial t^2}}$ Where $c = \sqrt{\rho/T}$

or $y_{xx} = c^2 y_{tt}$ is 1-D wave equation.