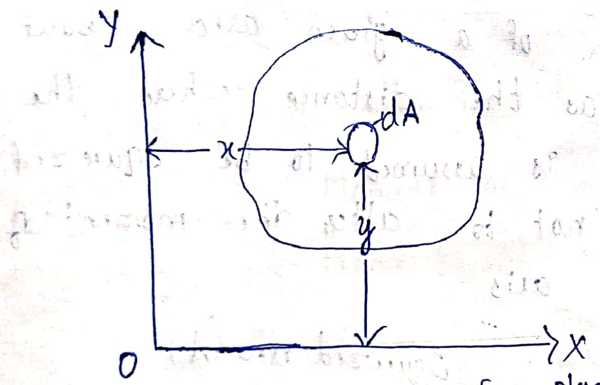
Moment of Inertia.

Syllabus:- Moment of inertia of an area, polar moment of inertia, Radius of gyration, ⊥r axis theorem and parallel axis theorem, Moment of inertia of □lar, circular, and Δlar areas from method of integration, Moment of inertia of composite areas, Numerical problems.

[Moment of inertia of an area is another important property of cross-sections of the members like beam column and many mechanical components. Determination of moment of inertia of the plane areas is of great importance in the study of subjects like strength of materials, structural designs, steel structures, and machine designs.]

Moment of inertia of an area (or) plane figure :-



of a plane fig.

Moment of inertia is the resistance offered to change in rotational motion of an area about an axis.

Consider a plane area as shown in fig. dA is an elemental area with co-ordinates x & y.

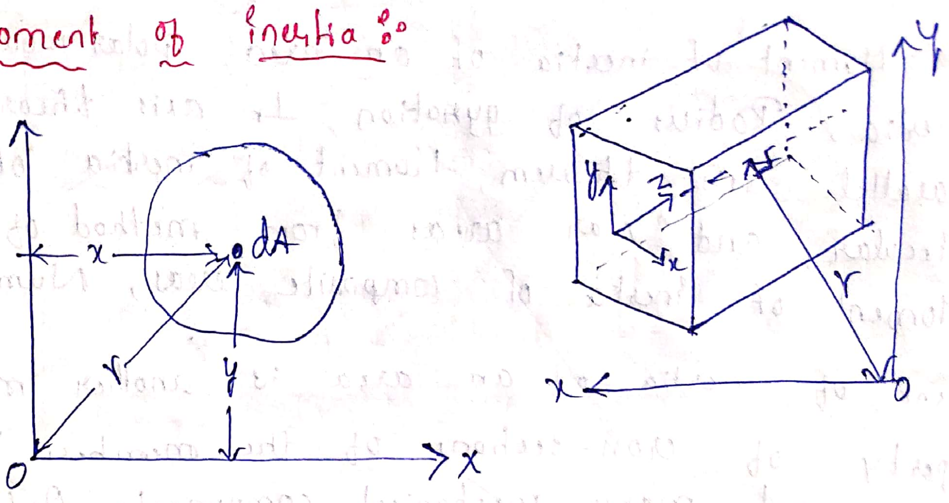
Moment of elemental area about x-axis (dA.y) = First moment of inertia. If first moment of inertia is again multiplied by ⊥r distance y, which gives dAy^2 is known as second moment of area (or) moment of inertia (or) moment of moment of area.

∴ Moment of inertia about x-axis = $I_{xx} = \sum y^2 dA$.

Moment of inertia about y-axis = $I_{yy} = \sum x^2 dA$

Unit of moment of inertia is denoted by mm^4 or cm^4 or m^4.

Polar moment of inertia :-

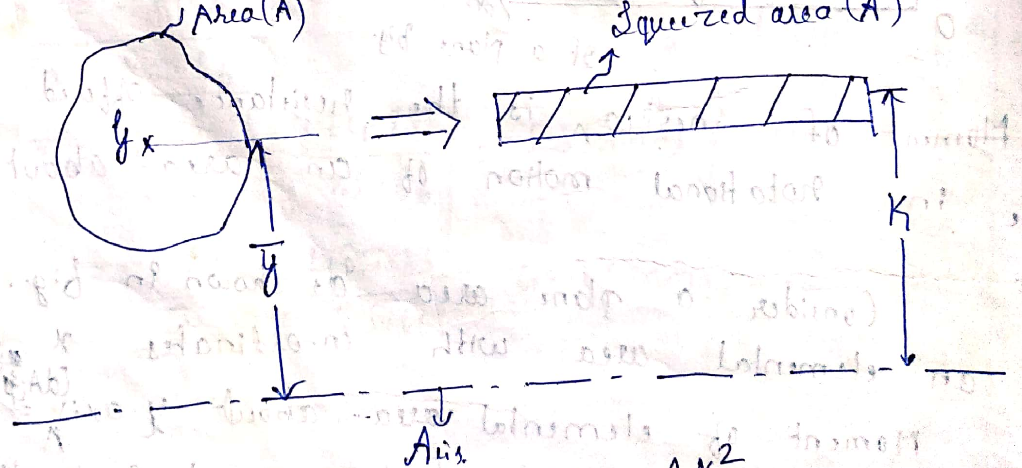


Moment of inertia about an axis ⊥r to the plane of an area is known as polar moment of inertia. It is noted by I_{zz}.

$$I_{zz} = \sum r^2 dA.$$

Radius of gyration :-

Radius of gyration of a given area, about an axis is defined as the distance where the whole area of the body is assumed to be squeezed and concentrated so as not to alter the moment of inertia about the given axis.



$$I = AK^2$$

Mathematically,

Radius of gyration $K = \sqrt{\frac{I}{A}}$

where I = Moment of Inertia
A = Area of body

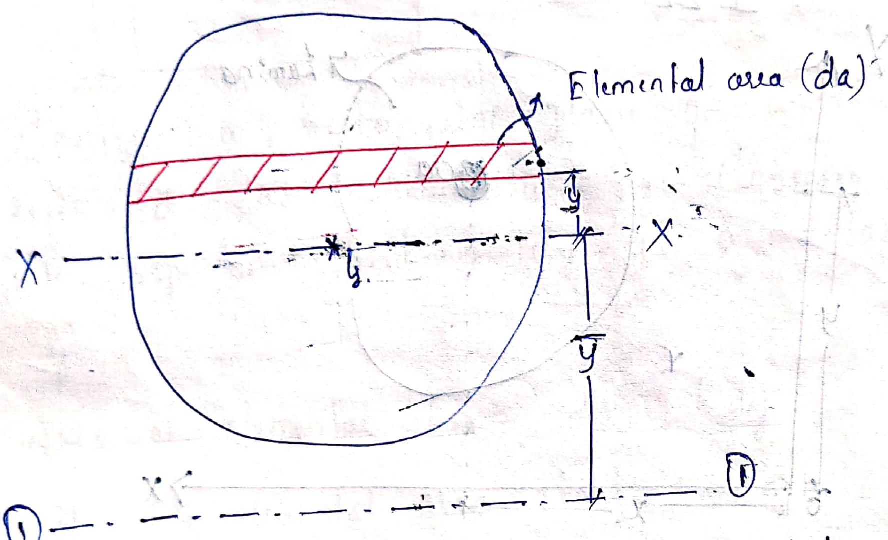
$$I = AK^2.$$

Theorems of moment of Inertia

1) Parallel axis theorem:

Statement: If the moment of inertia of a plane area about an axis through its C.G. is denoted by I_{xx}, then ~~M.I.~~ moment of the area about any reference axis ①-① parallel to x-axis and at a distance $\bar{y}$ from C.G. is given by

$$I_{1-1} = I_{xx} + A\bar{y}^2$$



Consider an elemental area da parallel to XX axis

Let I_{xx} - Moment of inertia about X-X axis.

I_{1-1} - Moment of inertia about reference axis ①-①.

A - Area of body.

$\bar{y}$ - Distance of C.G. of body from ①-①

[In fig. I_{xx}, I_{1-1} and elemental area are parallel to each other. Hence it is called parallel axis theorem]

Moment of Inertia of elemental area about ①-① $= \Sigma da\,(y+\bar{y})^2$

$$= \Sigma da\,(y^2+\bar{y}^2+2y\bar{y})$$

$$= \Sigma da\,y^2 + \Sigma da\,\bar{y}^2 + 2\Sigma da\,y\bar{y}$$

$$= \Sigma da\,\bar{y}^2 + \Sigma da\,y^2 + 2\bar{y}\,\Sigma da\,y$$

$$= \bar{y}^2\,\Sigma da + \Sigma da\,y^2 + 2\bar{y}\,\Sigma da\,y.$$

Now, $\Sigma da = A$

$\Sigma da\,y^2$ = M.I. about XX = I_{xx}

$\Sigma da\,y = 0$ (i.e. Moment of areas about centroid = 0)

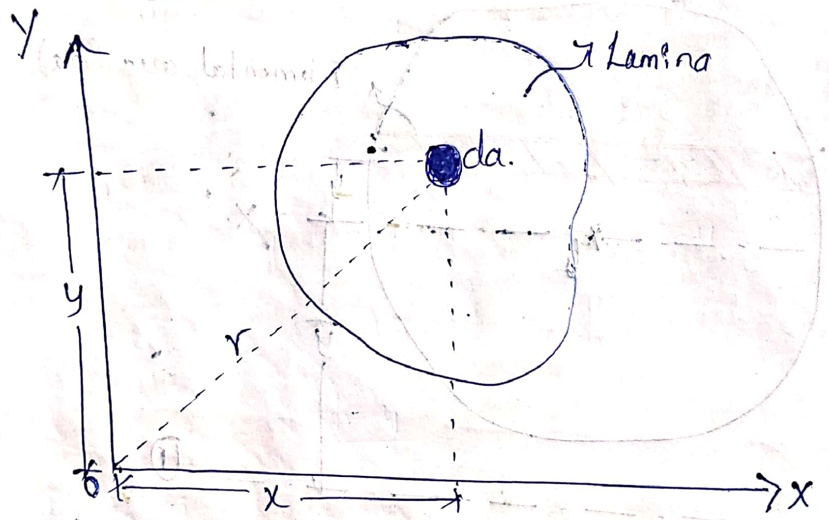
$\therefore \quad I_{1-1} = A\bar{y}^2 + I_{xx} + 0$

(or) $\boxed{I_{1-1} = I_{xx} + A\bar{y}^2}$

2.} Perpendicular axis theorem :-

Statement :- If I_{xx} and I_{yy} be the moment of inertia of the area about XX and YY axes respectively then moment of inertia about ZZ axis is given by

$$I_{zz} = I_{xx} + I_{yy}$$



Consider an elemental area 'da' at a distance 'r' from 'O'.

Moment of Inertia of element about Z-Z axis = $da\, r^2$

M.I. of whole area $I_{zz} = \int da\, r^2$

From fig. $r^2 = x^2 + y^2$

$$= \int da\,(x^2 + y^2)$$

$$I_{zz} = \int da\,x^2 + \int da\,y^2 \longrightarrow (1)$$

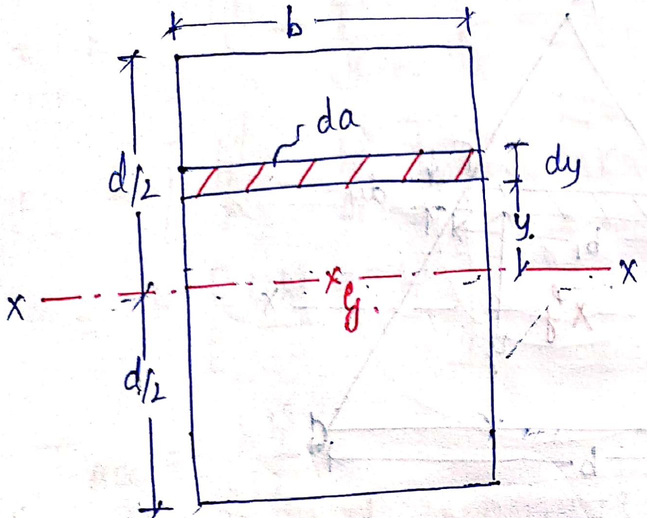
W.K.T,
$$\left.\begin{aligned} \int da\,x^2 &= I_{yy} \\ \int da\,y^2 &= I_{xx} \end{aligned}\right] \longrightarrow (2)$$

From (1) & (2)

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

M.I. of Regular Figures :

Moment of Inertia of ▭le about centroidal axis :



Consider a rectangular plane of width 'b' and depth 'd' as shown in fig. Consider an elemental strip of thickness dy at a distance 'y' from 'g'.

Area of elemental strip = da = b dy.

M.I. of this elemental area about xx axis =

$= da\, y^2 = (b\,dy)\, y^2 = by^2\, dy$

M.I. of whole rectangle $I_{xx} = \int_{-d/2}^{d/2} y^2 b\, dy$

$= b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$

$= \frac{b}{3}\left[\frac{d^3}{8} + \frac{d^3}{8}\right]$

$= \frac{b}{3}\left[\frac{2d^3}{8_4}\right]$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

Similarly, $\boxed{I_{yy} = \frac{db^3}{12}}$

M.I. of Triangle :-

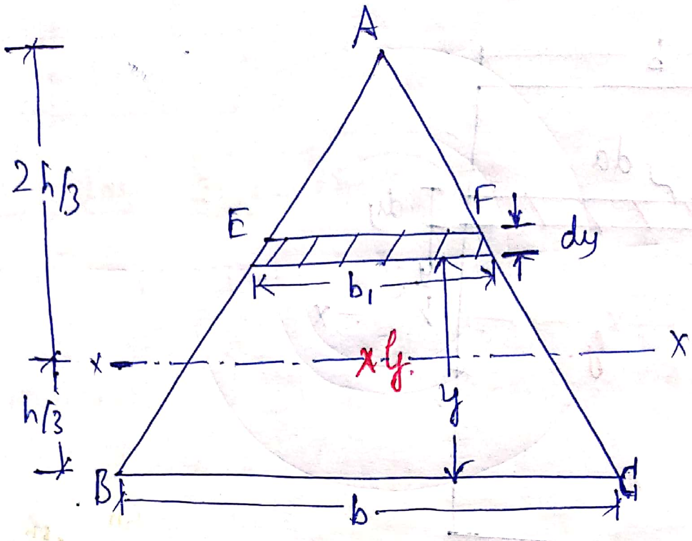


Fig. shows a triangle ABC of base 'b' and height 'h'. Consider an elemental strip EF of thickness 'dy' at a distance 'y' from base 'BC'

ΔAEF and ΔABC are similar Δles.

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \left(\frac{h-y}{h}\right) b$$

$$b_1 = \left(1 - \frac{y}{h}\right) b$$

M.I. of elemental strip about base ABC $= da\, y^2$

$$= (b_1\, dy)\, y^2$$

$$= \left\{\left(1-\frac{y}{h}\right) b\, dy\right\} y^2$$

$$= \left(y^2 - \frac{y^3}{h}\right) b\, dy$$

$\therefore$ M.I. of whole Δle about base ABC =

$$I_{AB} = \int_0^h \left(y^2 - \frac{y^3}{h}\right) b\, dy$$

$$= b \int_0^h \left(y^2 - \frac{y^3}{h}\right) dy.$$

$$= b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h$$

$$= b\left[\frac{h^3}{3} - \frac{h^4}{4h}\right]$$

$$= b\left[\frac{h^3}{3} - \frac{h^3}{4}\right]$$

$$= b\left[\frac{4h^3 - 3h^3}{12}\right]$$

$$I_{BC} = \frac{bh^3}{12}$$

M.I. about base $\boxed{I_{Base} = \frac{bh^3}{12}}$ $\frac{bh^3}{12} \times \frac{1}{2} \times bh$

NOTE :- To find M.I. about centroid G

W.K.T.,

$$I_{B-C} = I_{xx} + A\bar{y}^2$$

$$\frac{bh^3}{12} = I_{xx} + \left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2$$

$$I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18}$$

$$\boxed{I_{xx} = \frac{bh^3}{36}}$$

$|||^{ly}$ $I_{yy} = \frac{hb^3}{36}$

M.I. of circle :-

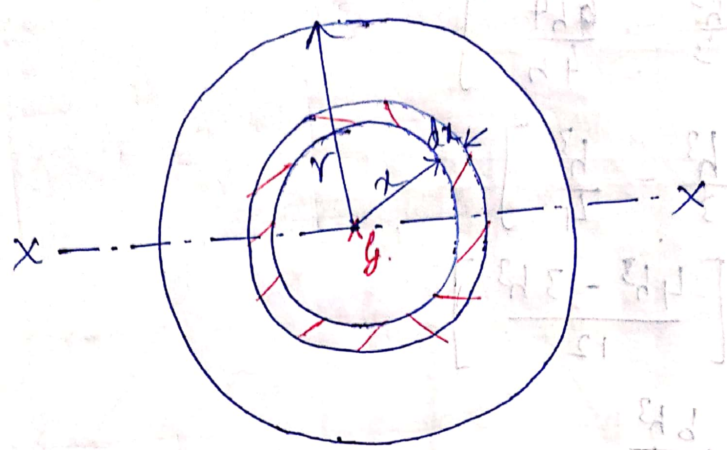


Fig. shows a circular plane of radius r. Consider an elemental strip of thickness 'dx' at a distance 'x' from the centre.

Area of elemental ring $da = (2\pi x)dx$

M.I. of this elemental strip from centre $= da\, x^2$

$$= (2\pi x\, dx)\, x^2$$

$$= 2\pi x^3 dx$$

$\therefore$ Moment of inertia of whole circle from centre $= I_{zz} = \int_0^r 2\pi x^3 dx$

$$= 2\pi \left[\frac{x^4}{4}\right]_0^r$$

$$= 2\pi \frac{r^4}{4}$$

$$\boxed{I_{zz} = \frac{\pi r^4}{2}} \rightarrow (1)$$

W.K.T.,

$$I_{zz} = I_{xx} + I_{yy}$$

But for circular lamina $I_{xx} = I_{yy}$

$$\therefore I_{zz} = I_{xx} + I_{xx}$$

$$I_{zz} = 2 I_{xx} \rightarrow (2)$$

From ① and ②

$$2 I_{xx} = \frac{\pi r^4}{2}$$

$$I_{xx} = \frac{\pi r^4}{4}$$

$$\therefore \boxed{I_{xx} = I_{yy} = \frac{\pi r^4}{4}}$$

or $r = \frac{d}{2}$

$$I_{xx} = I_{yy} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4$$

$$\boxed{I_{xx} = I_{yy} = \frac{\pi d^4}{64}}$$

M.I. of Semicircle :-

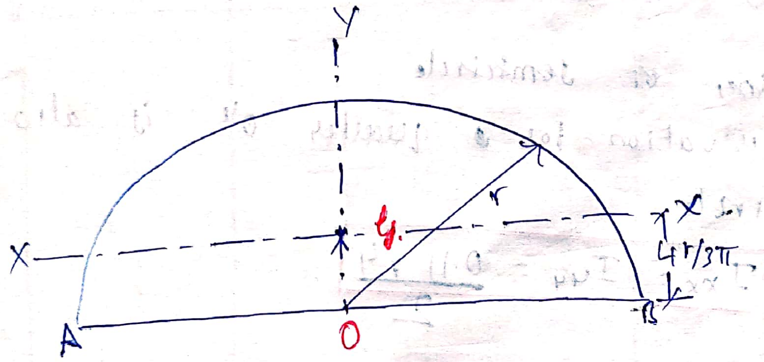


Fig. shows a semicircle of radius 'r'. The moment of Inertia of the semicircle about dia AB = $\frac{1}{2}$ M.I. of circle

$$\therefore I_{AB} = \frac{1}{2}\left(\frac{\pi r^4}{4}\right)$$

$$I_{AB} = \frac{\pi r^4}{8}$$

To find M.I. about centroidal axis W.K.T.

$$I_{AB} = I_{xx} + A\bar{y}^2$$

$$\frac{\pi r^4}{8} = I_{xx} + \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2$$

$$\frac{\pi r^4}{8} = I_{xx} + \frac{\cancel{\pi} r^2}{\cancel{2}}\left(\frac{\overset{8}{\cancel{16}} r^2}{9\pi^{\cancel{2}}}\right)$$

$$I_{xx} = \frac{\pi r^4}{8} - \frac{8 r^4}{9\pi} = 0.392 r^4 - 0.283 r^4$$

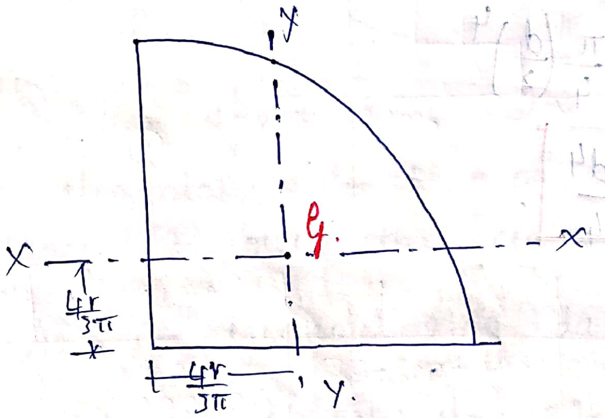
$$\boxed{I_{xx} = 0.11 r^4}$$

$$I_{yy} = \frac{1}{2} \times \text{M.I. of circle}$$

$$= \frac{1}{2} \times \frac{\pi r^4}{4}$$

$$\boxed{I_{yy} = \frac{\pi r^4}{8}}$$

M.I. of quarter circle :-

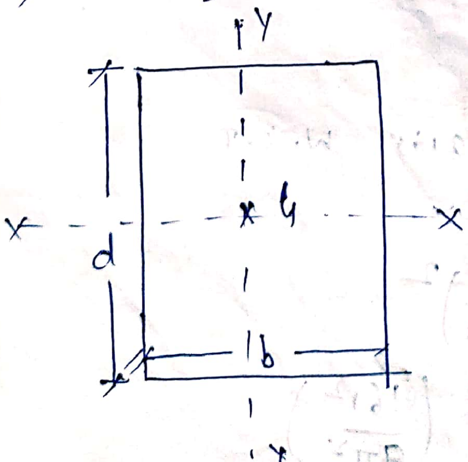


On the lines of semicircle

Derivation for quarter c'le is also same as for semicircle,

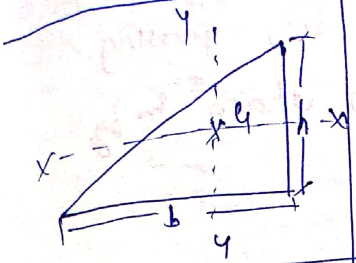
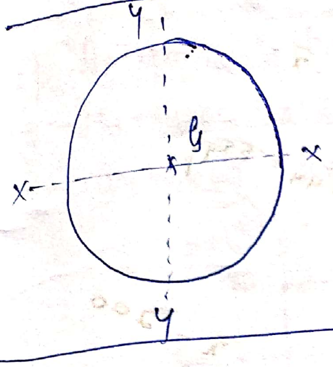
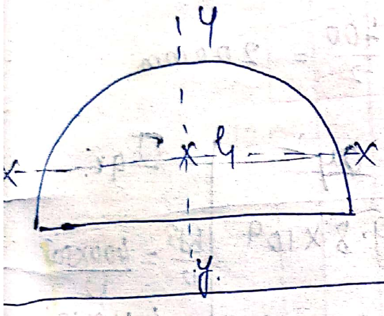
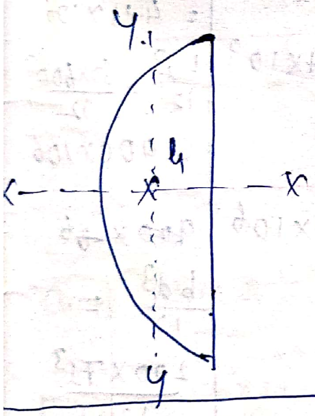
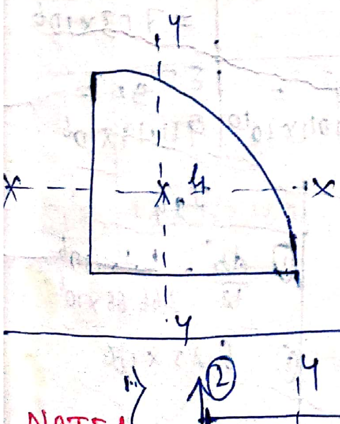
thus $\boxed{I_{xx} = I_{yy} = \frac{0.11\, r^4}{2}}$

List of formulae :-

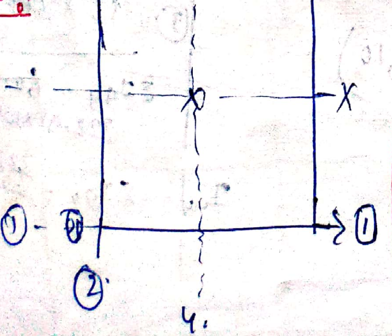
Component	I_{gx}	I_{gy}
1) Rectangle 	$\frac{bd^3}{12}$	$\frac{db^3}{12}$

NOTE 2) If the fig. is symmetric about y-y : $I_{yy} = \Sigma I_{gy}$

If the fig. is — " — x-x :- $I_{xx} = \Sigma I_{gx}$

Component	I_{gx}	I_{gy}
	$\frac{bh^3}{36}$	$\frac{hb^3}{36}$
	$\frac{\pi r^4}{4}$ or $\frac{\pi d^4}{64}$	$\frac{\pi r^4}{4}$ or $\frac{\pi d^4}{64}$
	$0.11 r^4$	$\frac{\pi r^4}{8}$
	$\frac{\pi r^4}{8}$	$0.11 r^4$
	$\frac{0.11 r^4}{2}$	$\frac{0.11 r^4}{2}$

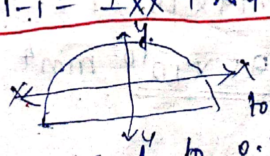
NOTE: i)



$$I_{1-1} = \Sigma I_{gx} + \Sigma a y^2 \qquad I_{2-2} = \Sigma I_{gy} + \Sigma a x^2$$

$$I_{1-1} = I_{xx} + A\bar{y}^2 \qquad I_{22} = I_{yy} + A\bar{x}^2$$

2



i) M.I of an axis ||led to the diametric axis is always equal to $0.11 r^4 = I_{xx}$

ii) M.I. of an axis ⊥ to diametric axis is always equal to $\frac{\pi r^4}{8} = I_{yy}$

Problems:-

1) Find the M.I. along the horizontal axis passing through the centroid of the section shown in fig.

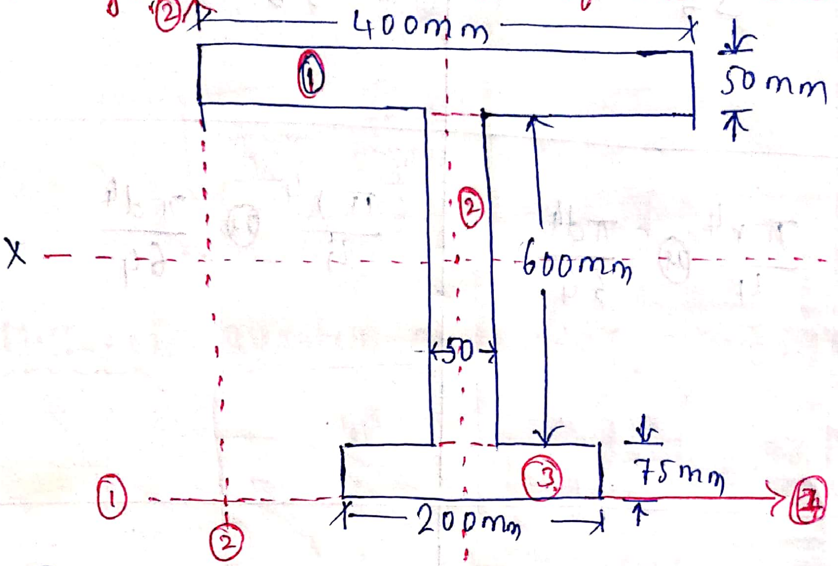


Fig. is symmetric about y - axis $\bar{x} = \frac{400}{2} = 200mm$

Components	Area(mm²)	y	ay	ay²	I_{gx}
□le ①	400×50 = 20000	$75+600+\frac{50}{2}$ = 700	14000000 = 14×10^6	9.8×10^9	$\frac{bd^3}{12} = \frac{400\times50^3}{12} = 4.16\times10^6$
□le ②	50×600 = 30000	$75+\frac{600}{2} = 375$	11.25×10^6	4.219×10^9	$\frac{bd^3}{12} = \frac{50\times600^3}{12} = 900\times10^6$
□le ③	200×75 = 15000	$\frac{75}{2} = 37.5$	562500	21.09×10^6	~~900×10⁶~~ $\frac{bd^3}{12} = \frac{200\times75^3}{12} = 7.03\times10^6$
	Σa = 65000		Σay = 25.81×10^6	Σay² = 1.404×10^{10}	ΣI_{gx} = 911.19×10^6

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{25.81\times10^6}{65000} = 397.07\ mm$$

$$I_{1-1} = \Sigma I_{gx} + \Sigma ay^2$$
$$= (911.19\times10^6) + (1.404\times10^{10})$$
$$= \underline{1.495\times10^{10}\ mm^4}$$

I_{gy}
① $\frac{db^3}{12}$ = ~~4.16×10⁶~~ 266.66×10^6
② 6.25×10^6
③ 50×10^6

ΣI_{gy} = ~~60.41×10⁶~~ $322.91\times10^6\ mm^4$

$I_{yy} = \Sigma I_{gy} = \frac{322.91\times10^6}{}$ ~~60.41×10⁶~~ mm^4

$\bar{x} =$

$I_{yy} = \Sigma I_{gy}$

$\bar{x} = 200$

W.K.T.,

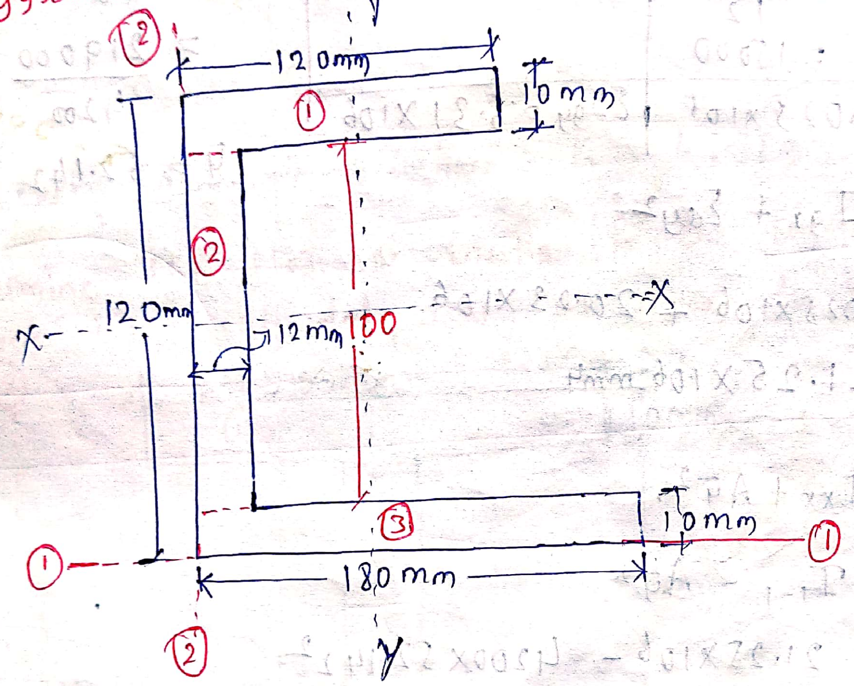
$$I_{1-1} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{1-1} - A\bar{y}^2$$

$$= 1.495\times10^{10} - (65000 \times 397.07^2)$$

$$\boxed{I_{xx} = 4.70\times10^{9}\ mm^4}$$

2) Feb 2003 Determine the M.I. of the ~~unequal~~ section about its centroidal axes. Calculate the least radius of gyration for the section as well.



Comp	Area (a) mm^2	x	y	ax	ay	ax^2	ay^2
1	$bd = 120\times10 = 1200$	$\frac{120}{2} = 60$	$10+100+\frac{10}{2} = 115$	72000	138×10^3	4.32×10^6	15.87×10^6
2	$bd = 12\times100 = 1200$	$\frac{12}{2} = 6$	$10+\frac{100}{2} = 60$	7200	72000	43200	4.32×10^6
3	$bd = 180\times10 = 1800$	$\frac{b}{2} = \frac{180}{2} = 90$	$\frac{10}{2} = 5$	162×10^3	9000	14.58×10^6	45000
$\Sigma a = 4200$				$\Sigma ax = 241200$	$\Sigma ay = 219000$	$\Sigma ax^2 = 18.94\times10^6$	$\Sigma ay^2 = 20.23\times10^6$

Component	I_{gx}	I_{gy}
$\square^{le}$ ①	$\frac{bd^3}{12} = \frac{120 \times 10^3}{12} = 10000$	1.44×10^6
$\square^{le}$ ②	$\frac{bd^3}{12} = \frac{12 \times 100^3}{12} = 1 \times 10^6$	14400
$\square^{le}$ ③	$\frac{bd^3}{12} = \frac{180 \times 10^3}{12} = 15000$	4.86×10^6
	$\Sigma I_{gx} = 1.025 \times 10^6$	$\Sigma I_{gy} = 6.31 \times 10^6$

$\bar{x} = \frac{\Sigma ax}{\Sigma a}$

$= \frac{241200}{4200}$

$\bar{x} = 57.428\ mm$

$\bar{y} = \frac{\Sigma ay}{\Sigma a}$

$= \frac{219000}{4200}$

$\bar{y} = 52.142$

$I_{1-1} = I_{gx} + \Sigma ay^2$

$= 1.025 \times 10^6 + 20.23 \times 10^6$

$I_{1-1} = 21.25 \times 10^6\ mm^4$

$I_{1-1} = I_{xx} + A\bar{y}^2$

$I_{xx} = I_{1-1} - A\bar{y}^2$

$= 21.25 \times 10^6 - 4200 \times 52.142^2$

$\boxed{I_{xx} = 9.831 \times 10^6\ mm^4}$

$I_{2-2} = I_{gy} + \Sigma ax^2$

$= 6.31 \times 10^6 + 18.94 \times 10^6$

$I_{2-2} = 25.25 \times 10^6\ mm^4$

$I_{2-2} = I_{yy} + A\bar{x}^2$

$I_{yy} = I_{22} - A\bar{x}^2$

$= 25.25 \times 10^6 - 4200 \times 57.428^2$

$\boxed{I_{yy} = 11.398 \times 10^6\ mm^4}$

Radius of gyration (K):

$$K = \sqrt{\frac{I}{A}}$$

$$K_x = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{9.831 \times 10^6}{4200}}$$

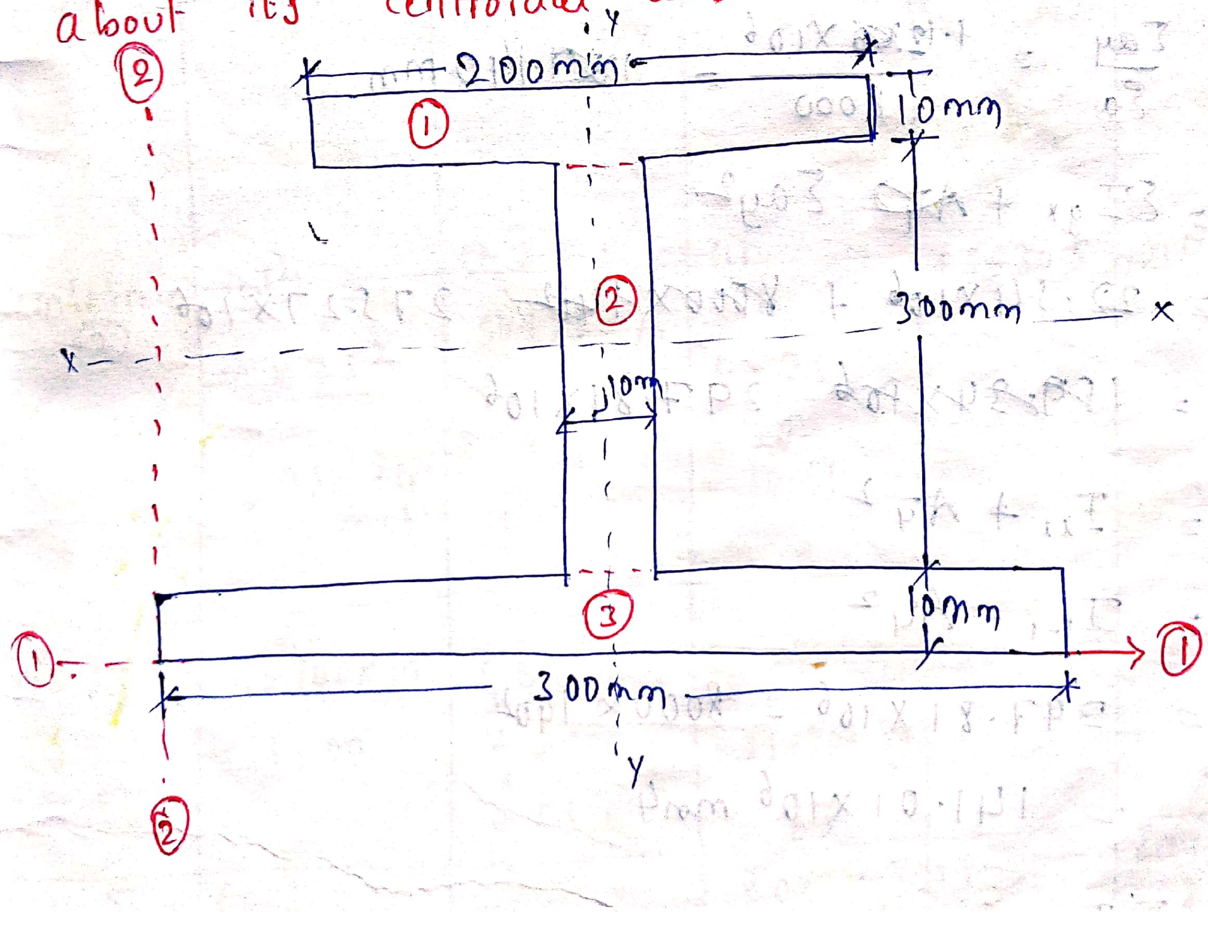
$$K_x = 48.38 \text{ mm}$$

$$K_y = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{11.398 \times 10^6}{4200}}$$

$$K_y = 52.09 \text{ mm}$$

Least radius of gyration = $K = 48.38$ mm.

3.5 Determine the M.I. of inertia of the unequal I-section about its centroidal axes as shown in fig.



Components	Area (a)	y	ay	ay^2	I_{gx}	I_{gy}
□le ①	$200 \times 10 = 2000$	$10+300+5 = 315$	630000	198.4×10^6	$\frac{bd^3}{12} = 16666.67$	$\frac{db^3}{12} = 6.67 \times 10^6$
□le ②.	$10 \times 300 = 3000$	$10+\frac{300}{2} = 160$	480000	76.8×10^6	22.5×10^6	25000
□le ③.	$300 \times 10 = 3000.$	$\frac{10}{2} = 5$	15000	75000	25000	22.5×10^6
$\Sigma a =$	8000		$\Sigma ay = \cancel{1.26 \times 10^6}$ 1.12×10^6	$\Sigma ay^2 = 275.27 \times 10^6$	$\Sigma I_{gx} = \cancel{2.54 \times 10^6}$ 22.54×10^6	$\Sigma I_{gy} = 29.19 \times 10^6$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{1.12 \times 10^6}{8000} = 140 \text{ mm}$$

$$I_{1-1} = \Sigma I_{gx} + \cancel{Ay^2}\ \Sigma ay^2$$

$$= 22.54 \times 10^6 + \cancel{8000 \times 140^2}\ 275.27 \times 10^6$$

$$I_{1-1} = \cancel{159.34 \times 10^6}\ 297.81 \times 10^6$$

$$I_{1-1} = I_{xx} + A\bar{y}^2$$

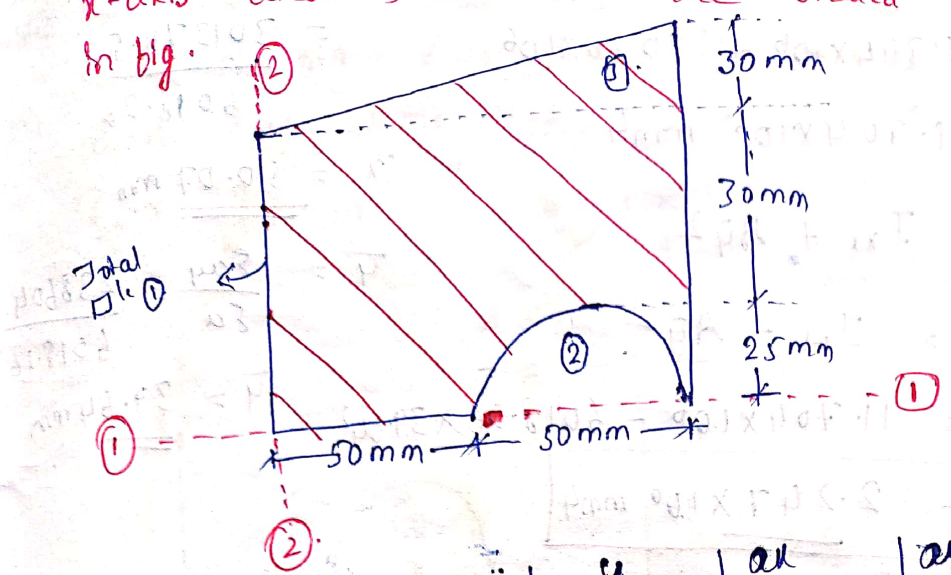
$$I_{xx} = I_{1-1} - A\bar{y}^2$$

$$= 297.81 \times 10^6 - 8000 \times 140^2$$

$$= 141.01 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \Sigma I_{gy} = 29.19 \times 10^6$$

4) Find the least radius of gyration about the x-axis and y-axis of the shaded area shown in fig.



Component	Area (a) mm^2	x	y	ax	ay	ax^2	ay^2
Total □le ①	$100 \times (55)$ 5500	50	$\frac{30+25}{2} = 27.5$	275000	151250	13.75×10^6	4.15×10^6
Semicircle ②	$\frac{\pi r^2}{2} = \frac{\pi \times 25^2}{2} = -981.74$ mm^2	$50 + r = 50+25=75$	$\frac{4r}{3\pi} = \frac{4 \times 25}{3\pi} = 10.61$	-73630.5	-10146	-5.52×10^6	-110516
Δle ③	$\frac{1}{2} \times 100 \times 30 = 1500$	$\frac{2}{3} b = \frac{2}{3} \times 100 = 66.67$	$25+30+\frac{h}{3} = 25+30+\frac{30}{3} = 65$	100005	97500	6.66×10^6	6.33×10^6
$\Sigma a = 6018.26$				$\Sigma ax = 301374.5$	$\Sigma ay = 238604$	$\Sigma ax^2 = 14.89 \times 10^6$	$\Sigma ay^2 = 10.36 \times 10^6$

Component	I_{gx}	I_{gy}
Total □le ①	$\frac{bd^3}{12} = \frac{100 \times 55^3}{12} = 1.38 \times 10^6$	$\frac{db^3}{12} = \frac{55 \times 100^3}{12} = 4.583 \times 10^6$
Semicircle	$-0.11r^4 = -42968.75$	$-\frac{\pi r^4}{8} = \frac{-\pi \times 25^4}{8} = \cancel{306796} = -153398$
Δle	$\frac{bh^3}{36} = \frac{100 \times 30^3}{36} = 75000$	$\frac{hb^3}{36} = \frac{30 \times 100^3}{36} = 833333.3$

$\Sigma I_{gx} = \cancel{1.337 \times 10^6}$ $\Sigma I_{gy} = \cancel{4.429 \times 10^6}$

$\Sigma I_{gx} = 1.344 \times 10^6$

$\Sigma I_{gy} = 5.26 \times 10^6$

$$I_{1-1} = \Sigma I_{yx} + \Sigma a y^2$$

$$= 1.344\times10^6 + 10.36\times10^6$$

$$I_{1-1} = 11.704\times10^6 \ mm^4$$

$$I_{1-1} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{1-1} - A\bar{y}^2$$

$$= 11.704\times10^6 - 6018.26\times39.64^2$$

$$\boxed{I_{xx} = 2.247\times10^6 \ mm^4}$$

$$\bar{x} = \frac{\Sigma a x}{\Sigma a} = \frac{301374.5}{6018.26}$$

$$\bar{x} = 50.07 \ mm$$

$$\bar{y} = \frac{\Sigma a y}{\Sigma a} = \frac{238604}{6018.26}$$

$$\bar{y} = 39.64 \ mm$$

$$I_{2-2} = \Sigma I_{gy} + \Sigma a x^2$$

$$= 5.26\times10^6 + \cancel{15.21}\ 14.89\times10^6$$

$$= 20.\cancel{47}\ 15\times10^6$$

$$I_{2-2} = I_{yy} + A\bar{x}^2$$

$$I_{yy} = I_{2-2} - A\bar{x}^2$$

$$= 20.\cancel{47}\ 15\times10^6 - 6018.26\times\cancel{39.6}\ 50.07^2$$

$$\boxed{I_{yy} = 5.\cancel{882}\ 06\times10^6 \ mm^4}$$

$$K_y = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{5.\cancel{88}\ 06 2\times10^6}{6018.26}}$$

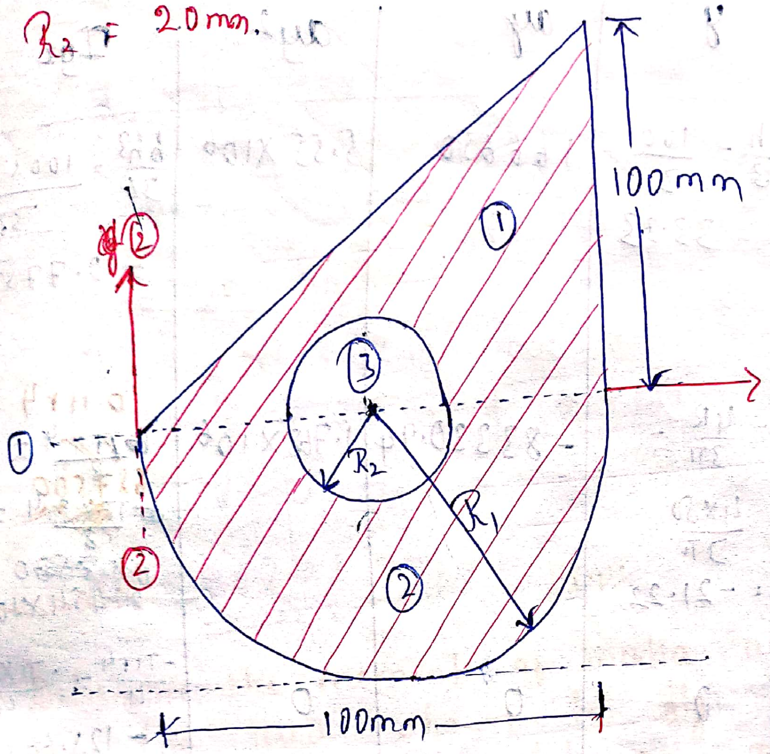
$$\boxed{K_y = 28.99 \ mm}$$

$$K_x = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{2.247\times10^6}{6018.26}}$$

$$\boxed{K_x = 19.32 \ mm}$$

$\therefore$ Least radius of gyration $= 19.32$ mm.

5.5 June/July 2011 (10M) Determine the second moment of area about horizontal centroidal axis for shaded area shown in fig. Also find the radius of gyration about the same axis. Take $R_1 = 50mm$ & $R_2 = 20mm$.



$R_1 = 50mm$, $R_2 = 20mm$.

Component	Area (a) mm^2	x (mm)	ax	ax^2	I_{gy}
Δ^{le} 1	$\frac{1}{2}bh = \frac{1}{2} \times 100 \times 100$ $= 5000$	$\frac{2}{3}b = \frac{2}{3} \times 100 = 66.67$	333350	22.22×10^6	$\frac{hb^3}{36} = \frac{100 \times 100^3}{36}$ $= 2.77 \times 10^6$
Semicircle 2	$\frac{\pi r^2}{2} = \frac{\pi \times 50^2}{2}$ $= 3927$	$r = 50$	196350	9.81×10^6	$\frac{\pi r^4}{8}$ ~~0.11r⁴~~ $= \frac{\pi \times 50^4}{8} =$ ~~687500~~ $= 2.454 \times 10^6$
Circle 3	$-\pi r^2 = -\pi \times 20^2$ $= -1256.63$	50	-62800	-3.14×10^6	$-\frac{\pi r^4}{4} = -\frac{\pi \times 20^4}{4}$ $= -125.66 \times 10^3$
$\Sigma a =$	7670.37	$\Sigma ax =$	466900	$\Sigma ax^2 = 28.9 \times 10^6$	$\Sigma I_{gy} = 3.33 \times 10^6$

$I_{\cancel{2}-\cancel{2}} = \Sigma A g_y + \Sigma ay^2$ C.G

$= \cancel{3.33 \times 10^6}$

Component	Area (a) mm^2	y	ay	ay^2	I_{gx}
Δ^{le} 1	$\frac{1}{2}bh = \frac{1}{2}\times 100\times 100$ $= 5000$	$\frac{h}{3} = \frac{100}{3}$ $= 33.33$	166650	5.55×10^6	$\frac{bh^3}{36} = \frac{100\times 100^3}{36}$ $= 2.77\times 10^6$
Semicircle 2	$\frac{\pi r^2}{2} = \frac{\pi\times 50^2}{2}$ $= 3927$	$-\frac{4R}{3\pi} = -\frac{4\times 50}{3\pi} = -21.22$	-83330.94	1.768×10^6	$0.11r^4$ $\cancel{\frac{\pi r^4}{8}} = \cancel{687509}$ $\cancel{\frac{\pi\times 50^4}{8}} =$ $\cancel{6827500}$ $\cancel{2.454\times 10^6}$
Circle 3	$-\pi r^2 = -\pi\times 20^2$ $= -1256.63$	0	0	0	$\frac{-\pi r^4}{4} = \frac{-\pi\times 20^4}{4}$ $= -125.66\times 10^3$
$\Sigma a =$	7670.37	$\Sigma ay =$	83319.06	$\Sigma ay^2 = 7.318\times 10^6$	$\Sigma I_{gx} = \cancel{8.098\times 10^6}$ 3.33×10^6

$I_{x-x} = \Sigma I_{gx} + \Sigma ay^2$

$= \cancel{8.098}\ 3.33 \times 10^6 + 7.318\times 10^6$

$= \cancel{12.41}\ 10.648 \times 10^6\ mm^4$

$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{83319.06}{7670.37}$

$\bar{y} = 10.86\ mm$

$I_{1-1} = I_{xx} + A\bar{y}^2$

$I_{xx} = I_{1-1} - A\bar{y}^2$

$= \cancel{12.41}\ 10.648 \times 10^6 - 7670.37\times 10.86^2$

$I_{x-x} = \cancel{11.505}\ 9.743 \times 10^6\ mm^4$

$K_x = \sqrt{\frac{I_{x-x}}{A}} = \sqrt{\frac{\cancel{11.505}\ 9.743\times 10^6}{7670.37}}$

$K_x = \cancel{38.72}\ mm$ $35.64\ mm$

$35.64\ mm$.

$$I_{2-2} = \Sigma I_{gy} + \Sigma a x^2$$

$$= 3.33\times 10^6 + 28.9\times 10^6$$

$$I_{2-2} = 32.23\times 10^6 \ mm^4$$

$$\bar{x} = \frac{\Sigma a x}{\Sigma a} = \frac{466900}{7670.37} = 60.87 mm$$

$$I_{2-2} = I_{yy} + A\bar{x}^2$$

$$I_{yy} = I_{2-2} - A\bar{x}^2$$

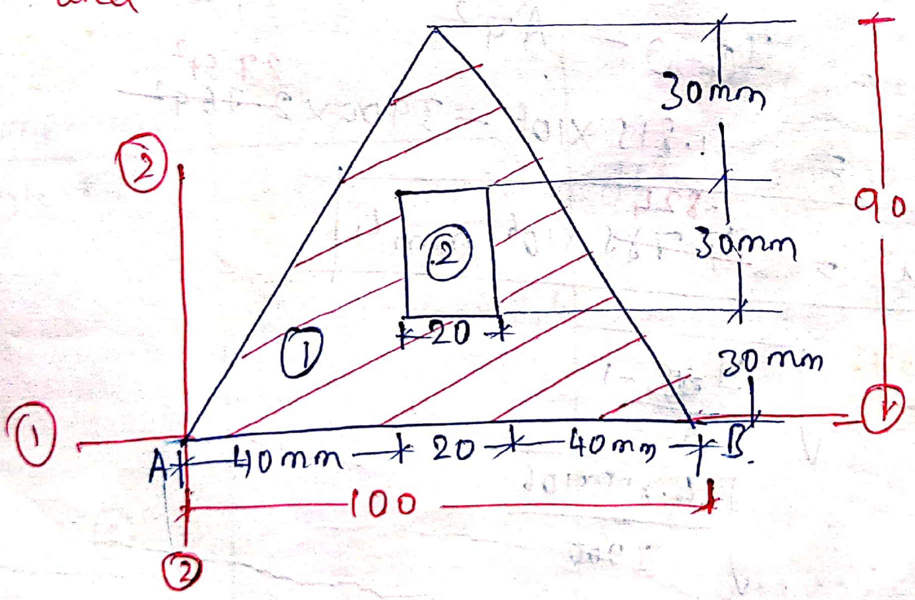
$$= 32.23\times 10^6 - 7670.37\times 60.87^2$$

$$I_{yy} = 3.81\times 10^6 \ mm^4$$

$$K_y = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{3.81\times 10^6}{7670.37}} =$$

$$K_y = 282.28 \ mm$$

6.) Determine the moment of inertia and radii of gyration of the area shown in fig. about the base AB and the centroidal axis parallel to AB. (DEC10) (12M)



Given data:

$$I_{1-1} = ?$$

$$I_{x-x} = ?$$

Component Area	Area (a) (mm^2)	y	ay	ay^2	I_{gx}
Total Δ^{le} ①	$\frac{1}{2}\times100\times90$ $=4500$	$\frac{h}{3}=\frac{90}{3}$ $=30.$	135000	4.05×10^6	$\cancel{\frac{bh^3}{12}}\rightarrow\frac{bh^3}{36}=$ $=\frac{100\times90^3}{36}$ $=2.025\times10^6$
$\square^{le}$ ②	-20×30 $=-600$	$30+\frac{d}{2}=$ $30+15$ $=45$	-27000	-1.215×10^6	$-\frac{bd^3}{12}=\frac{-20\times30^3}{12}$ $=-45000$
$\Sigma a=$ 3900		$\Sigma ay=$	108000	$\Sigma ay^2=2.835\times10^6$	$\Sigma I_{gx}=1.98\times10^6$

$$I_{①-①}=\Sigma I_{gx}+\Sigma ay^2$$

$$=1.98\times10^6+2.835\times10^6$$

$$\boxed{I_{①-①}=4.815\times10^6\ mm^4}$$

$$\bar{y}=\frac{\Sigma ay}{\Sigma a}$$

$$=\frac{108000}{3900}$$

$$=\cancel{2.769}\ mm$$

27.69

$$I_{①-①}=I_{xx}+A\bar{y}^2$$

$$I_{xx}=I_{①-①}-A\bar{y}^2$$

$$=4.815\times10^6-3900\times\cancel{2.769^2}\ 27.69^2$$

$$\boxed{I_{x-x}=\cancel{4.785}\ 1.824\times10^6\ mm^4}$$

$$K_{1-1}=\sqrt{\frac{I_{1-1}}{A}}$$

$$=\sqrt{\frac{4.815\times10^6}{3900}}$$

$$\boxed{K_{1-1}=35.13\ mm}$$

$$K_{x-x}=\sqrt{\frac{I_{xx}}{A}}=\sqrt{\frac{\cancel{4.785}\ 1.824\times10^6}{3900}}$$

21.62 mm.

$$\boxed{K_x=\cancel{35.02}\ mm}$$

Q.8. Find the polar moment of inertia of the section shown in fig. about an axis passing through its centroid and hence find polar radius of gyration.

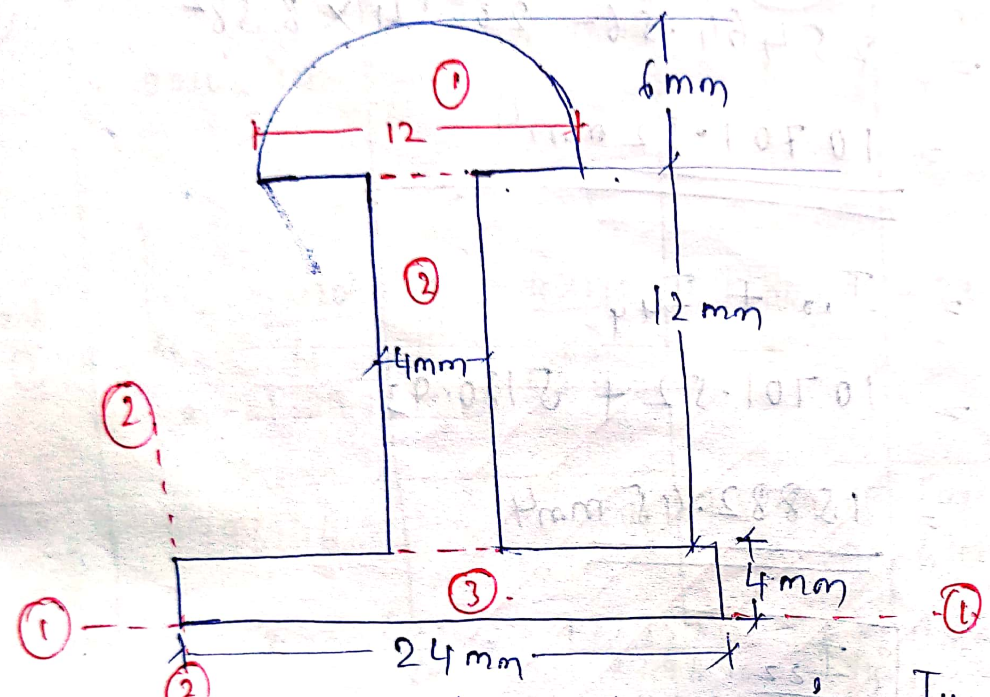


Fig. is symmetric about y-axis ∴ $I_{yy} = \Sigma I_{gy}$

Component	Area (a) mm^2	y	ay	ay^2	I_{gx}	I_{gy}
Δ^{le} ①	$\frac{\pi r^2}{2} = \frac{\pi \times 6^2}{2} = 56.54$	$4 + 12 + \frac{4\times 6}{3\pi} = 18.54$	1048.25	19434.58	$0.11 r^4 = 0.11 \times 6^4 = 142.56$	$\frac{\pi r^4}{8} = \frac{\pi \times 6^4}{8} = 508.93$
$\square^{le}$ ②	$4 \times 12 = 48$	$4 + 6 = 10$	480	4800	$\frac{bd^3}{12} = \frac{4 \times 12^3}{12} = 576$	$\frac{12 \times 4^3}{12} = 64$
$\square^{le}$ ③	$24 \times 4 = 96$	2	192	384	$\frac{bd^3}{12} = \frac{24 \times 4^3}{12} = 128$	$\frac{4 \times 24^3}{12} = 4608$
	$\Sigma a = 200.54$		$\Sigma ay = 1720.25$	$\Sigma ay^2 = 24618$	$\Sigma I_{gx} = 846.56$	$\Sigma I_{gy} = 5180.93$

$$I_{yy} = \Sigma I_{gy} = 5180.93\ mm^4$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{1720.25}{200.54} = 8.578\ mm$$

$$I_{1-1} = \Sigma I_{gx} + \Sigma a y^2$$

$$= 846.56 + 24618$$

$$= 25464.56 \text{ mm}^4$$

$$I_{1-1} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{1-1} - A\bar{y}^2$$

$$= 25464.56 - 200.54 \times 8.58^2$$

$$I_{xx} = 10701.52 \text{ mm}^4$$

$$I_{zz} = I_{xx} + I_{yy}$$

$$= 10701.52 + 5180.93$$

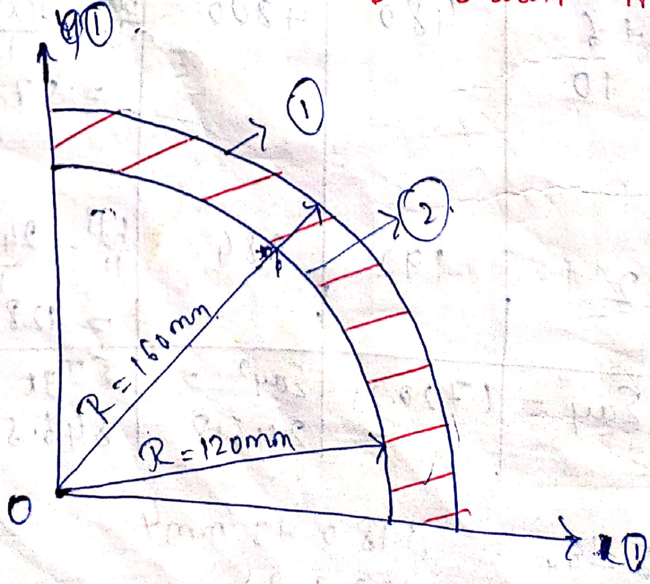
$$I_{zz} = 15882.45 \text{ mm}^4$$

$$K_z = \sqrt{\frac{I_{zz}}{A}}$$

$$= \sqrt{\frac{15882.45}{200.54}}$$

$$\boxed{K_z = 8.899 \text{ mm}}$$

Q.5 Compute moment of inertia of the shaded area about centroidal axis shown in fig.



Soln:- Given fig. is having both the centroidal ordinates in x & y directions same.. Hence computations are done on one axis.

Component No.	Area (mm²)	x	ax	ax^2	I_{gy}
Quadrant of ⊙le (1).	$\frac{\pi r^2}{4} = \frac{\pi \times 160^2}{4}$ $= 20106.19$	$\frac{4R}{3\pi} = \frac{4\times 160}{3\pi}$ $= 67.90\text{mm}$	1.365×10^6	92.679×10^6	$\frac{0.11r^4}{2} = \frac{0.11\times160^4}{2}$ $= 36.04\times10^6$
Quadrant of ⊙le (2).	$-\frac{\pi r^2}{4} = -\frac{\pi\times120^2}{4}$ $= -11309.73$	$\frac{4R}{3\pi} = \frac{4\times120}{3\pi}$ $= 50.93\text{mm}$	-576004.5	-29.33×10^6	$-\frac{0.11r^4}{2} = -\frac{0.11\times120^4}{2}$ $= -11.40\times10^6$
	$\Sigma a = 8796.46$		$\Sigma ax = 788995.5$	$\Sigma ax^2 = 63.35\times10^6$	$\Sigma I_{gx} = 24.64\times10^6$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{788995.5}{8796.46} = 89.7\text{mm}$$

$$I_{2-2} = \Sigma I_{gy} + \Sigma ax^2$$

$$= 24.64\times10^6 + 63.35\times10^6$$

$$\boxed{I_{2-2} = 87.99\times10^6\ \text{mm}^4}$$

$$I_{2-2} = I_{yy} + A\bar{x}^2$$

$$I_{yy} = I_{2-2} - A\bar{x}^2$$

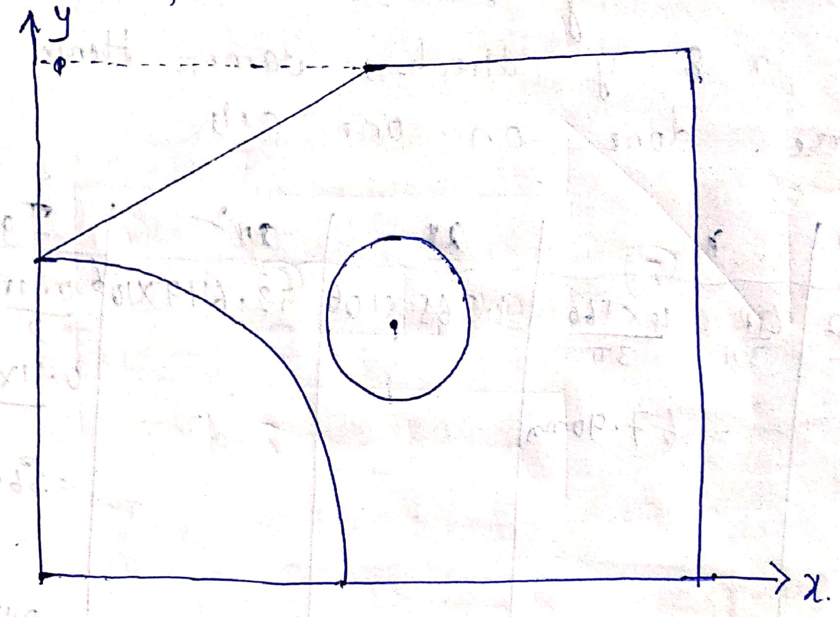
$$= 87.99\times10^6 - 8796.46\times89.7^2$$

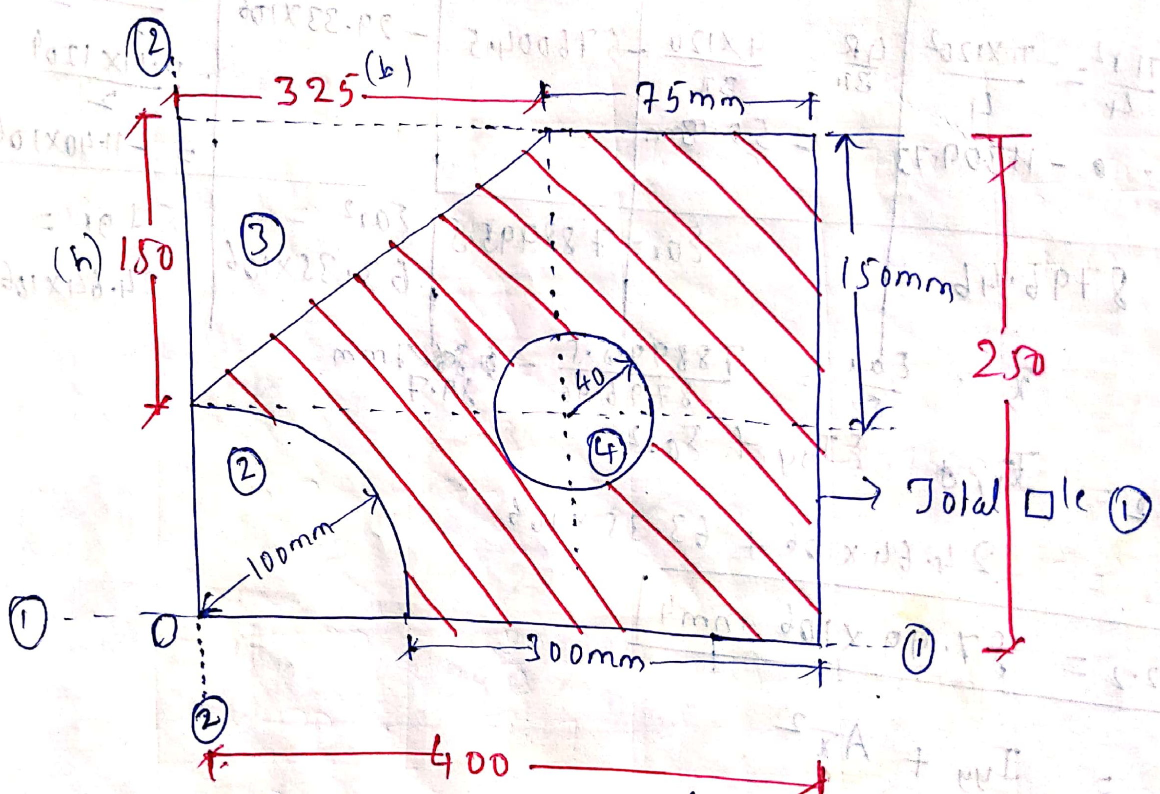
$$I_{yy} = 17.21\times10^6\ \text{mm}^4$$

$$\boxed{I_{xx} = I_{yy} = 17.21\times10^6\ \text{mm}^4}$$ [For quadrant of ⊙le $I_{xx} = I_{yy}$].

Since moment of inertia of quadrant of the circle on both the ~~directions~~ x & y are same. Therefore computations are done to calculate M.I. about any one axis.

10.) Find the polar moment of inertia of the plane lamina about point 'O'





I_{1-1} $\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{15.39 \times 10^6}{62744.45} =$

$I_{1-1} = \Sigma I_{gx} + \Sigma ay^2$

$= 481.861 \times 10^6 + 523.1 \times 10^6$

$I_{1-1} = 1.005 \times 10^9 \ mm^4$

$I_{2-2} = \Sigma I_{gy} + \Sigma ax^2 = 1.182 \times 10^9 + 3.169 \times 10^9$

$= 4.351 \times 10^9 \ mm^4$

$I_{zz} = I_{1-1} + I_{2-2} = 1.005 \times 10^9 + 4.351 \times 10^9$

$= 5.356 \times 10^9 \ mm^4$

Component	Area (a)	x	y	ax	ax^2	ay	ay^2
Total □le ①	$bd = 400\times250 = 100{,}000$	200	$\frac{250}{2} = 125$	20×10^6	4×10^9	12.5×10^6	1.5625×10^9
Quadrant of ○le ②	$-\frac{\pi r^2}{4} = -\frac{\pi\times100^2}{4} = -7854$	$\frac{4r}{3\pi} = \frac{4\times100}{3\pi} = 42.44$	$\frac{4r}{3\pi} = 42.44$	-333323.76	-14.14×10^6	-333323.76	-14.14×10^6
Δle ③	$-\frac{1}{2}bh = -\frac{1}{2}\times325\times150 = -24375$	$\frac{b}{3} = \frac{325}{3} = 108.33$	$100+\frac{2}{3}h = 100+\frac{2}{3}\times150 = 200$	-2.64×10^6	-286.05×10^6	-4875000	-975×10^6
○le ④	$-\pi r^2 = -\pi\times40^2 = -5026.55$	325	100	-1.633×10^6	-530.92×10^6	-502655	-50.26×10^6
	$\Sigma a = 62744.45$		$\Sigma ax =$	15.39×10^6	$\Sigma ax^2 = 3.169\times10^9$	$\Sigma ay = 6.789\times10^6$	$\Sigma ay^2 = 523.1\times10^6$

Component	I_{gx}	I_{gy}
Total □le	$+\frac{bd^3}{12} = \frac{400\times250^3}{12} = +520.83\times10^6$	$+\frac{db^3}{12} = \frac{250\times400^3}{12} = +1.333\times10^9$
Quadrant of ○le ②	$-\frac{0.11r^4}{2} = -5.5\times10^6$	$-\frac{0.11r^4}{2} = -5.5\times10^6$
Δle ③	$-\frac{bh^3}{36} = \frac{-325\times150^3}{36} = -30.46\times10^6$	$-\frac{hb^3}{36} = \frac{-150\times325^3}{36} = -143.03\times10^6$
○le ④	$-\frac{\pi r^4}{4} = -\frac{\pi\times40^4}{4} = -2.01\times10^6$	-2.01×10^6

$\Sigma I_{gx} = 482.86\times10^6$

$\Sigma I_{gy} = \cancel{1.185\times10^9}$ 1.182×10^9

11.) Find the polar radius of gyration of the shaded area about centroidal axes for the symmetrical fig. given below.

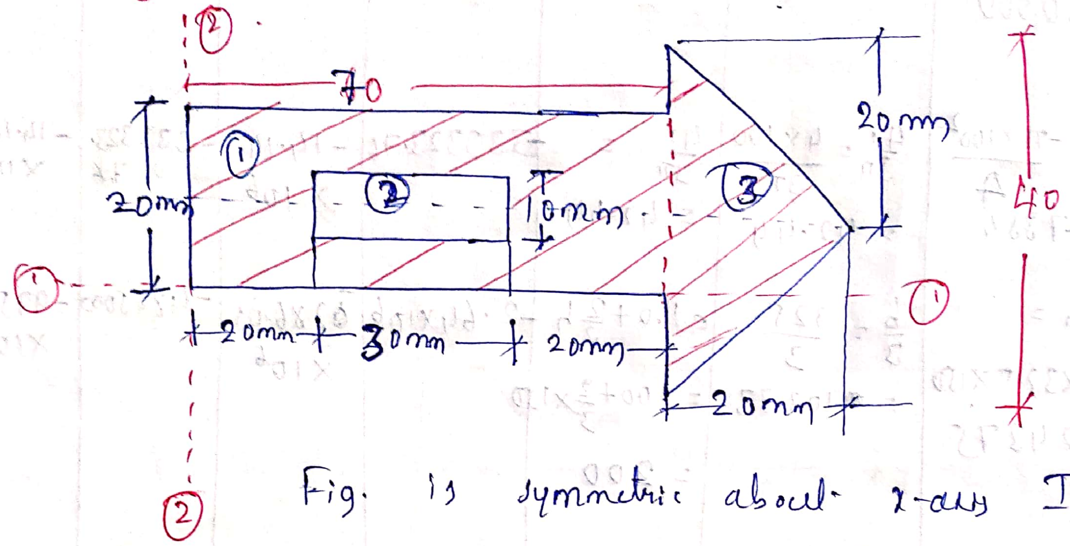


Fig. is symmetric about x-axis $I_{xx} = \Sigma I_{gx}$.

Component	Area (a)	x	ax	ax²	I_{gx}	I_{gy}
Total □le ①	70×20 = 1400	$\frac{70}{2} = 35$	49000	1.715×10^6	$\frac{bd^3}{12} = \frac{70\times20^3}{12} = 46.67\times10^3$	$\frac{db^3}{12} = \frac{20\times70^3}{12} = 571666.67$
□le ②	−(30×10) = −300	$20+\frac{30}{2} = 35$	−10500	−367500	$-\frac{30\times10^3}{12} = -2500$	$-\frac{10\times30^3}{12} = -22500$
△le ③	$\frac{1}{2}\times40\times20$ = 400	$70+\frac{20}{3} = 76.67$	30668	2.351×10^6	$bh^3/36 = \frac{20\times40^3}{36} = 35.55\times10^3$	$bh^3/36 = \frac{40\times20^3}{36} = 8888.89$
	Σa = 1500		Σax = 69168	$\Sigma ax^2 = 3.698\times10^6$	ΣI_{gx} = 79720	$\Sigma I_{gy} = 558.05\times10^3$

$I_{xx} = \Sigma I_{gx} = 79720 \text{ mm}^4$

$I_{2-2} = \Sigma I_{gy} + \Sigma ax^2$

$= 558.05\times10^3 + 3.698\times10^6$

$I_{2-2} = 4.256\times10^6 \text{ mm}^4$

$I_{2-2} = I_{yy} + A\bar{x}^2$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{69168}{1500} = 46.11 \text{ mm}$

$I_{yy} = I_{2-2} - A\bar{x}^2$

$= \cancel{558.05\times10^3}\ 4.256\times10^6 - 1500\times46.11^2$

$I_{yy} = 1.0668\times10^6 \text{ mm}^4$

$I_{zz} = I_{xx} + I_{yy}$

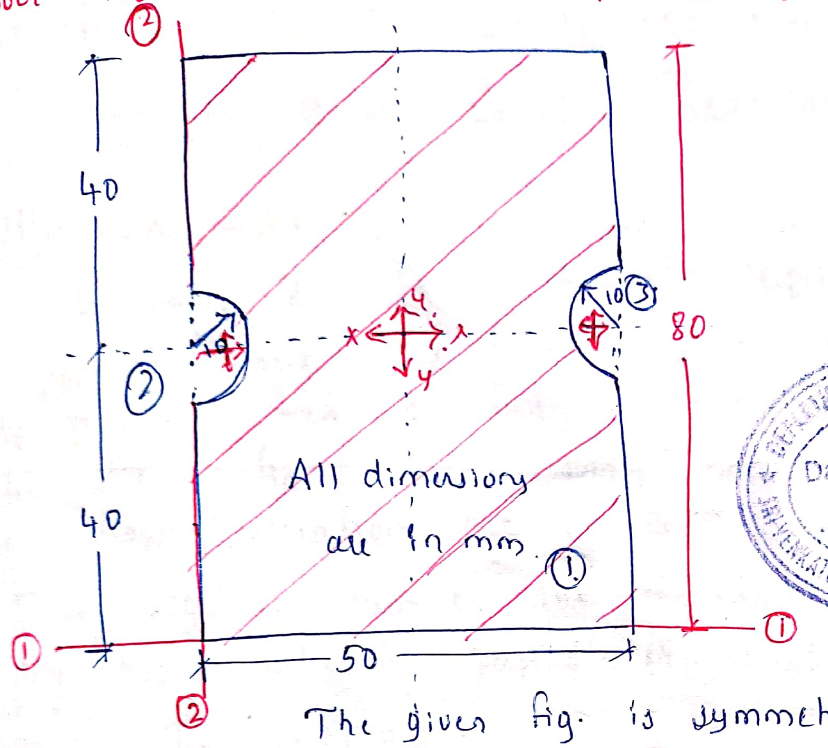
$= 79720 + 1.0668\times10^6$

$I_{zz} = 1.146\times10^6 \text{ mm}^4$

$K_z = \sqrt{\frac{I_{zz}}{A}}$

$\boxed{K_z = 27.64 \text{ mm}}$

7) Find the moment of inertia of the area shown in fig. about its centroidal axes parallel to co-ordinate axes



25/10/16

The given fig. is symmetric about x-axis & the centroidal x-axis for whole fig. & individual components are same

$\therefore$ $I_{xx} = \Sigma I_{gx}$

Fig. is also symmetric about y-axis but the centroidal y-axis for whole fig. & individual components are different $\therefore$ $I_{yy} = \Sigma I_{gy}$ is not applicable.

Component	Area (a) (mm^2)	x	ax	ax^2	I_{gx}	I_{gy}
Total □ ①	$50 \times 80 = 4000$	25	100×10^3	2.5×10^6	$bd^3/12 = \frac{50 \times 80^3}{12} = 2.133 \times 10^6$	$db^3/12 = 833.33 \times 10^3$
◗ ②	$-\frac{\pi \times 10^2}{2} = -157.07$	$\frac{4R}{3\pi} = \frac{4 \times 10}{3\pi} = 4.244$	-666.60	-2829.07	$-\frac{\pi r^4}{8} = -\frac{\pi \times 10^4}{8} = -3927$	$-0.11r^4 = -0.11 \times 10^4 = -1100$
◖ ③	-157.07	$50 - \frac{4R}{3\pi} = 45.75$	-7185.95	-328.75×10^3	$-\frac{\pi r^4}{8} = -3927$	$-0.11r^4 = -1100$
	$\Sigma a = 3685.86$	$\Sigma ax =$	$9214 7.45$	$\Sigma ax^2 = 2.168 \times 10^6$	$\Sigma I_{gx} = 2.125 \times 10^6$	$\Sigma I_{gy} = 831130.$

$$\boxed{I_{xx} = \Sigma I_{gx} = 2.125 \times 10^6 \ mm^4}$$

$I_{2-2} = \Sigma I_{gy} + \Sigma ax^2 = 831130 + 2.168 \times 10^6 = 2.999 \times 10^6$

$I_{2-2} = I_{yy} + A\bar{x}^2 \Rightarrow I_{yy} = I_{2-2} - A\bar{x}^2$ $\qquad \bar{x} = \frac{\Sigma ax}{\Sigma a} = 25mm$

$= 2.999 \times 10^6 - 3685.86 \times 25^2$

$$\boxed{I_{yy} = 695.337 \times 10^3 \ mm^4}$$