Rank of matrix by Elementary transformations :⇒

* The following are the Elementary row transformations of a matrix. The transformation can also be applied for columns.

1) Interchange of any two rows.
2) Multiplication of any row by a non-zero constant
3) Addition to any row a constant multiple of any other row.

Echelon form of a matrix :⇒

A non-zero matrix A is said to be in row echelon form if the following conditions satisfies.

a) All the zero rows are below non-zero rows.
b) The first non-zero entry in any non-zero row is 1

Rank of a matrix :⇒

The rank of a matrix A in its echelon form is Equal to the number of non-zero rows. It is denoted by $\rho(A)$

1) $A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 3 \\ 1 & 5 & 7 \end{bmatrix}$

$R_1 \longleftrightarrow R_2$

$A \sim \begin{bmatrix} -1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$

$R_2 \longrightarrow 2R_1 + R_2$, $R_3 \longrightarrow R_1 + R_3$

$A \sim \begin{bmatrix} -1 & 2 & 3 \\ 0 & 7 & 10 \\ 0 & 7 & 10 \end{bmatrix}$

$R_3 \longrightarrow R_3 - R_2$ gives

$$A \sim \begin{bmatrix} -1 & 2 & 3 \\ 0 & 7 & 10 \\ 0 & 0 & 0 \end{bmatrix}$$

$-R_1$ and $\frac{1}{7} R_2$

$$A \sim \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 10/7 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

I> Find the rank of the matrix

1> $$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

$R_2 \to -2R_1 + R_2$, $R_3 \to -R_1 + R_3$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$R_3 \to R_2 + R_3$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to (-1) R_2$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

2> $$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$A \sim \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_1$

$$A \sim \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$A \sim \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{1}{2} R_1$, $\frac{1}{2} R_2$

$$A \sim \begin{bmatrix} 1 & 3/2 & 5/2 & 2 \\ 0 & 1 & 3/2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

3) $$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_1 \longleftrightarrow R_2$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \rightarrow -2R_1 + R_2$, $R_3 \rightarrow R_3 - R_1$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \longleftrightarrow R_4$ $$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & -5 & -9 & 1 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$, $R_4 \to R_4 + 5R_2$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & -4 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$ and $-\frac{1}{4} R_3$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix A in the row echelon form is having three non-zero rows.

Thus, $\rho(A) = 3$

4> $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$ $\rho(A) = 4$

5> $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ $\rho(A) = 3$

6> $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ $\rho(A) = 2$

7> $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ $\rho(A) = 3$

Consistency of a system of linear Equations :⇒

→ Conditions for consistency and types of sol^{n}s -
consider a system of m Eq^{n}s in n unknowns represented in the matrix form $AX = B$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$$

'A' is called the coefficient matrix.
The matrix formed by appending to A an extra column consisting of the elements of B is called the augumented matrix denoted by $[A : B]$
That is, $[A : B] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & : & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & : & b_2 \\ \cdots & \cdots & \cdots & \cdots & & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & : & b_n \end{bmatrix}$

The system of Eq^{n}s represented by the matrix Eq^n $AX = B$ is consistent if $\rho(A) = \rho(A : B)$
Suppose, $\rho(A) = \rho(A : B) = r$, then the condition for two types of sol^n are as follows.

1) Unique sol^n :⇒ $\rho(A) = \rho(A : B) = r = n$, n being the no. of unknowns

2) Infinite sol^n :⇒ $\rho(A) = \rho(A : B) = r < n$
In this case $(n-r)$ unknowns can take arbitrary values, obviously $\rho(A) \neq \rho(A : B)$ implies that the system is inconsistent (donot process a solution)

1) Test for consistency and solve :⇒

$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8$$

Sol^n: $[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & -1 & 2 & : & 5 \\ 3 & 1 & 1 & : & 8 \end{bmatrix}$ is the augumented matrix

We now perform elementary row operations.

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$

$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -2 & 1 & : & -1 \\ 0 & -2 & -2 & : & -10 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -2 & 1 & : & -1 \\ 0 & 0 & -3 & : & -9 \end{bmatrix}$

Both A and $[A:B]$ matrices have all three rows non - zero

$\therefore \rho(A) = 3$ and $\rho[A:B] = 3$ i.e $r = 3$

Also, the no- of independent variables $n = 3$

Since $\rho(A) = \rho(A:B) = 3$ $(r = n = 3)$ the given system of Eq^{n}s is consistent and will have unique sol^n.

By the matrix $[A:B]$

$x + y + z = 6$ ——— (1)

$-2y + z = -1$ ——— (2)

$-3z = -9$ ——— (3)

from (3) $z = 3$

from (2) $-2y + 3 = -1 \Rightarrow y = 2$

from (1) $x + 2 + 3 = 6 \Rightarrow x = 1$

Thus $x = 1$, $y = 2$, $z = 3$ is the unique sol^n.

2) Test for consistency and solve

$x + 2y + 3z = 14$

$4x + 5y + 7z = 35$

$3x + 3y + 4z = 21$

Sol^n: $[A:B] = \begin{bmatrix} 1 & 2 & 3 & : & 14 \\ 4 & 5 & 7 & : & 35 \\ 3 & 3 & 4 & : & 21 \end{bmatrix}$

$R_2 \to R_2 - 4R_1$, $R_3 \to R_3 - 3R_1$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 3 & : & 14 \\ 0 & -3 & -5 & : & -21 \\ 0 & -3 & -5 & : & -21 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 3 & : & 14 \\ 0 & -3 & -5 & : & -21 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

we have $\rho(A) = 2$, $\rho(A:B) = 2$ Thus $r = 2$. Also $n = 3$
since $\rho(A) = \rho(A:B) = 2 < 3$. The system is consistent
and will have infinite solution.

we have. $x + 2y + 3z = 14$ —— ①

$-3y - 5z = -21$ —— ②

Let us take $z = k$ be arbitrary
from ② $\Rightarrow -3y - 5k = -21$ or $y = \frac{21-5k}{3} = 7 - \frac{5k}{3}$

Now from ① $\Rightarrow x + 2\left[7 - \left(\frac{5k}{3}\right)\right] + 3k = 14 \Rightarrow x = \frac{k}{3}$

Thus, $x = \frac{k}{3}$, $y = 7 - \frac{5k}{3}$, $z = k$ represents infinite solⁿ

3) Test for consistency and solve

$x - 4y + 7z = 14$

$3x + 8y - 2z = 13$

$7x - 8y + 26z = 5$

solⁿ: $[A:B] = \begin{bmatrix} 1 & -4 & 7 & : & 14 \\ 3 & 8 & -2 & : & 13 \\ 7 & -8 & 26 & : & 5 \end{bmatrix}$ is augumented matrix

$R_2 \rightarrow R_2 - 3R_1$, $R_3 \rightarrow R_3 - 7R_1$

$$[A:B] \sim \begin{bmatrix} 1 & -4 & 7 & : & 14 \\ 0 & 20 & -23 & : & -29 \\ 0 & 20 & -23 & : & -93 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

$$[A:B] \sim \begin{bmatrix} 1 & -4 & 7 & : & 14 \\ 0 & 20 & -23 & : & -29 \\ 0 & 0 & 0 & : & -64 \end{bmatrix}$$

we have $\rho(A) = 2$ and $\rho[A:B] = 3$
Since $\rho(A) \neq \rho(A:B)$ the system is inconsistent

4> Test for consistency and solve.

$$5x_1 + x_2 + 3x_3 = 20$$
$$2x_1 + 5x_2 + 2x_3 = 18$$
$$3x_1 + 2x_2 + x_3 = 14$$

Sol^n: $[A:B] = \begin{bmatrix} 5 & 1 & 3 & : 20 \\ 2 & 5 & 2 & : 18 \\ 3 & 2 & 1 & : 14 \end{bmatrix}$ is the augmented matrix

$R_2 \rightarrow 5R_2 - 2R_1$, $R_3 \rightarrow 5R_3 - 3R_1$

$$[A:B] = \begin{bmatrix} 5 & 1 & 3 & : 20 \\ 0 & 23 & 4 & : 50 \\ 0 & 7 & -4 & : 10 \end{bmatrix}$$

$R_3 \rightarrow 23R_3 - 7R_2$

$$[A:B] = \begin{bmatrix} 5 & 1 & 3 & : 20 \\ 0 & 23 & 4 & : 50 \\ 0 & 0 & -120 & : -120 \end{bmatrix}$$

we have $\rho(A) = 3$, $\rho(A:B) = 3$ i.e $r = 3$ Also $n = 3$
since $\rho(A) = \rho(A:B) = 3$ $(r = n = 3)$
the system is consistent and will have unique solution.

we now have, $5x_1 + x_2 + 3x_3 = 20$ —— (1)
$23x_2 + 4x_3 = 50$ —— (2)
$-120x_3 = -120$ —— (3)

from (3), $x_3 = 1$
from (2) we get $x_2 = 2$
from (1) $x_1 = 3$
Thus $x_1 = 3$, $x_2 = 2$, $x_3 = 1$ is the unique sol^n

5> Test the consistency and solve

$$x + 2y + 2z = 1$$
$$2x + y + z = 2$$
$$3x + 2y + 2z = 3$$
$$y + z = 0$$

$x = 1$
$y = -k$
$z = k$

6> S.T the following system of Eq^{n}s doesnot posses any sol^n : $5x+3y+7z=5$

$3x+26y+2z=9$

$7x+2y+10z=5$

7> Investigate the values of λ and μ such that the system of Eq^{n}s. $x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=\mu$ may have

a> Unique sol^n. b> Infinite sol^n c> No sol^n

sol^n: $[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$ is augumented matrix

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$

$R_3 \to R_3 - R_2$

$[A-B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$

a> Unique sol^n :⇒ we must have $\rho(A) = \rho[A:B] = 3$.
$\rho(A)$ will be 3 if $(\lambda-3)=0$ since the other entries in the last row of A are zero.
If $(\lambda-3) \neq 0$ or $\lambda \neq 3$ irrespective of the value of μ, $\rho[A:B]$ will also be 3.
Hence, the system will have unique sol^n if $\lambda \neq 3$

b> Infinite sol^n :⇒ Here we have $n=3$ and we need $\rho(A) = \rho(A:B) = r < 3$ we must have $r=2$ since first row and second row are non-zero
$\rho(A) = \rho(A:B) = 2$ only when the last row of $[A:B]$ is completely zero. This is possible if $\lambda-3=0$, $\mu-10=0$.

Hence the system will have infinite sol^n if $\lambda = 3$ and $\mu = 10$

c) No sol^n :→ we must have $\rho(A) = \rho[A:B]$. By case (a) $\rho(A) = 3$ if $\lambda = 3$ and hence if $\lambda = 3$ we obtain $\rho(A) = 2$.
If we impose $(\mu - 10) \neq 0$. then $\rho[A:B]$ will be 3
Hence, the system has no sol^n if $\lambda = 3$ & $\mu \neq 10$

(6) Find for what value of K the system of Eq^{n}s

$$x + y + z = 1$$
$$x + 2y + 4z = K$$
$$x + 4y + 10z = K^2$$

possesses a sol^n. solve completely in each case

sol^n: $$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & K \\ 1 & 4 & 10 & K^2 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & K-1 \\ 0 & 3 & 9 & K^2-1 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & K-1 \\ 0 & 0 & 0 & K^2-3K+2 \end{array}\right]$$

$\rho(A) = 2$ and for the system to be consistent we must have $\rho[A:B]$ also 2. This is possible if $K^2 - 3K + 2 = 0$ $\therefore K = 1, K = 2$
Hence we conclude that the system possesses a sol^n if $K = 1, 2$
since $\rho(A) = [A:B] = 2 < 3$ for the cases $K = 1, 2$ the system will have infinite sol^n and the same are as follows.
case i) $K = 1$. The system of Eq^{n}s are

$x+y+z=1$ —— (1)

$y+3z=0$ —— (2)

Let $z=k_1$ be arbitrary

$\therefore$ from (2) $y=-3k_1$ and from (1) $x=1+2k_2$

Case ii> $K=2$. The system of Eqns are

$x+y+z=1$ —— (3)

$y+3z=1$ —— (4)

Let $z=K_1$ be arbitrary.

from (4) $y=1-3k_2$ and from (3) $x=2k_2$

Thus, $x=1+2k_1$, $y=-3k_1$, $z=k_1$

and $x=2k_2$, $y=1-3k_2$, $z=k_2$

Gauss - Elimination method :⇒

1> Solve the following system of Eqns by Gauss-Elimin -ation method.

$x+y+z=9$

$x-2y+3z=8$

$2x+y-z=3$

Soln: The augumented matrix of the system is

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

$R_3 \to R_2 + (-3)R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

Hence we have, $x+y+z=9$

$-3y+2z=-1$

$$11z = 44$$
$$z = 4$$
$$-3y + 8 = -1, \quad y = 3 \quad \text{Also} \quad x = 2$$

Thus, $x = 2$, $y = 3$, $z = 4$ is the required sol^n

2> solve by Gauss Elimination method

$$2x + y + 4z = 12$$
$$4x + 11y - z = 33$$
$$8x - 3y + 2z = 20$$

sol^n.

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 4 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 9 & -9 & 9 \\ 0 & -7 & -14 & -28 \end{array}\right]$$

$1/9\, R_2$, $-1/7\, R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 2 & 4 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$[A:B] \sim \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 3 & 3 \end{array}\right]$$

Hence we have,

$$2x + y + 4z = 12$$
$$y - z = 1$$
$$3z = 3$$

$\therefore z = 1$, $y = 2$ and $x = 3$

3> solve by Gauss Elimination method.

$$x_1 - 2x_2 + 3x_3 = 2$$
$$3x_1 - x_2 + 4x_3 = 4$$
$$2x_1 + x_2 - 2x_3 = 5$$

$$[A:B] = \begin{bmatrix} 1 & -2 & 3 & : & 2 \\ 3 & -1 & 4 & : & 4 \\ 2 & 1 & -2 & : & 5 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$$[A:B] \sim \begin{bmatrix} 1 & -2 & 3 & : & 2 \\ 0 & 5 & -5 & : & -2 \\ 0 & 5 & -8 & : & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$[A:B] \sim \begin{bmatrix} 1 & -2 & 3 & : & 2 \\ 0 & 5 & -5 & : & -2 \\ 0 & 0 & -3 & : & 3 \end{bmatrix}$$

Hence we have, $x_1 - 2x_2 + 3x_3 = 2$

$5x_2 - 5x_3 = -2$

$-3x_3 = 3$

$x_3 = -1$

$5x_2 + 5 = -2$, $x_2 = -7/5 = -1.4$

$x_1 - 2(-1.4) - 3 = 2$, $x_1 = 2.2$

4> Apply Gauss-Elimination method

$2x + 5y + 7z = 52$

$2x + y - z = 0$

$x + y + z = 9$

sol^n: $[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 2 & 1 & -1 & : & 0 \\ 2 & 5 & 7 & : & 52 \end{bmatrix}$ $R_1 \leftrightarrow R_3$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 2R_1$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 3 & 5 & : & 34 \end{bmatrix}$$

$R_3 \to R_3 + 3R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 0 & -4 & : & -20 \end{bmatrix}$$

Hence we have, $x + y + z = 9$

$-y - 3z = -18$

$-4z = -20$

$z = 5, \quad y = 3, \quad x = 1$

Gauss - Jordan Elimination method

1) Solve the following system of Eq^ns by Gauss - Jordan method.

$x + y + z = 9$

$x - 2y + 3z = 8$

$2x + y - z = 3$

Sol^n: $[A : B] = \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 1 & -2 & 3 & : & 8 \\ 2 & 1 & -1 & : & 3 \end{bmatrix}$

$R_2 \to R_2 - R_1, \quad R_3 \to R_2 - 2R_1$

$[A : B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & -1 & -3 & : & -15 \end{bmatrix}$

$R_1 \to 3R_1 + R_2, \quad R_3 \to R_2 - 3R_3$

$[A : B] \sim \begin{bmatrix} 3 & 0 & 5 & : & 26 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 11 & : & 44 \end{bmatrix}$

$R_3 \to R_3 / 11$

$[A : B] \sim \begin{bmatrix} 3 & 0 & 5 & : & 26 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$

$R_1 \to R_1 - 5R_3, \quad R_2 \to R_2 - 2R_3$

$[A : B] \sim \begin{bmatrix} 3 & 0 & 0 & : & 6 \\ 0 & -3 & 0 & : & -9 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$

Hence, we have $3x = 6, \ -3y = -9, \ z = 4$

Thus, $x = 2, \ y = 3, \ z = 4$

2) Apply Gauss - Jordan method to solve the system of Eq^ns

$2x + 5y + 7z = 52$
$2x + y - z = 0$
$x + y + z = 9$

Sol^n: $[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 2 & 1 & -1 & : & 0 \\ 2 & 5 & 7 & : & 52 \end{bmatrix}$

$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - 2R_1$

$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 3 & 5 & : & 34 \end{bmatrix}$

$R_1 \to R_2 + R_1, \quad R_3 \to 3R_2 + R_3$

$[A:B] \sim \begin{bmatrix} 1 & 0 & -2 & : & -9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 0 & -4 & : & -20 \end{bmatrix}$

$R_3 \to -\frac{1}{4} R_3$

$\sim \begin{bmatrix} 1 & 0 & -2 & : & -9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$

$R_1 \to R_1 + 2R_3, \quad R_2 \to R_2 + 3R_3$

$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & -1 & 0 & : & -3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$

Hence we have $x = 1$, $\cancel{-}y = \cancel{-}3$, $z = 5$

3) Apply Gauss-Jordan method to solve the following system of Equations.

$2x_1 + x_2 + 3x_3 = 1$
$4x_1 + 4x_2 + 7x_3 = 1$
$2x_1 + 5x_2 + 9x_3 = 3$

Sol^n. $[A:B] = \begin{bmatrix} 2 & 1 & 3 & : & 1 \\ 4 & 4 & 7 & : & 1 \\ 2 & 5 & 9 & : & 3 \end{bmatrix}$

$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc:c} 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 4 & 6 & 2 \end{array}\right]$$

$R_1 \rightarrow 2R_1 - R_2$, $R_3 \rightarrow R_3 - 2R_2$

$$\sim \left[\begin{array}{ccc:c} 4 & 0 & 5 & 3 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 4 & 4 \end{array}\right]$$

$R_3 \longrightarrow \frac{1}{4} R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 4 & 0 & 5 & 3 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_3$, $R_1 \rightarrow R_1 - 5R_3$

$$\sim \left[\begin{array}{ccc:c} 4 & 0 & 0 & -2 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Hence we have

$4x_1 = -2$, $2x_2 = -2$, $x_3 = 1$

Thus $x_1 = -0.5$, $x_2 = -1$, $x_3 = 1$

Gauss-seidel method :⇒

1) solve the following system of Eqⁿs by Gauss-seidel method

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

solⁿ. The given system of Eqⁿs are diagonally dominant and the Eqⁿs are put in the following form.

$$x = \frac{1}{10}[12 - y - z] \quad \text{——} (1)$$

$$y = \frac{1}{10}[12 - x - z] \quad \text{——} (2)$$

$$z = \frac{1}{10}[12 - x - y] \quad \text{——} (3)$$

Let us start with the trial solⁿ $x = 0, y = 0, z = 0$

First iteration :=>

$$x^{(1)} = \frac{1}{10}[12-0-0] = 1.2$$

$$y^{(1)} = \frac{1}{10}[12-1.2-0] = 1.08$$

$$z^{(1)} = \frac{1}{10}[12-1.2-1.08] = 0.972$$

Second iteration :=>

$$x^{(2)} = \frac{1}{10}[12-1.08-0.972] = 0.9948$$

$$y^{(2)} = \frac{1}{10}[12-0.9948-0.972] = 1.00332$$

$$z^{(2)} = \frac{1}{10}[12-0.9948-1.00332] = 1.000188$$

Third iteration :=>

$$x^{(3)} = \frac{1}{10}[12-1.00332-1.000188] = 0.99965$$

$$y^{(3)} = \frac{1}{10}[12-0.99965-1.000188] = 1.00002$$

$$z^{(3)} = \frac{1}{10}[12-0.99965-1.00002] = 1.00003$$

Fourth iteration :>

$$x^{(4)} = \frac{1}{10}[12-1.00002-1.00003] = 0.999995 = 1$$

$$y^{(4)} = \frac{1}{10}[12-1-1.00003] = 0.999997 \approx 1$$

$$z^{(4)} = \frac{1}{10}[12-1-1] = 1$$

Then $x = 1,\ y = 1,\ z = 1$

24 Solve the following system of eq^{n}s by Gauss-Seidel method to obtain the final sol^n correct to three places of decimals.

$x + y + 54z = 110$

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

Sol^n: The given system of eq^{n}s is not diagonally dominant and hence we have to first rearrange the given system of eq^{n}s as follows.

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

These eq^{n}s are now written as

$$x = \frac{1}{27}[85 - 6y + z] \quad \text{—— (1)}$$

$$y = \frac{1}{15}[72 - 6x - 2z] \quad \text{—— (2)}$$

$$z = \frac{1}{54}[110 - x - y] \quad \text{—— (3)}$$

1st iteration :→

we start with trial sol^n $x=0, y=0, z=0$

$$x^{(1)} = \frac{1}{27}[85 - 0 + 0] = 3.14815$$

$$y^{(1)} = \frac{1}{15}[72 - 6(3.14815) - 0] = 3.54074$$

$$z^{(1)} = \frac{1}{54}[110 - 3.14815 - 3.54074] = 1.91317$$

2nd iteration :→

$$x^{(2)} = \frac{1}{27}[85 - 6(3.54074) + 1.91317] = 2.43218$$

$$y^{(2)} = \frac{1}{15}[72 - 6(2.43218) - 2(1.91317)] = 3.57204$$

$$z^{(2)} = \frac{1}{54}[110 - 2.43218 - 3.57204] = 1.92585$$

3rd iteration :→

$$x^{(3)} = \frac{1}{27}[85-6(3.57204)+1.92585] = 2.42569$$

$$y^{(3)} = \frac{1}{15}[72-6(2.42569)-2(1.92585)] = 3.57294$$

$$z^{(3)} = \frac{1}{54}[110-2.42569-3.57294] = 1.92595$$

4th iteration :→

$$x^{(4)} = \frac{1}{27}[85-6(3.57204)+1.92585] = 2.42549$$

$$y^{(4)} = \frac{1}{15}[72-6(2.42549)-2(1.92595)] = 3.57301$$

$$z^{(4)} = \frac{1}{54}[110-2.42549-3.57301] = 1.92595$$

Thus $x = 2.426$, $y = 3.573$, 1.926

3) solve by Gauss-seidel method.

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

sol^n:

$$x = \frac{1}{20}[17 - y + 2z]$$

$$y = \frac{1}{20}[-18 - 3x + z]$$

$$z = \frac{1}{20}[25 - 2x + 3y]$$

we start with the trial sol^n. $x=0$, $y=0$, $z=0$

1st iteration :

$$x^{(1)} = \frac{17}{20} = 0.85$$

$$y^{(1)} = \frac{1}{20}[-18-3(0.85)] = -1.0275$$

$$z^{(1)} = \frac{1}{20}[25-2(0.85)+3(-1.0275)] = 1.0109$$

second iteration :→

$$x^{(2)} = \frac{1}{20}[17-(-1.0275)+2(1.0109)] = 1.0025$$

$$y^{(2)} = \frac{1}{20}[-18-3(1.0025)+1.0109] = -0.9998$$

$$z^{(2)} = \frac{1}{20}[25-2(1.0025)+3(-0.9998)] = 0.9998$$

3rd iteration :→

$$x^{(3)} = \frac{1}{20}[17-(-0.9998)+2(0.9998)] = 0.99997 \approx 1$$

$$y^{(3)} = \frac{1}{20}[-18-3(0.99997)+0.9998] = -1.0000055 \approx -1$$

$$z^{(3)} = \frac{1}{20}[25-2(0.99997)+3(-1.0000055)] = 1.0000022 \approx 1$$

Thus $x = 1,\ y = -1,\ z = 1$

4> Apply Gauss-seidel iteration method to solve

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

~~com~~ Carryout 4 iterations taking the initial approxima-tion to the sol^ns $(1, 0, 3)$

sol^n:

$$x = \frac{1}{5}[12 - 2y - z]$$

$$y = \frac{1}{4}[15 - x - 2z]$$

$$z = \frac{1}{5}[20 - x - 2y]$$

By data $x^{(0)} = 1,\ y^{(0)} = 0,\ z^{(0)} = 3$

1st iteration :→

$$x^{(1)} = \frac{1}{5}[12 - 2(0) - 3] = 1.8$$

$$y^{(1)} = \frac{1}{4}[15 - 1.8 - 2(3)] = 1.8$$

$$z^{(1)} = \frac{1}{5}[20 - 1.8 - 2(1.8)] = 2.92$$

2nd iteration :⇒

$x^{(2)} = \frac{1}{5}[12 - 2(1.8) - 2.92] = 1.096$

$y^{(2)} = \frac{1}{4}[15 - 1.096 - 2(2.92)] = 2.016$

$z^{(2)} = \frac{1}{5}[20 - 1.096 - 2(2.016)] = 2.9744$

3rd iteration :⇒

$x^{(3)} = \frac{1}{5}[12 - 2(2.016) - 2.9744] = 0.99872$

$y^{(3)} = \frac{1}{4}[15 - 0.99872 - 2(2.9744)] = 2.01312$

$z^{(3)} = \frac{1}{5}[20 - 0.99872 - 2(2.01312)] = 2.995$

4th iteration :⇒

$x^{(4)} = \frac{1}{5}[12 - 2(2.01312) - 2.995] = 0.995752$

$y^{(4)} = \frac{1}{4}[15 - 0.995752 - 2(2.995)] = 2.003562$

$z^{(4)} = \frac{1}{5}[20 - 0.995752 - 2(2.003562)] = 2.9994248$

$x = 0.9958, \quad y = 2.0036, \quad z = 2.9994$

Eigen values and Eigen vectors of a square matrix :⇒

Defⁿ :⇒ Given a square matrix A, if there exists a scalar λ and a non-zero column matrix X such that $AX = \lambda X$, then λ is called an eigen value of A and X is called an eigen vector of A corresponding to an eigen value λ.

Rayleigh's power method :→

1) Find the largest eigen value and corresponding eigen vector of the matrix A by the power method given that

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Sol^n: Let $X^{(0)} = [1,0,0]'$ be the initial Eigen vector.

$$AX^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda^{(1)}.X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5\begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda^{(2)}.X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8\begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \lambda^{(3)}X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \begin{bmatrix} 2.93 \\ 0 \\ 2.86 \end{bmatrix} = 2.93\begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} = \lambda^{(4)}X^{(4)}$$

$$AX^{(4)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} = \begin{bmatrix} 2.98 \\ 0 \\ 2.96 \end{bmatrix} = 2.98\begin{bmatrix} 1 \\ 0 \\ 0.99 \end{bmatrix} = \lambda^{(5)}.X^{(5)}$$

$$AX^{(5)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.99 \end{bmatrix} = \begin{bmatrix} 2.99 \\ 0 \\ 2.98 \end{bmatrix} = 2.99\begin{bmatrix} 1 \\ 0 \\ 0.997 \end{bmatrix} = \lambda^{(6)}.X^{(6)}$$

$$AX^{(6)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.997 \end{bmatrix} = \begin{bmatrix} 2.997 \\ 0 \\ 2.994 \end{bmatrix} = 2.997\begin{bmatrix} 1 \\ 0 \\ 0.999 \end{bmatrix} = \lambda^{(7)}.X^{(7)}$$

Thus the largest eigen value is approximately 3 and the corresponding eigen vector is $[1,0,1]'$

2) Find the dominant eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ by power method taking the initial eigen vector as $[1,1,1]'$

Sol:- By data $x^{(0)} = [1,1,1]'$ is the initial eigen vector

$$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \lambda^{(1)}.x^{(1)}$$

$$Ax^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 7.37 \\ -2.67 \\ 4.01 \end{bmatrix} = 7.34\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \lambda^{(2)}.x^{(2)}$$

$$Ax^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 7.82 \\ -3.63 \\ 4.01 \end{bmatrix} = 7.82\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \lambda^{(3)}.x^{(3)}$$

$$Ax^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \begin{bmatrix} 7.94 \\ -3.89 \\ 3.99 \end{bmatrix} = 7.94\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \lambda^{(4)}.x^{(4)}$$

$$Ax^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 7.98 \\ -3.97 \\ 3.99 \end{bmatrix} = 7.98\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(5)}.x^{(5)}$$

$$Ax^{(5)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 4 \end{bmatrix} = 8\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(6)}.x^{(6)}$$

Thus the dominant eigen value is 8 and corresponding eigen vector is $[1, -0.5, 0.5]'$

3> Using Rayleigh's power method find numerically the largest eigen value and corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

sol^n: let $X^{(0)} = [1, 0, 0]'$ be the initial eigen vector

$$AX^{(0)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 25 \\ 1 \\ 2 \end{bmatrix} = 25\begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} = \lambda^{(1)}.X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 25.2 \\ 1.12 \\ 1.68 \end{bmatrix} = 25.2\begin{bmatrix} 1 \\ 0.04 \\ 0.07 \end{bmatrix} = \lambda^{(2)}.X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 0.04 \\ 0.07 \end{bmatrix} = \begin{bmatrix} 25.18 \\ 1.12 \\ 1.72 \end{bmatrix} = 25.18\begin{bmatrix} 1 \\ 0.04 \\ 0.07 \end{bmatrix} = \lambda^{(3)}.X^{(3)}$$

we obrewe that $X^{(2)} = X^{(3)}$

4> Find the numerically largest eigen value and the correspo-nding eigen vector of the matrix

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

by taking the initial approximation to the eigen vector as $[1, 0.8, -0.8]^T$ perform 5 iterations

sol^n: By data $X^{(0)} = [1, 0.8, -0.8]^T$ is the initial eigen vector

Hence we have

$$AX^{(0)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix} = \begin{bmatrix} 5.6 \\ 5.2 \\ -5.2 \end{bmatrix} = 5.6\begin{bmatrix} 1 \\ 0.93 \\ -0.93 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0.93 \\ -0.93 \end{bmatrix} = \begin{bmatrix} 5.86 \\ 5.72 \\ -5.72 \end{bmatrix} = 5.86\begin{bmatrix} 1 \\ 0.98 \\ -0.98 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.98 \\ -0.98 \end{bmatrix} = \begin{bmatrix} 5.96 \\ 5.92 \\ -5.92 \end{bmatrix} = 5.96 \begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix} = \begin{bmatrix} 5.98 \\ 5.96 \\ -5.96 \end{bmatrix} = 5.98 \begin{bmatrix} 1 \\ 0.997 \\ -0.997 \end{bmatrix} = \lambda^{(4)} X^{(4)}$$

$$AX^{(4)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.997 \\ -0.997 \end{bmatrix} = \begin{bmatrix} 5.994 \\ 5.988 \\ -5.988 \end{bmatrix} = 5.994 \begin{bmatrix} 1 \\ 0.999 \\ -0.999 \end{bmatrix} = \lambda^{(5)} X^{(5)}$$

Thus after 5 iterations are numerically largen eigen value is 5.994 and corresponding eigen vector is $[1, 0.999, -0.999]'$

5) Find the largest eigen value and the corresponding Eigen vector of the matrix A by using the power method by taking initial vector as $[1, 1, 1]'$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Sol: By data $X^{(0)} = [1, 1, 1]^T$ Hence we have

$$AX^{(0)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix} = 4 \begin{bmatrix} 0.75 \\ -1 \\ 0.75 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.75 \\ -1 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 2.5 \\ -3.5 \\ 2.5 \end{bmatrix} = 3.5 \begin{bmatrix} 0.71 \\ -1 \\ 0.71 \end{bmatrix} = \lambda^{(4)} X^{(4)}$$

$$AX^{(4)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.71 \\ -1 \\ 0.71 \end{bmatrix} = \begin{bmatrix} 2.42 \\ -3.42 \\ 2.42 \end{bmatrix} = 3.42 \begin{bmatrix} 0.708 \\ -1 \\ 0.708 \end{bmatrix} = \lambda^{(5)} X^{(5)}$$

$$Ax^{(5)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.708 \\ -1 \\ 0.708 \end{bmatrix} = \begin{bmatrix} 2.416 \\ -3.416 \\ 2.416 \end{bmatrix} = 3.416 \begin{bmatrix} 0.7073 \\ -1 \\ 0.7073 \end{bmatrix} = \lambda^{(6)} x^{(6)}$$

$$Ax^{(6)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.7073 \\ -1 \\ 0.7073 \end{bmatrix} = \begin{bmatrix} 2.4146 \\ -3.4146 \\ 2.4146 \end{bmatrix} = 3.4146 \begin{bmatrix} 0.7071 \\ -1 \\ 0.7071 \end{bmatrix} = \lambda^{(7)} x^{(7)}$$

Hence the largest eigen value is 3.4146 and the corresponding eigen vector is $[0.7071, -1, 0.7071]'$

6) $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ take $x_0 = [1, 0, 0]'$

Ans : $\lambda = 4$, $[1, 0.5, 0]'$