

Module-3

Analysis of system of forces

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Mechanics and its classification

Mechanics:- It is a branch of physical science which deals with the state of rest (or) motion of a body under the action of forces.

Mechanics

① Statics

② Dynamics

Kinematics

Kinetics

① Statics:- It is a branch of physical science which deals with study of state of rest of a body under the action of external forces.

② Dynamics:- It is a branch of physical science which deals with study of state of motion of a body under the action of external forces.

i) Kinematics:- It is a branch of dynamics in which the force causing motion will not be considered.

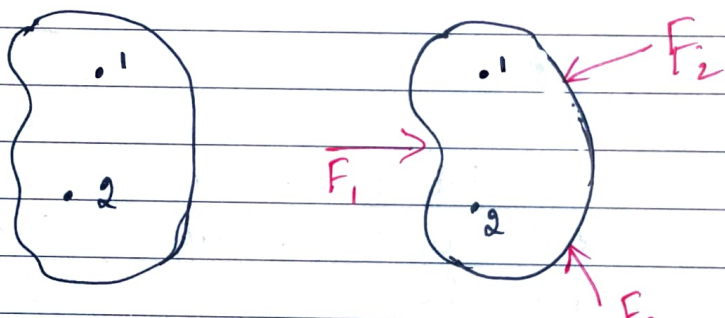
ii) Kinetics:- It is a branch of dynamics in which the force causing motion will be considered.

Basic definitions / Basic idealizations

Particle:- A body with a infinitely small volume & negligible mass.

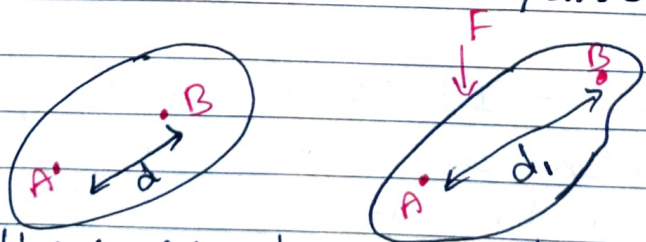
Body:- Assemblage of a number of particle.

Rigid body:- The body with its particles remaining at a fixed distance even after applying external force.



All particles in a rigid body remains at a fixed distance one another even after the application of forces. Thus there is no change in either shape (or) dimensions of the body.

Deformable body:- After applying the external force on a body with its particles the distance between the particles will vary.



If there is change in distance between any two particles of the body after application of external force then it is called as deformable body.

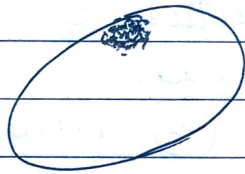
Scalar Quantity:- The Quantity which has only magnitude and no direction.

Eg:- Mass, Density, Volume, Distance.

Vector Quantity:- The Quantity which has both magnitude & direction.

Eg:- Velocity, Acceleration, Displacement, Force etc.,

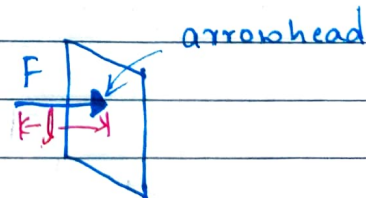
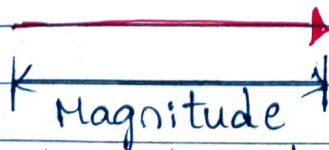
Continuum:- The continuous distribution of the particles in a body without any intermolecular space.



Force and its characteristics

Force:- It is an external agency which will tend to change of state of rest (or) motion of a body.
(or)

Force is an external agent which changes (or) tends to change the state of rest (or) motion of an object. It is a vector quantity [has both magnitude & direction]. The SI unit of force is Newton (N).



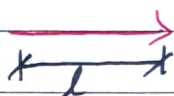
Every force is characterised by its:-

- ① Magnitude:- length of the vector represents the magnitude of force.

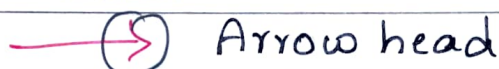
Eg:- If the magnitude of the force is higher then the length of the vector is higher.



If the magnitude of the force is lesser then the length of the vector is lesser.

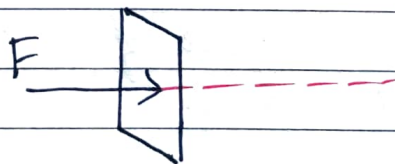


② Direction:- The direction of the force is represented by an arrow head.



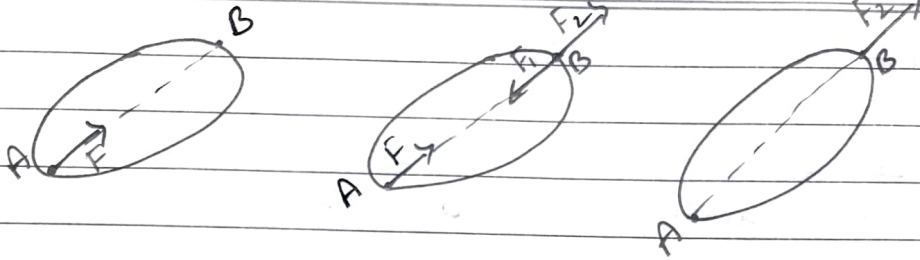
③ Point of application:- Point at which the force ^{acting} is applied. Both the direction and point of application define its line of action.

④ Line of action:- The line along which the force acts.



Principle of transmissibility of forces

This principle states that a force can be transmitted from one point to another point along the same line of action such that the effect produced by the force on a body remains unchanged.



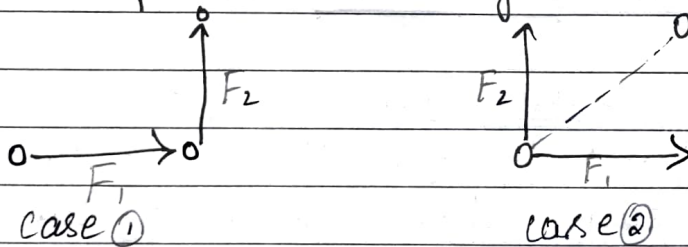
Consider a rigid body subjected to a force F at point 'A'. According to the principle of transmissibility, the force ' F ' can be transmitted to a new point 'B' along the same line of action such that the net effect on the body remains unchanged.

i.e., $F = F_1 = F_2$

∴ $F - F_1 + F_2 = \Sigma F$

Principle of Superposition of forces

The net effect of a system of forces on a rigid body is equal to the combined effect of individual forces on the rigid body.



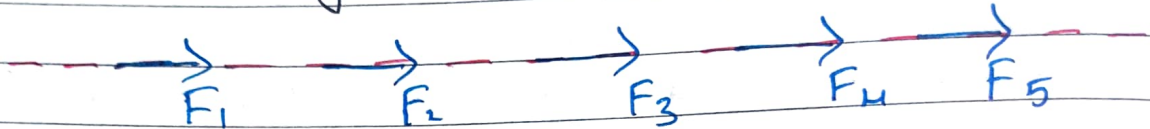
Classification of force system (or) types of system of forces

If two (or) more forces are acting on a body (or) on a member (or) on a particle constitute a force system.

Types

① Collinear force system

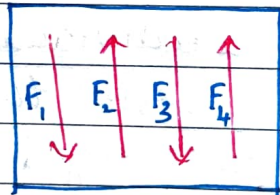
Forces acting in a single plane



i.e., All the forces acting along the same line
(or) line of action of forces lying in the same plane. line.

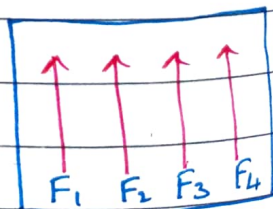
② Coplanar parallel force system

All the forces lying in the same plane & all the forces are parallel to each other.



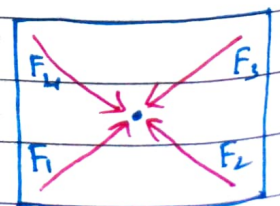
③ Coplanar parallel like force system

All the forces lying in the same plane parallel to each other and acting along the same direction.



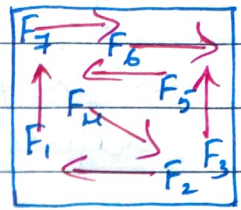
④ Coplanar concurrent force system

All the forces are lying in the same plane and line of action of all the forces passing through a single point.



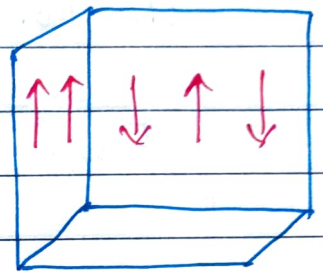
⑤ Coplanar non-concurrent force system

All the forces are lying in the same plane & are not parallel to each other and their line of action will not meet at a single point.



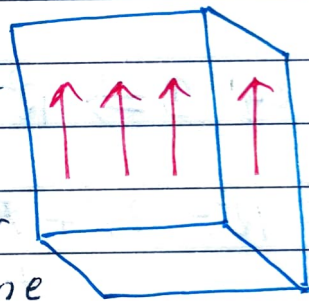
⑥ Non-coplanar parallel force system

The forces are lying in different planes but all the forces are parallel to each other.



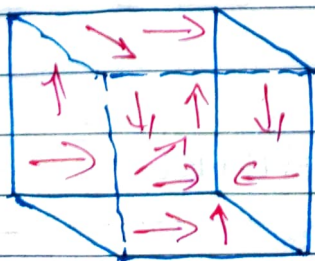
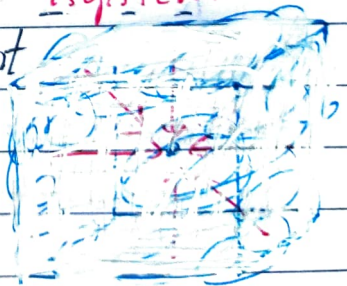
⑦ Non-coplanar parallel like force system

The forces are lying in different planes but all the forces are parallel to each other and their line of action is along the same direction.



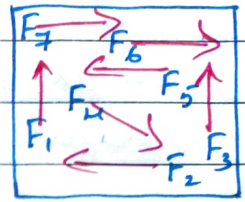
⑧ Non-coplanar concurrent force system

The forces will be lying in the different planes, not parallel to each other and not meeting at a single point.



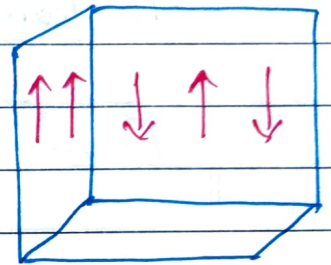
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All the forces are lying in the same plane & are not parallel to each other and their line of action will not meet at a single point.



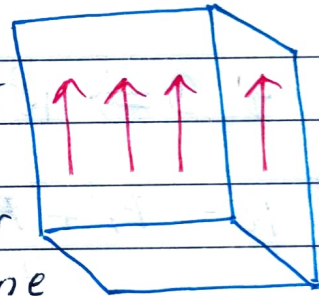
⑥ Non-coplanar parallel force system

The forces are lying in different planes but all the forces are parallel to each other.



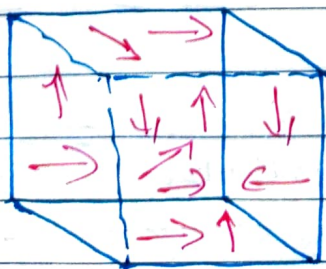
⑦ Non-coplanar parallel like force system

The forces are lying in different planes but all the forces are parallel to each other and their line of action is along the same direction.



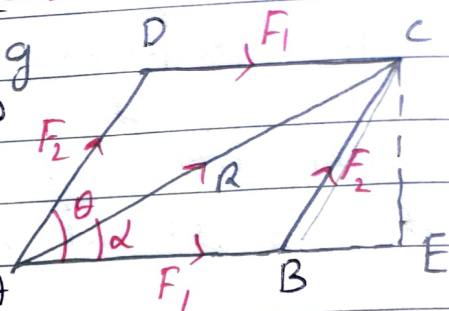
⑧ Non-coplanar concurrent force system

The forces will be lying in the different planes, not parallel to each other and not meeting at a single point.



Parallelogram Law of forces

If two forces F_1 & F_2 acting at a point represented by two adjacent sides of a parallelogram then their resultant is represented by the diagonal of parallelogram passing through the point of intersection of both the forces.



Magnitude of the resultant force (R)

From $\Delta^e CEA$, apply Pythagoras theorem

$$R^2 = AE^2 + CE^2$$

$$R^2 = (AB + BE)^2 + CE^2 \rightarrow \text{①}$$

$$\text{eqn ①} \Rightarrow R^2 = (F_1 + F_2 \cos \theta)^2 + F_2^2 \sin^2 \theta$$

$$R^2 = F_1^2 + F_2^2 \cos^2 \theta + 2F_1 F_2 \cos \theta + F_2^2 \sin^2 \theta$$

$$R^2 = F_1^2 + F_2^2 (\cos^2 \theta + \sin^2 \theta) + 2F_1 F_2 \cos \theta$$

$$R^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta$$

But from $\Delta^e CEB$

$$\cos \theta = \frac{BE}{F_2}$$

$$BE = F_2 \cos \theta$$

$$\sin \theta = \frac{CE}{F_2}$$

$$CE = F_2 \sin \theta$$

$$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta}$$

Direction of the resultant force (α)

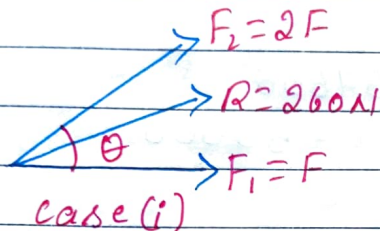
From $\Delta^e CEA$

$$\tan \alpha = \frac{CE}{AE} = \frac{CE}{AB + BE} = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\therefore \alpha = \tan^{-1} \left[\frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta} \right]$$

- ① The resultant of two forces, one of which is double the other is 260N. If the direction of the larger force is reversed and the other one remains unaltered, the resultant reduces to 180N. Determine the magnitude of the forces and the angle between the two forces.

Solⁿ:-

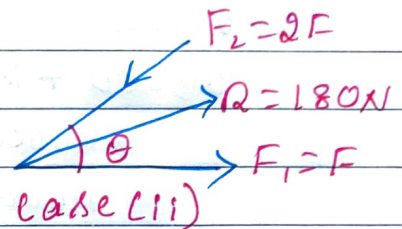


$$F_1 = F = ?$$

$$F_2 = 2F = ?$$

$$\theta = ?$$

$$R = 260 \text{ N}$$



$$F_1 = F = ?$$

$$F_2 = 2F = ?$$

$$\theta = ?$$

$$R = 180 \text{ N}$$

case (i):-

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$260 = \sqrt{F^2 + 2F^2 + 2(F)(2F) \cos \theta}$$

Squaring on both sides

$$260^2 = F^2 + 4F^2 + 4F^2 \cos \theta$$

$$5F^2 + 4F^2 \cos \theta = 67600 \rightarrow \textcircled{1}$$

case (ii):-

$$180 = \sqrt{F^2 + (-2F)^2 + 2F(-2F) \cos \theta}$$

$$180 = \sqrt{F^2 + 4F^2 - 4F^2 \cos \theta}$$

squaring both sides

$$5F^2 - 4F^2 \cos \theta = 32400 \rightarrow \textcircled{2}$$

Adding eqⁿ ① & eqⁿ ②

$$5F^2 + 4F^2 \cos \theta = 67600$$

$$5F^2 - 4F^2 \cos \theta = 32400$$

$$10F^2 = 10,000$$

$$\therefore F^2 = 1000$$

$$\therefore F = 100 \text{ N}$$

$$F_1 = 100 \text{ N} \quad \& \quad F_2 = 200 \text{ N}$$

Substitute the value of Force 'F' in eqⁿ ①

$$5F^2 + 4F^2 \cos \theta = 67600$$

$$5(100)^2 + 4(100)^2 \cos \theta = 67600$$

$$50,000 + 40,000 \cos \theta = 67600$$

$$40000 \cos \theta = 67600 - 50000$$

$$40000 \cos \theta = 17600$$

$$\cos \theta = \frac{17600}{40000}$$

$$\cos \theta = 0.44$$

$$\theta = \cos^{-1}(0.44)$$

$$\theta = 63.9^\circ$$

- ② The sum of two resultant forces P & Q is 500N & their resultant is 400N. If the resultant is perpendicular to 'P', Find P, Q and the angle b/w P & Q.

Solⁿ:-
 $P + Q = 500 \text{ N} \rightarrow \text{①}$
 $R = 400 \text{ N}$

From fig. $Q^2 = P^2 + R^2$

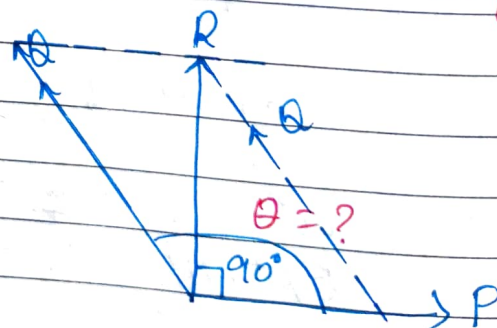
$$\therefore Q^2 - P^2 = R^2$$

$$(Q+P)(Q-P) = R^2$$

$$500(Q-P) = 400^2$$

$$Q-P = \frac{400^2}{500}$$

$$Q-P = 320 \rightarrow \text{②}$$



Adding eqn ① & ②

$$Q + P = 500$$

$$Q - P = 320$$

$$2Q = 820$$

$$Q = 410$$

$$\therefore P = 90 \text{ N}$$

WKT, $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

$$400 = \sqrt{90^2 + 410^2 + 2 \times 90 \times 410 \times \cos \theta}$$

$$400 = \sqrt{176200 + 73800 \cos \theta}$$

Square on both sides

$$160000 = 176200 + 73800 \cos \theta$$

$$73800 \cos \theta = -176200 + 160000$$

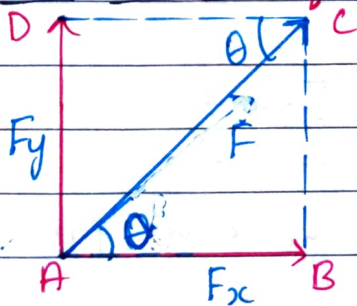
$$\cos \theta = -0.219$$

$$\theta = \cos^{-1}(-0.219)$$

$$\theta = 102.6^\circ$$

⑧ Resolution and composition of forces

① Resolution of forces



From $\triangle ABC$

$$\cos \theta = \frac{F_x}{F}$$

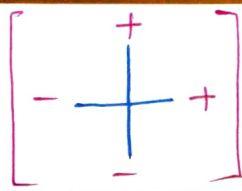
$$F_x = F \cos \theta$$

From $\triangle ADC$

$$\sin \theta = \frac{F_y}{F}$$

$$F_y = F \sin \theta$$

Splitting up of a force into two rectangular components (H^{le} and V^{le}) is called resolution of a force.



Sign convention

② Composition of forces

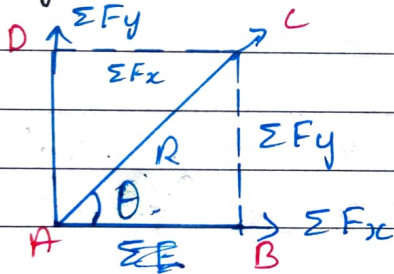
Combining 2 or more forces into a single resultant force is called composition of forces.

$$\Sigma F_{xc} = F_1 + F_2 \cos \theta_2 - F_3 \cos(90 - \theta_3) - F_4 \cos \theta_4$$

$$\Sigma F_{xc} = F_1 + F_2 \cos \theta_2 - F_3 \sin \theta_3 - F_4 \cos \theta_4$$

$$\Sigma F_y = -F_2 \sin \theta_2 - F_3 \sin(90 - \theta_3) + F_4 \sin \theta_4 + F_5$$

$$\Sigma F_y = -F_2 \sin \theta_2 - F_3 \cos \theta_3 + F_4 \sin \theta_4 + F_5$$



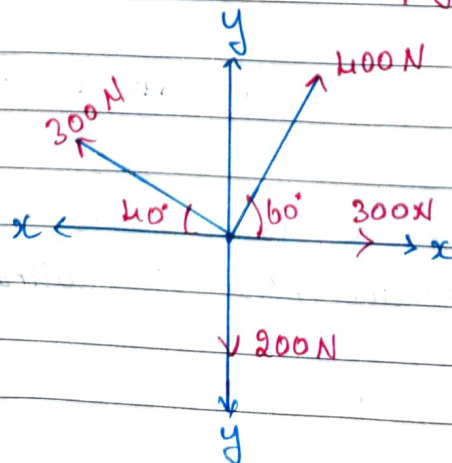
$$R^2 = \Sigma F_x^2 + \Sigma F_y^2$$

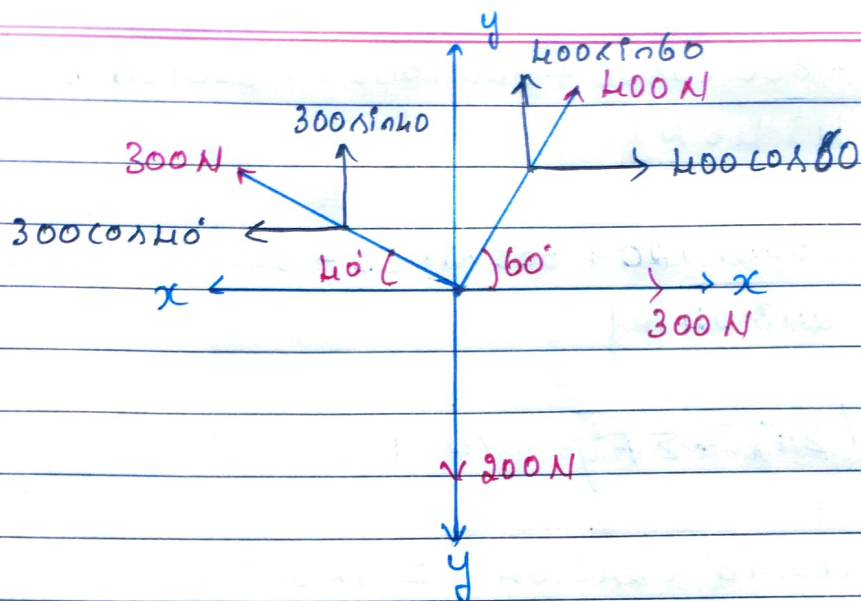
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right]$$

① Find the resultant of the coplanar concurrent force system shown in figure.





$$\Sigma F_x = -300 \cos 40^\circ + 400 \cos 60^\circ + 300$$

$$\boxed{\Sigma F_x = 270.187 \text{ N}}$$

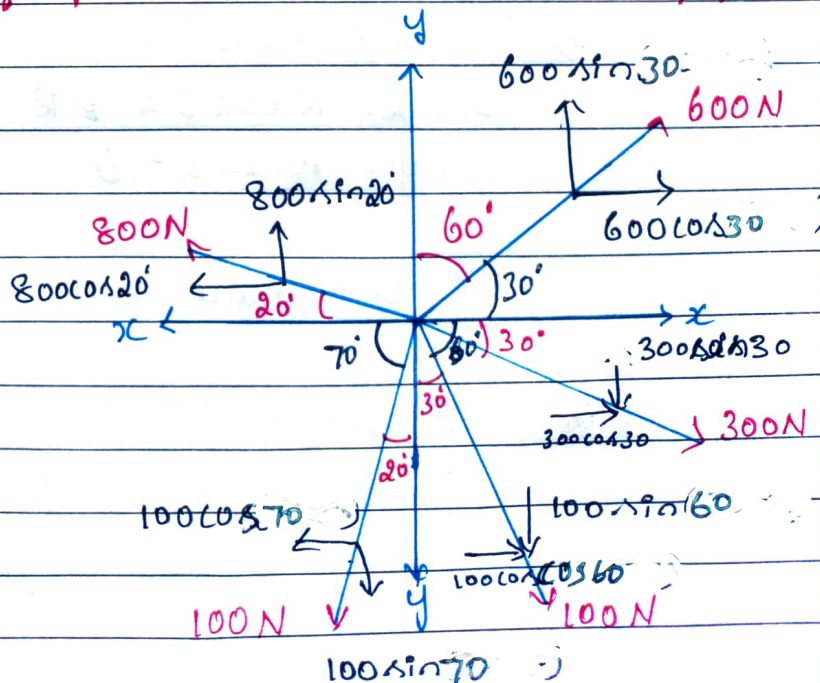
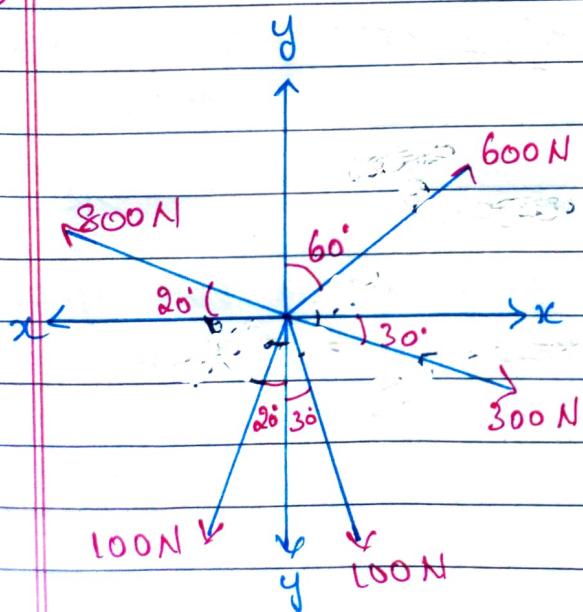
$$\Sigma F_y = 300 \sin 40^\circ + 400 \sin 60^\circ - 200$$

$$\boxed{\Sigma F_y = 339.246 \text{ N}}$$

$$\therefore R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{270.187^2 + 339.246^2} = \underline{\underline{433.7 \text{ N}}}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[\frac{339.246}{270.187} \right] = \underline{\underline{51.47^\circ}}$$

② Find the resultant of force system shown in figure.



$$\Sigma F_x = -800 \cos 20 + 600 \cos 30 + 300 \cos 30 + 100 \cos 60 - 100 \cos 70$$

$$\Sigma F_x = 43.46 \text{ N}$$

$$\Sigma F_y = 800 \sin 20 + 600 \sin 30 - 300 \sin 30 - 100 \sin 60 - 100 \sin 70$$

$$\Sigma F_y = 243.04 \text{ N}$$

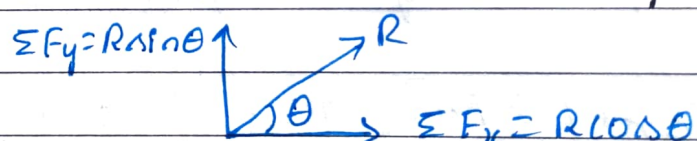
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$R = \sqrt{43.46^2 + 243.04^2} = \underline{\underline{246.9 \text{ N}}}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[\frac{243.04}{43.46} \right] = \underline{\underline{80^\circ}}$$

Note:-

1) If the magnitude ^{& direction} of the ^{resultant} is given in the problem then $\Sigma F_x = R \cos \theta$ & $\Sigma F_y = R \sin \theta$

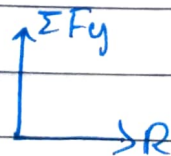


If 'R' makes an angle ' θ ' with x-axis then $\Sigma F_x = R \cos \theta$ & $\Sigma F_y = R \sin \theta$

2) If the resultant is given along the x-axis.

$$\Sigma F_x = R \cos 0 = \underline{\underline{R}}$$

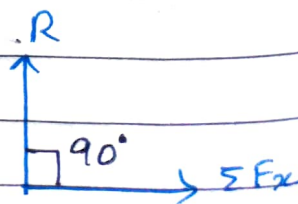
$$\Sigma F_y = R \sin 0 = \underline{\underline{0}}$$



3) If the resultant is given along the y-axis

$$\Sigma F_x = R \cos 90 = \underline{\underline{0}}$$

$$\Sigma F_y = R \sin 90 = \underline{\underline{R}}$$



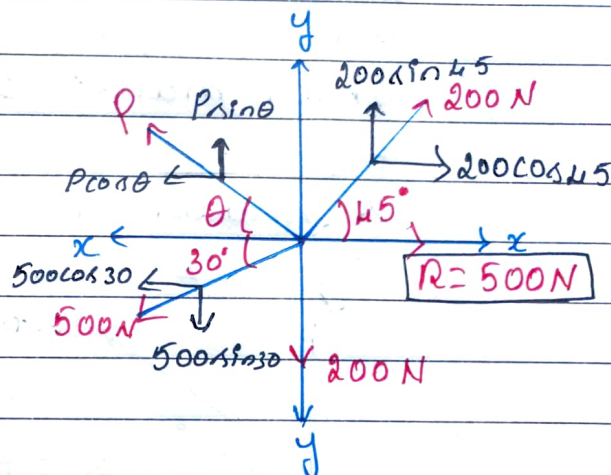
- ③ Four coplanar forces acting at a point are shown in figure. One of the forces is unknown & its magnitude is shown by 'P'. The resultant has a magnitude of 500 N and is acting along x-axis. Determine the unknown force P and its inclination with the x-axis.

Solⁿ:- Resultant lies in x-axis

$$\therefore \Sigma F_x = R ; \Sigma F_y = 0 ;$$

$$\therefore \Sigma F_x = R = 500 \text{ N}$$

$$\Sigma F_y = 0$$



$$\Sigma F_x = 500 \text{ N}$$

$$- P \cos \theta + 200 \cos 45 - 500 \cos 30 = 500 \text{ N}$$

$$P \cos \theta = -791.6 \text{ N} \rightarrow \textcircled{1}$$

$$\Sigma F_y = 0$$

$$- 500 \sin 30 + P \sin \theta + 200 \sin 45 - 200 = 0$$

$$P \sin \theta = 308.58 \text{ N} \rightarrow \textcircled{2}$$

Squaring and adding eqⁿ ① & ②

$$P^2 \sin^2 \theta + P^2 \cos^2 \theta = 308.58^2 + (-791.6)^2$$

$$P^2 (\sin^2 \theta + \cos^2 \theta) = 721852.17$$

$$P = 849.61 \text{ N}$$

Substitute 'P' value in eqⁿ ①

$$\text{eqⁿ ①} \Rightarrow 849.61 \cos \theta = -791.6$$

$$\cos \theta = -0.931$$

$$\theta = \cos^{-1}(-0.931)$$

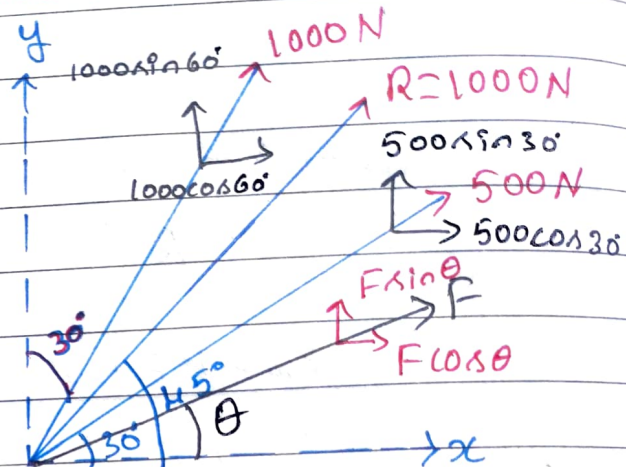
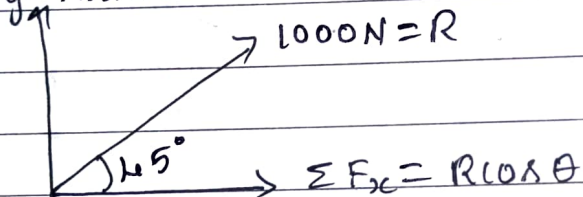
$$\theta = 21.40^\circ$$

- ④ Two forces acting on a body are 500N & 1000N as shown in figure. Determine the 3rd force 'F' such that the resultant of all the three forces is 1000N directed at 45° to the x-axis

Soln:-

Assume,
Let 'F' be the 3rd force which makes an angle θ with the x-axis as shown in figure.

$$\Sigma F_y = R \sin \theta$$



$$\Sigma F_x = R \cos \theta = 1000 \cos 45 = 707.10 \text{ N}$$

$$\Sigma F_y = R \sin \theta = 1000 \sin 45 = 707.10 \text{ N}$$

consider

$$\Sigma F_x = 707.10 \text{ N}$$

$$F \cos \theta + 500 \cos 30 + 1000 \cos 60 = 707.10$$

$$F \cos \theta = -225.91 \rightarrow \textcircled{1}$$

consider

$$\Sigma F_y = 707.10 \text{ N}$$

$$F \sin \theta + 500 \sin 30 + 1000 \sin 60 = 707.10$$

$$F \sin \theta = -409 \rightarrow \textcircled{2}$$

Squaring & adding eqn $\textcircled{1}$ & $\textcircled{2}$

$$F^2 \cos^2 \theta + F^2 \sin^2 \theta = 225.91^2 + 409^2$$

$$F^2 (\sin^2 \theta + \cos^2 \theta) = 218316.32$$

$$F = 467.24 \text{ N}$$

Divide eqn ② by ①

$$\frac{F \sin \theta}{F \cos \theta} = \frac{409}{225.91}$$

$$\tan \theta = \frac{409}{225.91}$$

$$\theta = \tan^{-1}(1.81)$$

$$\theta = 61.08^\circ$$

- ⑤ 2kN force is the resultant of system of forces acting up along y-axis as shown in fig. Determine the value of F & θ .

Soln:-

$$\uparrow R \quad \Sigma F_y = 2 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0$$

As Resultant lies in y-axis

$$\Sigma F_y = 2 \text{ kN} \quad \& \quad \Sigma F_x = 0$$

considering, $\Sigma F_y = 2 \text{ kN}$

$$\therefore -2.4 \sin 30^\circ + F \sin \theta = 2$$

$$F \sin \theta = 3.2 \rightarrow \text{①}$$

Taking $\Sigma F_x = 0$

$$-5 + 2.4 \cos 30^\circ + F \cos \theta = 0$$

$$F \cos \theta = 2.92 \text{ kN} \rightarrow \text{②}$$

Squaring & adding eqn ① & ②

$$F^2 \sin^2 \theta + F^2 \cos^2 \theta = 3.2^2 + 2.92^2$$

$$F^2 (\sin^2 \theta + \cos^2 \theta) = 18.76$$

$$F = 4.33 \text{ kN}$$

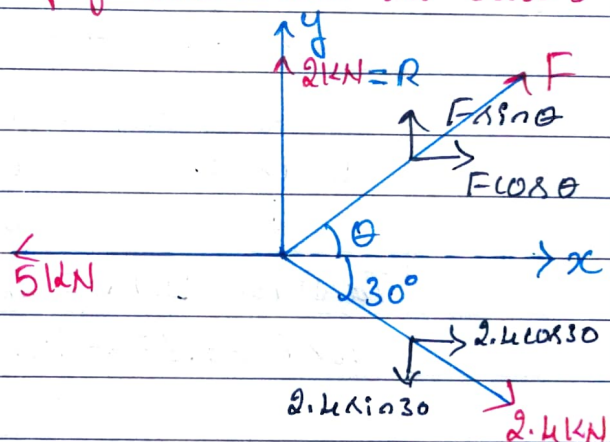
Divide eqn ① by ②

$$\frac{F \sin \theta}{F \cos \theta} = \frac{3.2}{2.92}$$

$$\tan \theta = \frac{3.2}{2.92}$$

$$\theta = \tan^{-1}(3.2/2.92)$$

$$\theta = 47.605^\circ$$

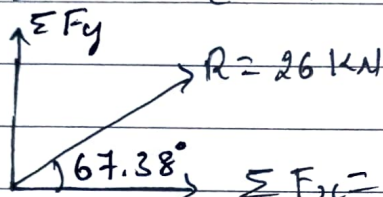


- ⑥ 26 kN force is the resultant of two forces, one of which is as shown in figure. Determine the other force.

Solⁿ :- From figure

$$\theta_1 = \tan^{-1} (12/5) = 67.38^\circ$$

$$\theta_2 = \tan^{-1} (3/4) = 36.87^\circ$$



$$\Sigma F_x = R \cos \theta$$

$$= 26 \cos 67.38^\circ$$

$$\Sigma F_x = 10 \text{ kN}$$

$$\Sigma F_y = R \sin \theta = 26 \sin 67.38^\circ = 24 \text{ kN}$$

Considering, $\Sigma F_x = 10 \text{ kN}$

$$10 \cos 36.87^\circ + P \cos \theta = 10$$

$$P \cos \theta = 2 \rightarrow \textcircled{1}$$

Taking, $\Sigma F_y = 24 \text{ kN}$

$$10 \sin 36.87^\circ + P \sin \theta = 24$$

$$P \sin \theta = 18 \rightarrow \textcircled{2}$$

Squaring & adding eqⁿ ① & ②

$$P^2 \cos^2 \theta + P^2 \sin^2 \theta = 2^2 + 18^2$$

$$P^2 (\sin^2 \theta + \cos^2 \theta) = 328$$

$$P = 18.11 \text{ kN}$$

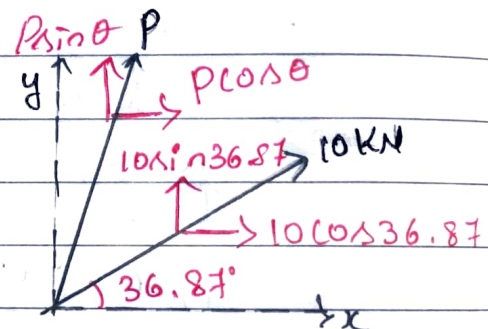
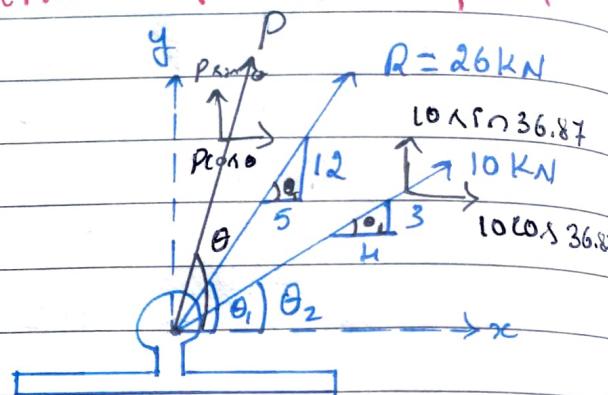
Divided eqⁿ ② by ①

$$\frac{P \sin \theta}{P \cos \theta} = \frac{18}{2}$$

$$\tan \theta = 9$$

$$\theta = \tan^{-1} (9)$$

$$\theta = 83.66^\circ$$



- ⑦ The resultant of a force system on a bracket as shown in figure is acting vertically upwards. If the angle 30° between the two like forces is fixed, find the angle ' θ ' & magnitude of the force.

Sol:- An resultant lies in y-axis

$$R = \Sigma F_y$$

$$\rightarrow \Sigma F_x = 0$$

considering $\Sigma F_x = 0$

$$-2 \cos 60 + 4 \cos (30 + \theta) + 4 \cos \theta = 0$$

$$-1 + 4 \cos (30 + \theta) + 4 \cos \theta = 0$$

$$4 [\cos (30 + \theta) + \cos \theta] = 1$$

$$\cos (30 + \theta) + \cos \theta = \frac{1}{4}$$

$$2 \cos \left(\frac{30 + \theta + \theta}{2} \right) \cos \left(\frac{30 + \theta - \theta}{2} \right) = \frac{1}{4}$$

$$\left[\because \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$$

$$2 \cos \left(\frac{30 + 2\theta}{2} \right) \cos 15 = \frac{1}{4}$$

$$1.931 \cos \left(\frac{30 + 2\theta}{2} \right) = \frac{1}{4}$$

$$1.931 \cos (15 + \theta) = \frac{1}{4}$$

$$\cos (15 + \theta) = 0.129$$

$$15 + \theta = 82.56^\circ$$

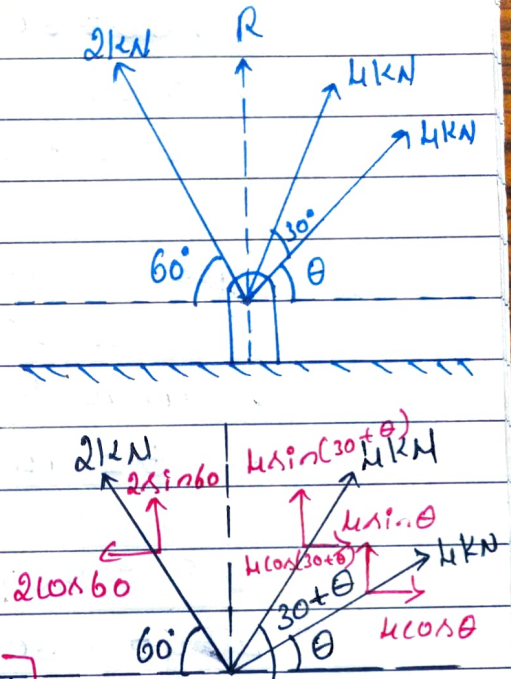
$$\theta = 67.5^\circ$$

Taking $\Sigma F_y = R$

$$2 \sin 60 + 4 \sin (30 + 67.5) + 4 \sin 67.5 = R$$

$$1.73 + 3.96 + 3.69 = R$$

$$R = 9.39 \text{ kN}$$



- ⑧ Determine the resultant force acting on the structure at point 'O' both in magnitude and direction.

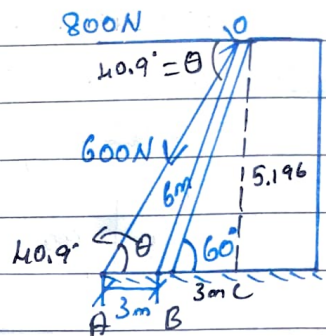
Solⁿ:- From $\Delta^{le} OBC$

$$\cos 60 = BC/6$$

$$BC = 6 \cos 60 = \underline{3\text{ m}}$$

$$\sin 60 = CO/6$$

$$CO = 6 \sin 60 = \underline{5.196\text{ m}}$$

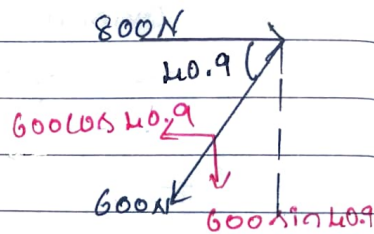


From figure $AC = AB + BC = 3 + 3 = \underline{6\text{ m}}$

From $\Delta^{le} OCA$

$$\tan \theta = 5.196/6$$

$$\theta = \tan^{-1} (5.196/6) = \underline{40.9^\circ}$$



$$\Sigma F_x = 800 - 600 \cos 40.9 = \underline{346.48\text{ N}}$$

$$\Sigma F_y = -600 \sin 40.9 = \underline{-392.84\text{ N}}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{346.48^2 + 392.84^2}$$

$$\boxed{R = 523.708\text{ N}}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right]$$

$$\theta = \tan^{-1} \left[\frac{392.84}{346.48} \right]$$

$$\boxed{\theta = 48.58^\circ}$$

- ⑨ Three forces acting at the centre of gravity of a block are shown in figure. The direction of 300 N forces may vary but the angle b/w them is always 120° . Determine the value of ' θ ' for which the resultant of the three forces is directed parallel to the plane. also find resultant of force.

Solⁿ: As resultant lies in x-axis

$$\Sigma F_x = R \quad \& \quad \Sigma F_y = 0$$

considering $\Sigma F_y = 0$

$$500 \sin 30 + 300 \sin (40 + \theta) + 300 \sin \theta = 0$$

$$500 \sin 30 + 300 [\sin (40 + \theta) + \sin \theta] = 0$$

$$300 [\sin (40 + \theta) + \sin \theta] = -250$$

$$\sin (40 + \theta) + \sin \theta = -0.833$$

$$2 \sin \left(\frac{40 + \theta + \theta}{2} \right) \cos \left(\frac{40 + \theta - \theta}{2} \right) = -0.833$$

$$\sin (A+B) = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\sin \left[\frac{(40 + 2\theta)}{2} \right] \cos (20) = -0.416$$

$$\sin \left(\frac{2(20 + \theta)}{2} \right) \cos 20 = -0.416$$

$$\sin (20 + \theta) \cos 20 = -0.416$$

$$\sin (20 + \theta) \times 0.94 = -0.416$$

$$\sin (20 + \theta) = -0.443$$

$$20 + \theta = \sin^{-1}(-0.443)$$

$$20 + \theta = -26.3$$

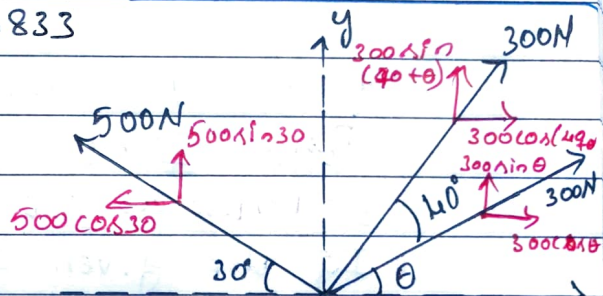
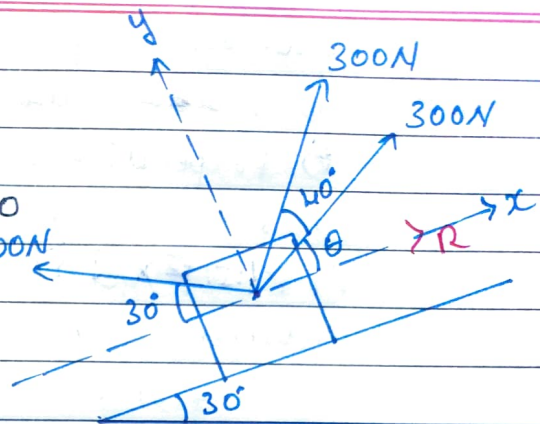
$$\theta = -46.3^\circ$$

Taking $\Sigma F_x = R$

$$-500 \cos 30 + 300 \cos (40 - 46.3) + 300 \cos (-46.3) = R$$

$$-433.01 + 298.18 + 207.26 = R$$

$$R = 72.43 \text{ N}$$



Moment of force [M]

The measure of rotational effect of a force on a body is called moment of force.

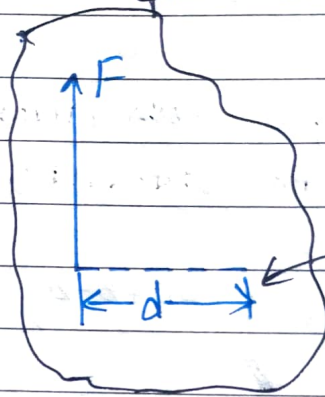
It is calculated as the product of the magnitude of the force & the perpendicular distance b/w the line of action of the forces and the point (or) axis of rotation.



Moment of force depends on two main factors

- 1) Magnitude of the force $[F]$
- 2) Perpendicular distance from the line of action of force $[d]$

$$\therefore M = F \times d$$

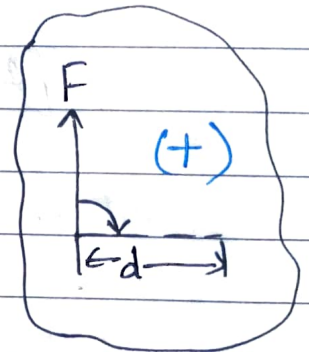


Point of rotation

Types of moment

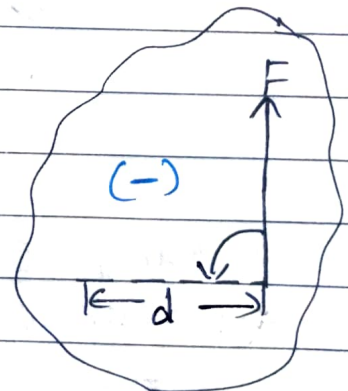
① Clockwise moment

If the given force tends to rotate the body in clockwise direction, then it is said to be a clockwise moment and taken to be +ve.

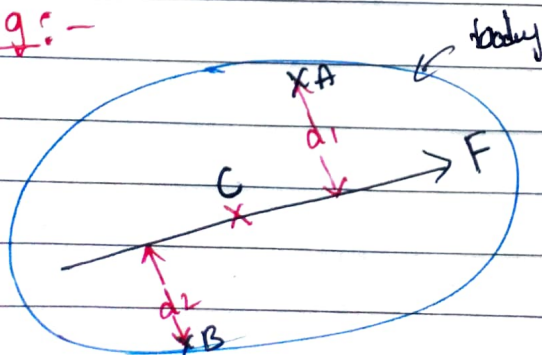


② Anticlockwise moment

If the given force tends to rotate the body in anticlockwise direction, then it is said to be an anticlockwise moment and taken to be -ve.



E.g.:-



$$M_A = -Fd$$

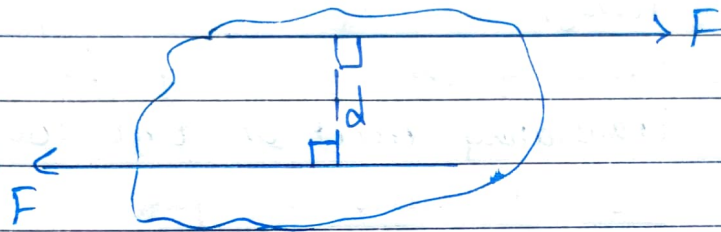
$$M_B = +Fd$$

$$M_C = F \times 0 = 0$$

Couple

Two parallel forces equal in magnitude and opposite direction and separated by a definite distance, constitutes a couple.

The sum of the forces forming a couple in any direction is zero. i.e., The translatory effect of the couple is zero.



Moment of a couple

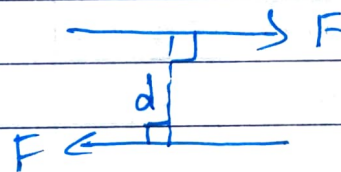
magnitude of

The moment of a couple is the product of, one of the forces and the perpendicular distance between the forces.

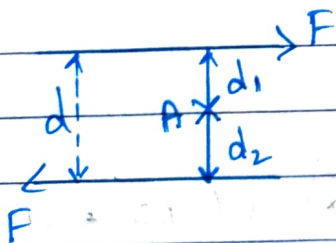
$$\text{i.e., } M = F \times d$$

S.I. Unit:- N-m; kN-m;

N-m; kN-mm;



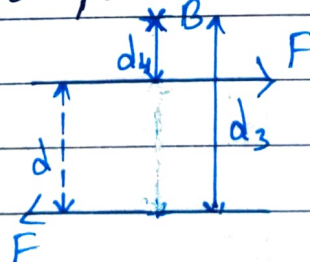
Consider two forces of magnitude 'F' and the perpendicular distance between two forces be 'd'. Now, moment of these forces about a point is given by.



$$M_A = Fd_1 + Fd_2$$

$$= F(d_1 + d_2)$$

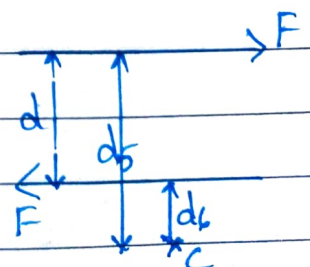
$$M_A = Fd$$



$$M_B = Fd_3 - Fd_4$$

$$= F(d_3 - d_4)$$

$$M_B = Fd$$



$$M_C = Fd_5 - Fd_6$$

$$= F(d_5 - d_6)$$

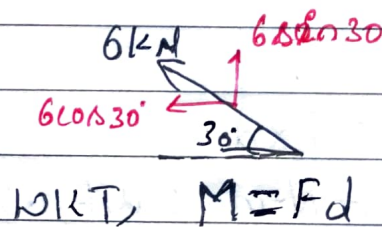
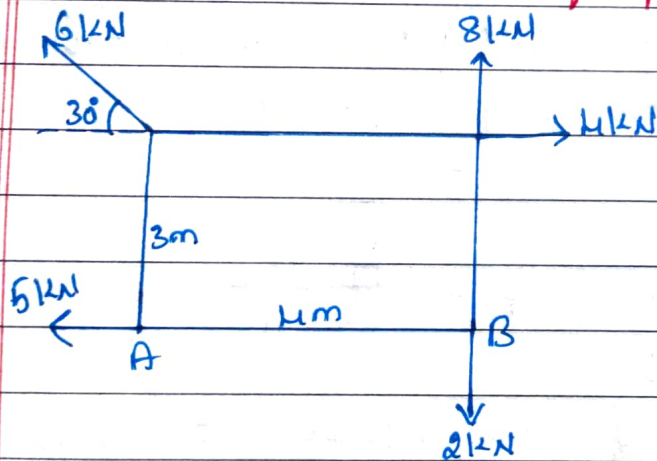
$$M_C = Fd$$

Characteristics of Couple

- ① Two equal & opposite parallel forces are required to form a couple.
- ② The magnitude of the moment of the couple = product of the magnitude of one of the forces and moment arm [Perpendicular distance between the two forces].
- ③ Resultant of the forces of the couple is zero.
- ④ The translatory effect of the couple is zero.

Problems on moment of force

- ① Determine moment of forces at points 'A' & 'B'.



Solⁿ:- Moment about 'A'

$$M_A = -6 \cos 30^\circ \times 3 + 6 \sin 30^\circ \times 0 - (8 \times 4) + (4 \times 3) + (2 \times 4) - (5 \times 0)$$

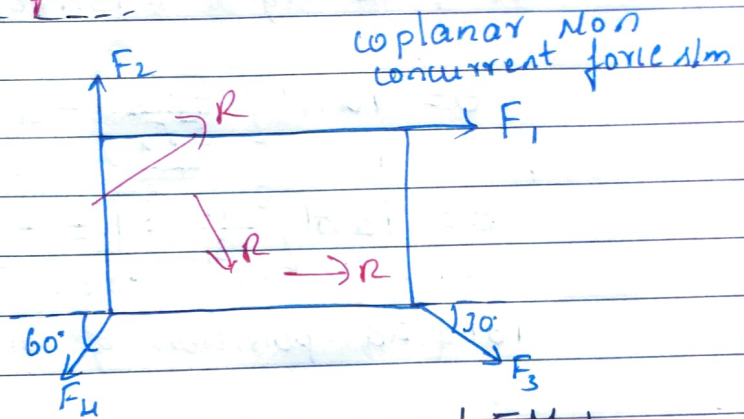
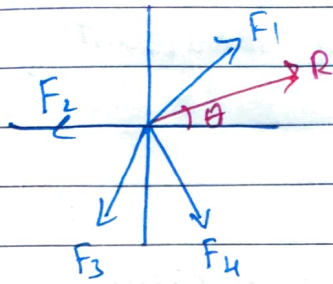
$$M_A = -27.58 \text{ kN-m}$$

Moment about 'B'

$$M_B = -6 \cos 30^\circ \times 3 + 6 \sin 30^\circ \times 4 + (8 \times 0) + (4 \times 3) + (5 \times 0) + (2 \times 0)$$

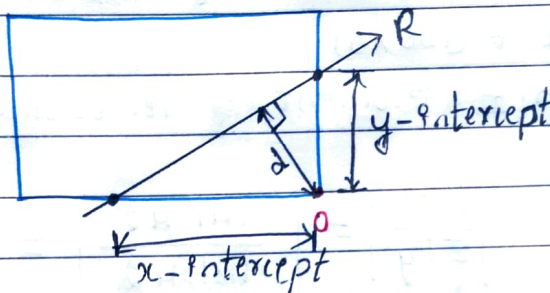
$$M_B = 8.411 \text{ kN-m}$$

Position of the resultant forces



$$x\text{-intercept} = \left| \frac{\sum M}{\sum F_y} \right|$$

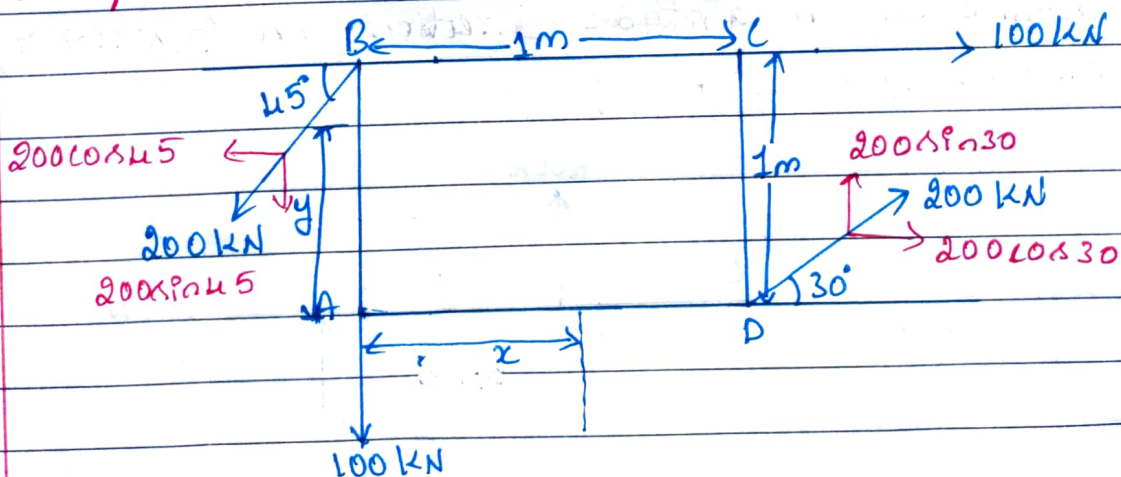
$$y\text{-intercept} = \left| \frac{\sum M}{\sum F_x} \right|$$



$$d = \left| \frac{\sum M}{R} \right| \quad \text{Mode}$$

Problems on coplanar non-concurrent system of forces

- ① A rigid plate ABCD is subjected to forces as shown in figure. Compute the magnitude, direction and line of action of the resultant of the system with reference to the point A.



Soln:-

$$\sum F_x = -200 \cos 45 + 100 + 200 \cos 30 = 131.78 \text{ kN}$$

$$\sum F_y = -200 \sin 45 + 200 \sin 30 - 100 = -141.42 \text{ kN}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{131.78^2 + 141.42^2} = \underline{193.3 \text{ kN}} = \text{Magnitude of resultant of force}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right] = \text{direction of resultant of forces}$$

$$\theta = \tan^{-1} \left[\frac{-141.42}{131.78} \right] = \underline{-47.02^\circ}$$

To find position of resultant of system of forces

$$\Sigma M_A = -200 \cos 45^\circ \times 1 + 200 \sin 45^\circ \times 0 + (100 \times 1) + 200 \cos 30^\circ \times 0 - 200 \sin 30^\circ \times 1 + (100 \times 0)$$

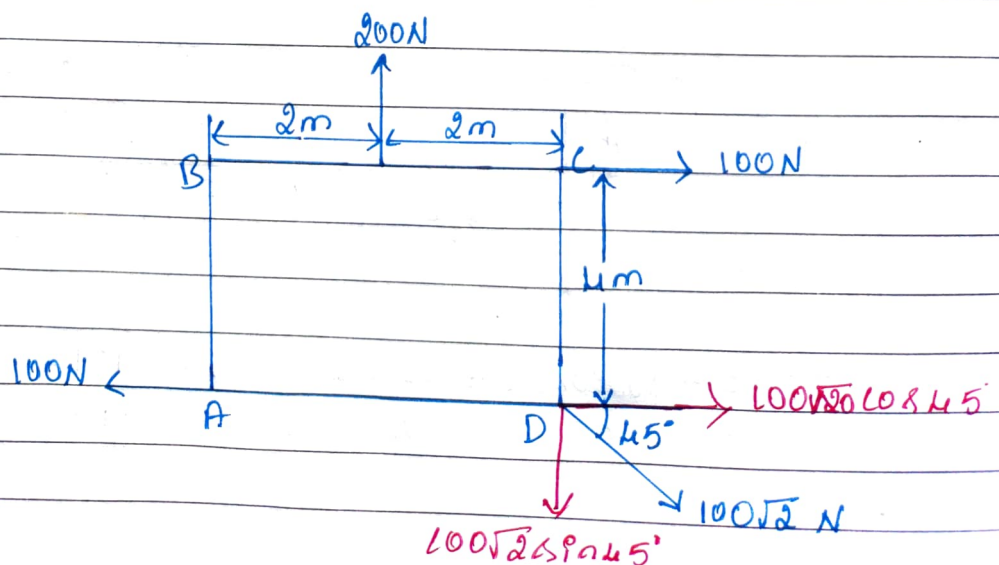
$$\Sigma M_A = \underline{-141.42 \text{ kN-m}} = \text{Moment of resultant of forces.}$$

$$\text{WKT, } x\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_y} \right| = \left| \frac{-141.42}{141.42} \right| = \underline{1 \text{ m}}$$

$$y\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_x} \right| = \left| \frac{-141.42}{131.78} \right| = \underline{1.073 \text{ m}}$$

$$d = \left| \frac{\Sigma M}{R} \right| = \left| \frac{-141.42}{193.3} \right| = \underline{0.732 \text{ m}}$$

- ② For the non-concurrent coplanar system shown in fig. determine the magnitude, direction and position of the resultant force with reference to A.



Solⁿ - $\Sigma F_x = -100 + 100 + 100\sqrt{2} \cos 45^\circ = 100 \text{ N}$
 $\Sigma F_y = 200 - 100\sqrt{2} \sin 45^\circ = 100 \text{ N}$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{100^2 + 100^2} = 141.42 \text{ N}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[\frac{100}{100} \right] = 45^\circ$$

To find the position of the resultant force at point 'A'

$$\Sigma M_A = (100 \times 0) - (200 \times 2) + (100 \times 4) + (100\sqrt{2} \cos 45^\circ \times 0) + (100\sqrt{2} \sin 45^\circ \times 4)$$

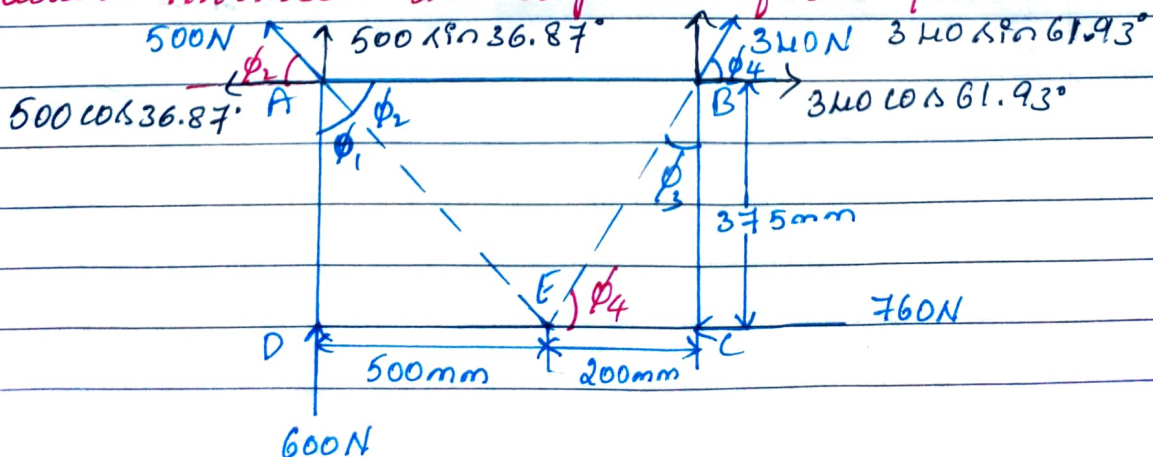
$$\Sigma M_A = 400 \text{ N-m}$$

$$\text{WKT, } x\text{-intercept} = \left| \frac{\Sigma M_A}{\Sigma F_y} \right| = \left| \frac{400}{100} \right| = 4 \text{ m}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_B}{\Sigma F_x} \right| = \left| \frac{400}{100} \right| = 4 \text{ m}$$

$$d = \left| \frac{\Sigma M_O}{R} \right| = \left| \frac{400}{141.42} \right| = 2.82 \text{ m}$$

- ③ Four forces act on a 700mm x 375mm plate as shown in figure. Find the resultant of these forces, and locate the point where the line of action of the resultant intersects the edge AB of the plate.



1) :- From $\Delta^{\text{Ic}} BFC \Rightarrow \tan \phi_4 = \frac{375}{200} \Rightarrow \phi_4 = \tan^{-1} \left(\frac{375}{200} \right)$

From $\Delta^{\text{Ic}} ADE \Rightarrow \phi_4 = 61.93^\circ$
 $\Rightarrow \phi_2 = \tan^{-1} \left(\frac{375}{500} \right) = 36.87^\circ$

$$\Sigma F_x = -500 \cos 36.87 + 340 \cos 61.93 - 760 = -1000 \text{ N}$$

$$\Sigma F_y = 600 + 500 \sin 36.87 + 340 \sin 61.93 = 1200 \text{ N}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{1000^2 + 1200^2} = 1562 \text{ N}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[\frac{1200}{-1000} \right] = -50.19^\circ$$

W.K.T, $\Sigma M_A = 0$

$$\Sigma M_A = (600 \times 0) + (500 \times 0) + (340 \cos 61.93 \times 0) - [340 \sin 61.93 \times 700] + (760 \times 375)$$

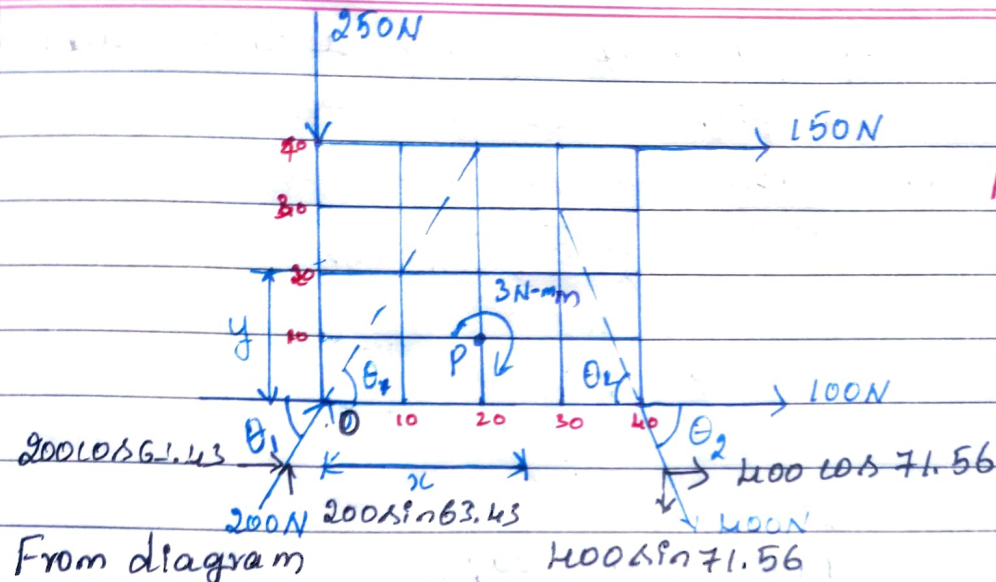
$$\Sigma M_A = 74,995 \text{ N-m}$$

$$x\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_y} \right| = \left| \frac{74995}{1200} \right| = 62.496 \text{ mm}$$

$$y\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_x} \right| = \left| \frac{74995}{1000} \right| = 74.995 \text{ mm}$$

$$d = \left| \frac{\Sigma M}{R} \right| = \left| \frac{74995}{1562} \right| = 48.01 \text{ m}$$

2) Determine the position of resultant of a force system if the external moment $M = 3 \text{ N-m}$ at the point $P(20, 10)$ as shown in figure.



$$\theta_2 = \tan^{-1} \left(\frac{30}{10} \right) = 71.56$$

$$\theta_1 = \tan^{-1} \left(\frac{40}{10} \right) = 63.43^\circ$$

$$\Sigma F_x = 200 \cos 63.43 + 150 + 100 + 400 \cos 71.56 = 465.98 \text{ N}$$

$$\Sigma F_y = 200 \sin 63.43 - 250 - 400 \sin 71.56 = -450.58 \text{ N}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(465.98)^2 + (-450.58)^2} = 648.19 \text{ N}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[\frac{-450.58}{465.98} \right] = -44.03^\circ$$

$$\Sigma M_o = (200 \times 0) + (250 \times 0) + (150 \times 40) + (100 \times 0) + (400 \cos 71.56 \times 0) + (400 \sin 71.56 \times 40) + 3$$

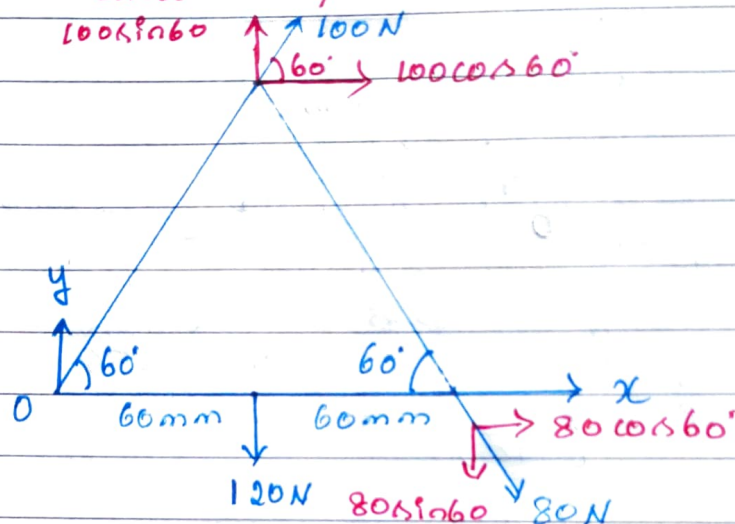
$$\Sigma M_o = 21181.48 - 7520 \text{ N-mm}$$

$$\text{WKT, } x\text{-intercept} = \left| \frac{\Sigma F_y}{\Sigma F_x} \right| = \left| \frac{21181.48 - 7520}{465.98} \right| = 16.68 \text{ mm}$$

$$y\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_x} \right| = \left| \frac{21181.48 - 7520}{465.98} \right| = 16.13 \text{ mm}$$

$$d = \left| \frac{\Sigma M}{R} \right| = \left| \frac{21181.48 - 7520}{648.19} \right| = 11.60 \text{ mm}$$

- ⑤ The system of forces acting on a triangular plate is as shown in figure. Determine the magnitude, direction of the resultant and its position with respect to 'O'.



Solⁿ:- WKT,

$$\Sigma F_x = 100 \cos 60^\circ + 80 \cos 60^\circ = 90 \text{ N}$$

$$\Sigma F_y = 100 \sin 60^\circ - 80 \sin 60^\circ - 120 = -102.68 \text{ N}$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{90^2 + 102.68^2} = 136.54 \text{ N}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[\frac{-102.68}{90} \right] = -48.57^\circ$$

$$\Sigma M_O = 100 \sin 60^\circ \times 60 + 100 \cos 60^\circ \times 60 + (80 \cos 60^\circ \times 0) + (80 \sin 60^\circ \times 120) + (120 \times 60) + (100 \times 0)$$

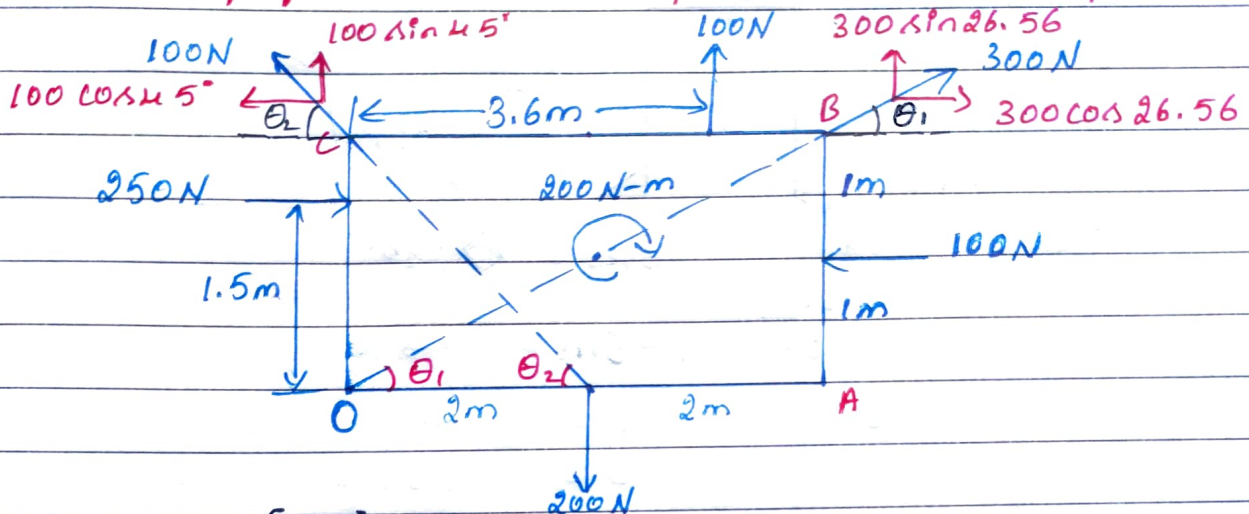
$$\Sigma M_O = 15,517.7 \text{ N-mm}$$

$$x\text{-intercept} = \left| \frac{\Sigma M_O}{\Sigma F_y} \right| = \left| \frac{15517.7}{102.68} \right| = 151.12 \text{ mm}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_O}{\Sigma F_x} \right| = \left| \frac{15517.7}{90} \right| = 172.41 \text{ mm}$$

$$d = \left| \frac{\Sigma M_O}{R} \right| = \left| \frac{15517.7}{136.54} \right| = 113.64 \text{ mm}$$

Determine magnitude, direction & point of application of the resultant force for the system of coplanar forces shown in figure. Locate the position with respect to 'O'.



$$\theta_1 = \tan^{-1} \left[\frac{2}{4} \right] = 26.56^\circ$$

$$\theta_2 = \tan^{-1} \left[\frac{2}{2} \right] = 45^\circ$$

$$\Sigma F_x = 250 - 100 \cos 45 + 300 \cos 26.56 - 100 = 347.6 \text{ N}$$

$$\Sigma F_y = 100 \sin 45 + 100 + 300 \sin 26.56 - 200 = 104.85 \text{ N}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{347.6^2 + 104.85^2} = 363 \text{ N}$$

$$\theta = \tan^{-1} \left[\frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[\frac{104.85}{347.6} \right] = 16.78^\circ$$

$$\Sigma M_o = (250 \times 1.5) - (100 \cos 45 \times 2) - (100 \times 3.6) + (300 \times 0) - (100 \times 1) + 200 + (200 \times 2) - (100 \sin 45 \times 0) - (300 \sin 26.56 \times 4) + (300 \cos 26.56 \times 2)$$

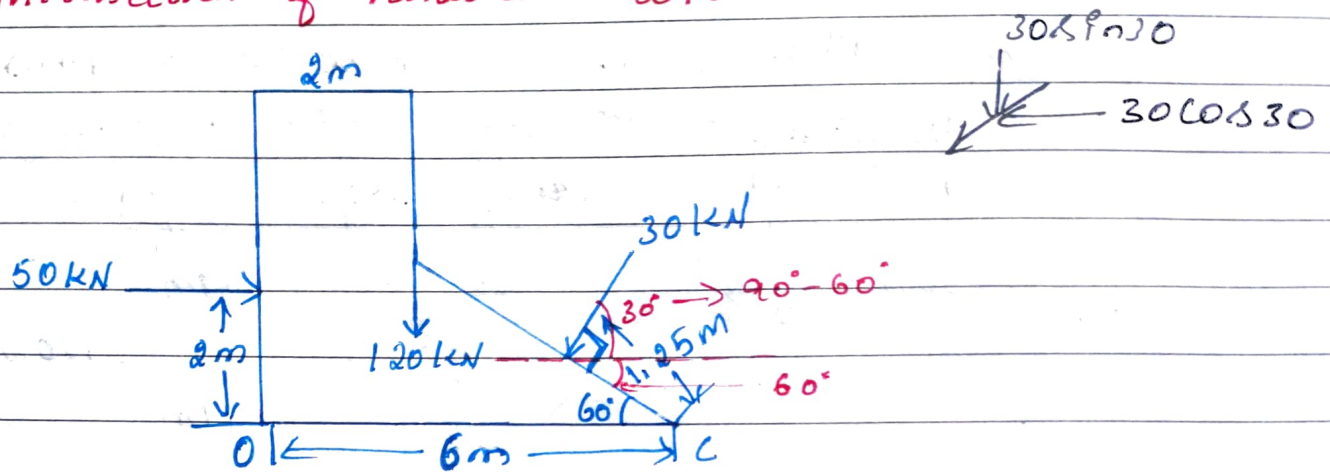
$$\Sigma M_o = 373.7 \text{ N-m}$$

$$x\text{-intercept} = \left| \frac{\Sigma M_o}{\Sigma F_y} \right| = \left| \frac{373.7}{104.85} \right| = 3.56 \text{ m}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_o}{\Sigma F_x} \right| = \left| \frac{373.7}{347.6} \right| = 1.07 \text{ m}$$

$$d = \left| \frac{\Sigma M_o}{R} \right| = \left| \frac{373.7}{363} \right| = 1.02 \text{ m}$$

- ⑦ A dam section is shown in figure. Determine the point of intersection of resultant with the base.



$$\Sigma F_x = 50 - 30 \cos 30 = 24 \text{ kN}$$

$$\Sigma F_y = -120 - 30 \sin 30 = -135 \text{ kN}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{24^2 + 135^2} = 137.11 \text{ kN}$$

$$\theta = \tan^{-1} \left(\frac{\Sigma F_y}{\Sigma F_x} \right) = \tan^{-1} \left(\frac{-135}{24} \right) = 79.91^\circ$$

Locate the point of intersection

$$\Sigma M_c = (50 \times 2) - (120 \times 4) - (30 \times 1.25) = -417.5 \text{ kN-m}$$

$$x\text{-intercept} = \left| \frac{\Sigma M_c}{\Sigma F_y} \right| = \left| \frac{-417.5}{-135} \right| = 3.09 \text{ m}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_c}{\Sigma F_x} \right| = \left| \frac{-417.5}{24} \right| = 17.39 \text{ m}$$

$$d = \left| \frac{\Sigma M}{R} \right| = \left| \frac{-417.5}{137.11} \right| = 3.04 \text{ m}$$

Imp

Varignon's theorem of momentsStatement:-

The algebraic sum of moments of system of forces about a point is equal to moment of their resultant about the same point.

Proof:-

$$F_1 d_1 + F_2 d_2 = R d$$

$$R d = F_1 d_1 + F_2 d_2$$

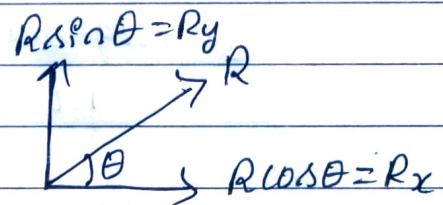
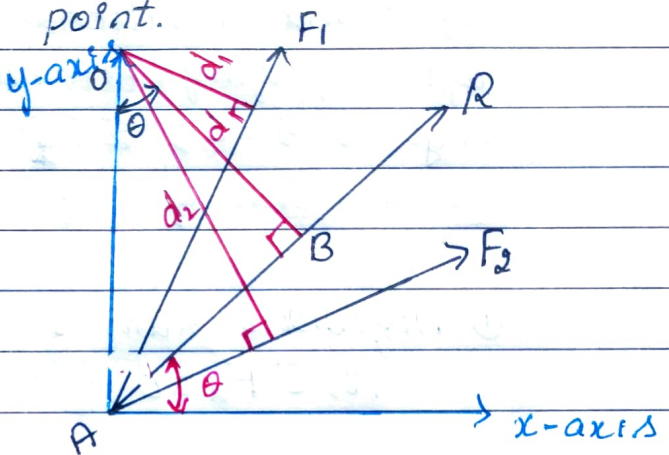
From $\triangle OBA$

$$\cos \theta = \frac{d}{AO} \Rightarrow d = AO \cdot \cos \theta$$

$$\therefore R (AO \cdot \cos \theta) = R d$$

$$R d = AO \cdot (R \cos \theta)$$

$$R d = AO \cdot (R_x) \rightarrow (1)$$

III^{ly}

$$F_1 d_1 = AO [F_{1x}] \rightarrow (2)$$

$$F_2 d_2 = AO [F_{2x}] \rightarrow (3)$$

Add eqⁿ (2) & (3)

$$\Rightarrow F_1 d_1 + F_2 d_2 = AO [F_{1x}] + AO [F_{2x}]$$

$$= AO [F_{1x} + F_{2x}]$$

$$F_1 d_1 + F_2 d_2 = AO \cdot (R_x) \rightarrow (4)$$

equate eqⁿ (1) & (4)

$$\boxed{R d = F_1 d_1 + F_2 d_2}$$

$F_{1x} + F_{2x} = R_x$
 $\therefore$ Sum of H^{le} components of system of forces is equal to H^{le} component of resultant force.

Equilibrium and conditions of equilibrium

Equilibrium

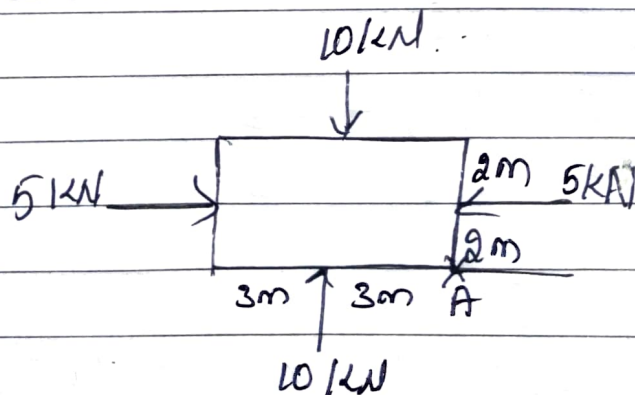
A body is said to be in equilibrium when there is no translatory effect and rotational effect even after applying external forces.

$$\Sigma F_y = +5 - 5 = 0; \quad \Sigma F_x = 10 - 10 = 0;$$

Conditions of Equilibrium

- ① Algebraic sum of forces in x-direction is zero
i.e., $\Sigma F_x = 0$.
- ② Algebraic sum of forces in y-direction is zero
i.e., $\Sigma F_y = 0$.
- ③ Algebraic sum of moments of system of forces is zero.
i.e., $\Sigma M = 0$.

Eg:-



$$\Sigma F_x = +10 - 10 = 0$$

$$\Sigma F_y = +5 - 5 = 0$$

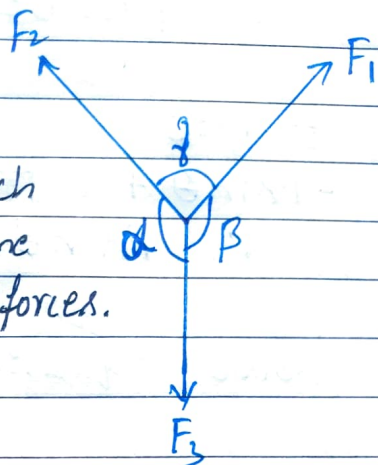
$$\Sigma M_A = (10 \times 2) - (5 \times 3) - (10 \times 2) + (5 \times 3) = 0$$

V. Imp

Lami's theoremStatement

If three forces acting at a point are in equilibrium then each force is directly proportional to sine of angle between the other two forces.

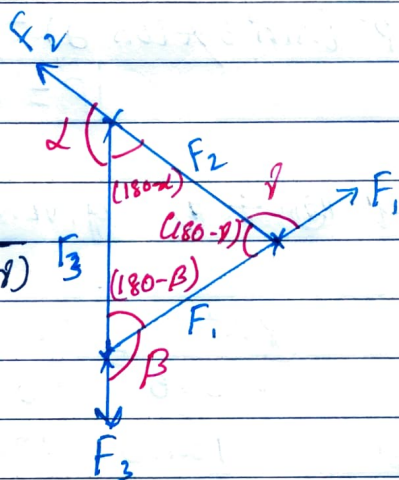
$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

Applying sine rule

$$\frac{F_1}{\sin(180-\alpha)} = \frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\gamma)}$$

$$\therefore \sin(180-\theta) = \sin \theta$$

$$\boxed{\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}}$$

Problems on equilibrium of forces

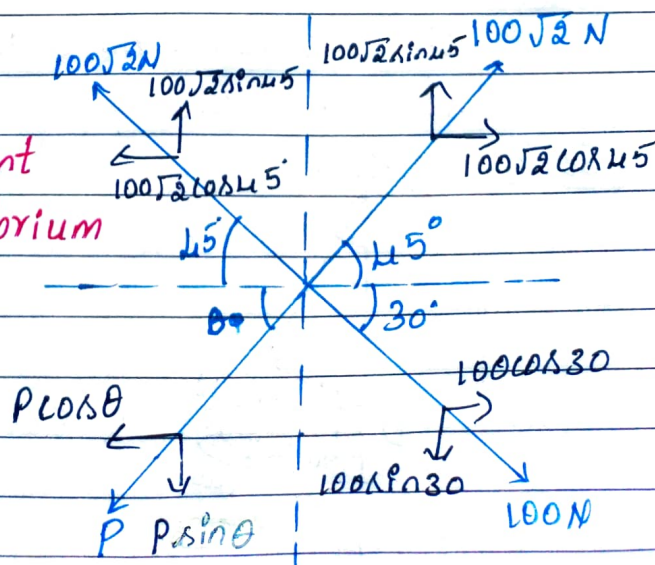
- ① Determine the magnitude and direction of force P , which keeps the concurrent system of forces in equilibrium

Sol:-

Applying eq^m condition

$$\sum F_{xc} = 0$$

$$-100\sqrt{2}\cos 45^\circ + 100\sqrt{2}\cos 45^\circ + 100\cos 30^\circ - P\cos \theta = 0$$



$$\therefore P \cos \theta = 86.6 \text{ N} \rightarrow \textcircled{1}$$

$$\Sigma F_y = 0$$

$$-P \sin \theta + 100\sqrt{2} \sin 45^\circ + 100\sqrt{2} \sin 45^\circ - 100 \sin 30^\circ = 0$$

$$\therefore P \sin \theta = 150 \text{ N} \rightarrow \textcircled{2}$$

Solve Squaring and adding eqn ① & ②

$$P^2 \sin^2 \theta + P^2 \cos^2 \theta = 150^2 + 86.6^2$$

$$P^2 (\sin^2 \theta + \cos^2 \theta) = 29999.56$$

$$P = 173.2 \text{ N}$$

Eqn ② $\div$ ① gives

$$\frac{P \sin \theta}{P \cos \theta} = \frac{150}{86.6}$$

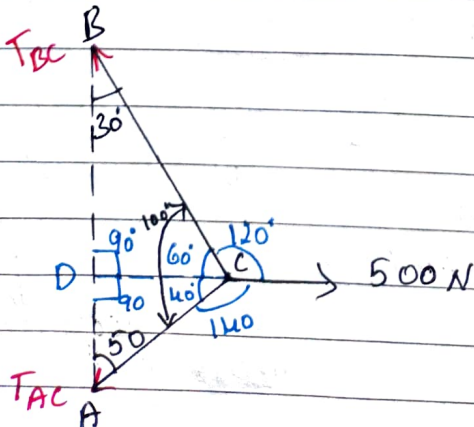
$$\tan \theta = \frac{150}{86.6}$$

$$\theta = \tan^{-1} \left(\frac{150}{86.6} \right)$$

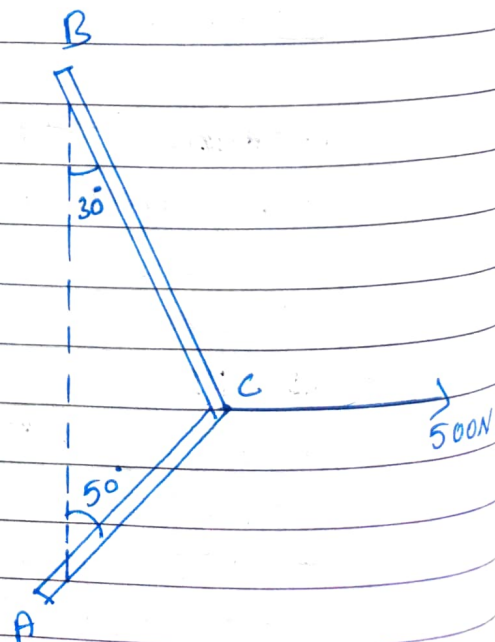
$$\theta = 60^\circ$$

- ② Determine the forces induced in the members AC & BC of the structure shown in figure.

Soln:-



T - Tensile force



Applying Lami's Theorem

$$\frac{500}{\sin 100} = \frac{T_{BC}}{\sin 140} = \frac{T_{AC}}{\sin 120}$$

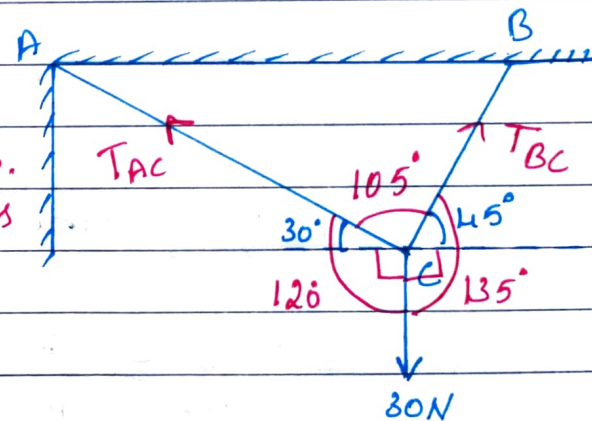
consider, $\frac{T_{BC}}{\sin 140} = \frac{500}{\sin 100}$

$$\therefore T_{BC} = 326.35 \text{ N}$$

consider, $\frac{T_{AC}}{\sin 120} = \frac{500}{\sin 100}$

$$\therefore T_{AC} = 439.69 \text{ N}$$

- ③ Two cables are connected at A & B as shown in figure. A force of 30 kN is applied at C. Determine the forces in the cables CA & CB.



Sol:- Applying Lami's theorem

$$\frac{T_{AC}}{\sin 135} = \frac{30}{\sin 105} = \frac{T_{BC}}{\sin 120}$$

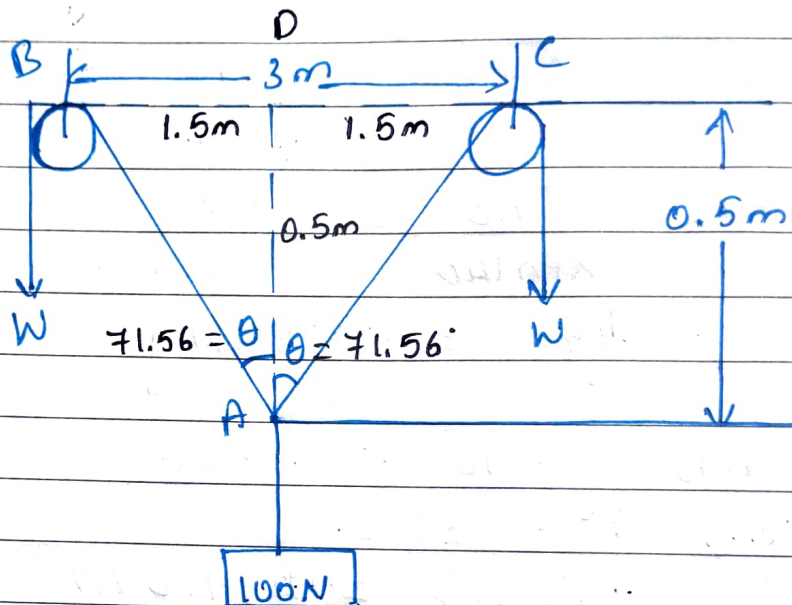
consider, $\frac{T_{AC}}{\sin 135} = \frac{30}{\sin 105}$

$$\therefore T_{AC} = 21.96 \text{ kN}$$

consider, $\frac{T_{BC}}{\sin 120} = \frac{30}{\sin 105}$

$$T_{BC} = 26.89 \text{ kN}$$

- ④ Find the value of w , which is required to maintain equilibrium configuration as shown in figure.



From Figure, $\tan \theta = \frac{1.5}{0.5}$

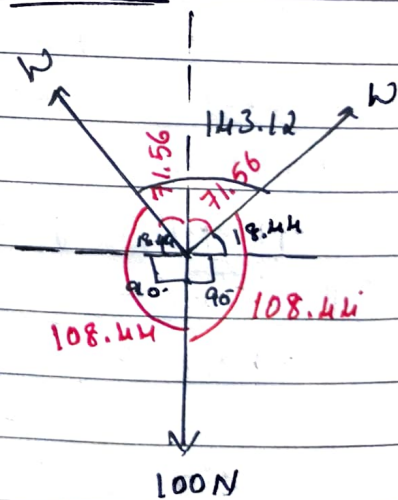
$$\therefore \theta = \tan^{-1} \left(\frac{1.5}{0.5} \right) = 71.56^\circ$$

Applying sine rule

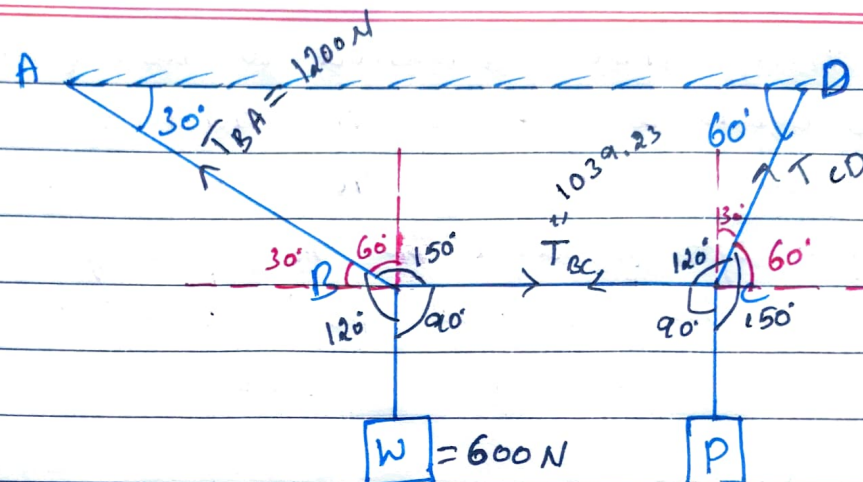
$$\frac{100}{\sin 143.12} = \frac{w}{\sin 108.44} = \frac{w}{\sin 108.44}$$

$$\therefore w = \frac{100 \sin 108.44}{\sin 143.12}$$

$$w = 158.10 \text{ N}$$



- ⑤ Figure below shows a rope supporting two loads w & P . If BC is horizontal and $w = 600 \text{ N}$, determine the load P . Also, find the tensile forces developed in the different segments of the rope.



Solⁿ:- Consider the joint 'B' & apply Lami's theorem

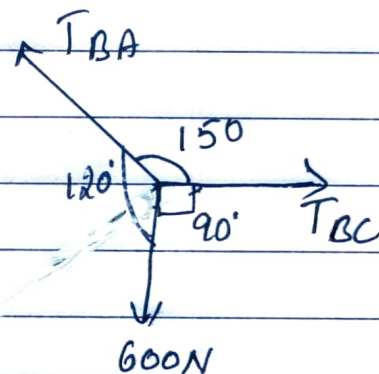
$$\frac{T_{BA}}{\sin 90^\circ} = \frac{600}{\sin 150^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

consider, $\frac{T_{BA}}{\sin 90^\circ} = \frac{600}{\sin 150^\circ}$

$$\therefore \boxed{T_{BA} = 1200 \text{ N}}$$

consider, $\frac{T_{BC}}{\sin 120^\circ} = \frac{600}{\sin 150^\circ}$

$$\therefore \boxed{T_{BC} = 1039.23 \text{ N}}$$



consider the joint 'C' & apply Lami's theorem

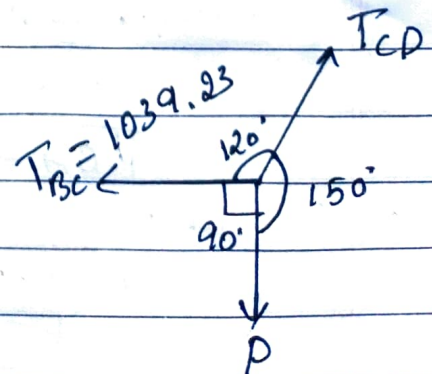
$$\frac{T_{CD}}{\sin 90^\circ} = \frac{1039.23}{\sin 150^\circ} = \frac{P}{\sin 120^\circ}$$

consider, $\frac{T_{CD}}{\sin 90^\circ} = \frac{1039.23}{\sin 150^\circ}$

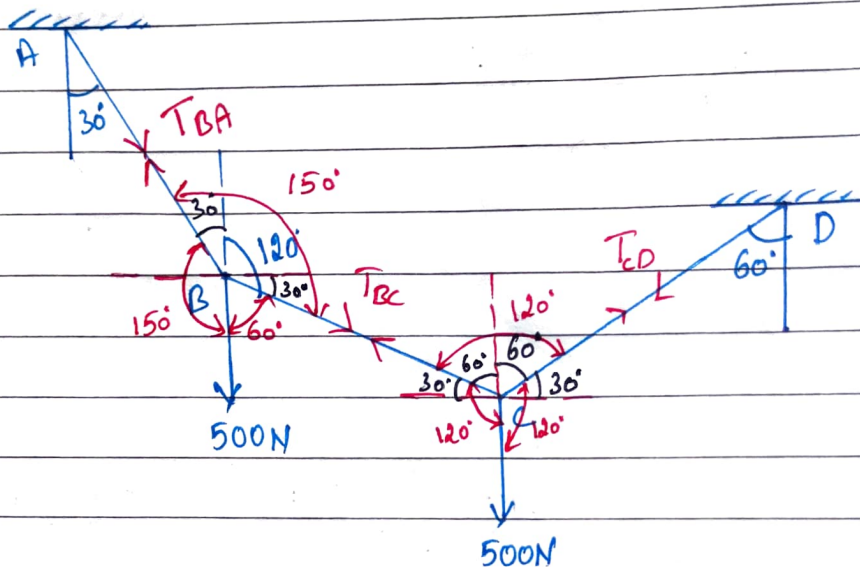
$$\therefore \boxed{T_{CD} = 2078.46 \text{ N}}$$

consider, $\frac{P}{\sin 120^\circ} = \frac{1039.23}{\sin 150^\circ}$

$$\therefore \boxed{P = 1800 \text{ N}}$$



A string ABCD attached to two fixed points A & D has two equal weights 500N attached to it at B and C. The weights rest with portions AB and CD inclined at angles of 30° and 60° respectively with the vertical. Find the tensions in the portions AB, BC and CD of the string. The inclination of BC with vertical is 120° .



consider joint 'B' & Apply Lami's theorem

$$\text{WKT, } \frac{T_{BA}}{\sin 60} = \frac{500}{\sin 150} = \frac{T_{BC}}{\sin 150}$$

$$\text{consider, } \frac{T_{BA}}{\sin 60} = \frac{500}{\sin 150}$$

$$\therefore T_{BA} = 866.02 \text{ N}$$

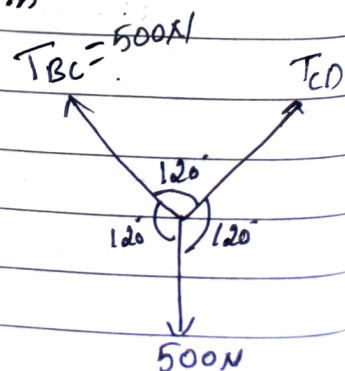
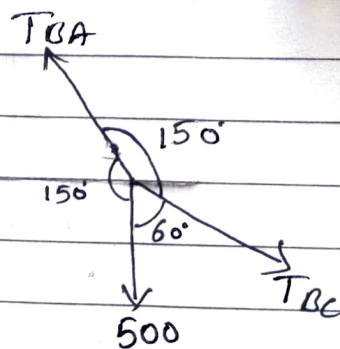
$$\text{consider, } \frac{T_{BC}}{\sin 150} = \frac{500}{\sin 150}$$

$$\therefore T_{BC} = 500 \text{ N}$$

consider joint 'C' & Apply Lami's theorem

$$\text{WKT, } \frac{500}{\sin 120} = \frac{500}{\sin 120} = \frac{T_{CD}}{\sin 120}$$

$$\therefore T_{CD} = 500 \text{ N}$$

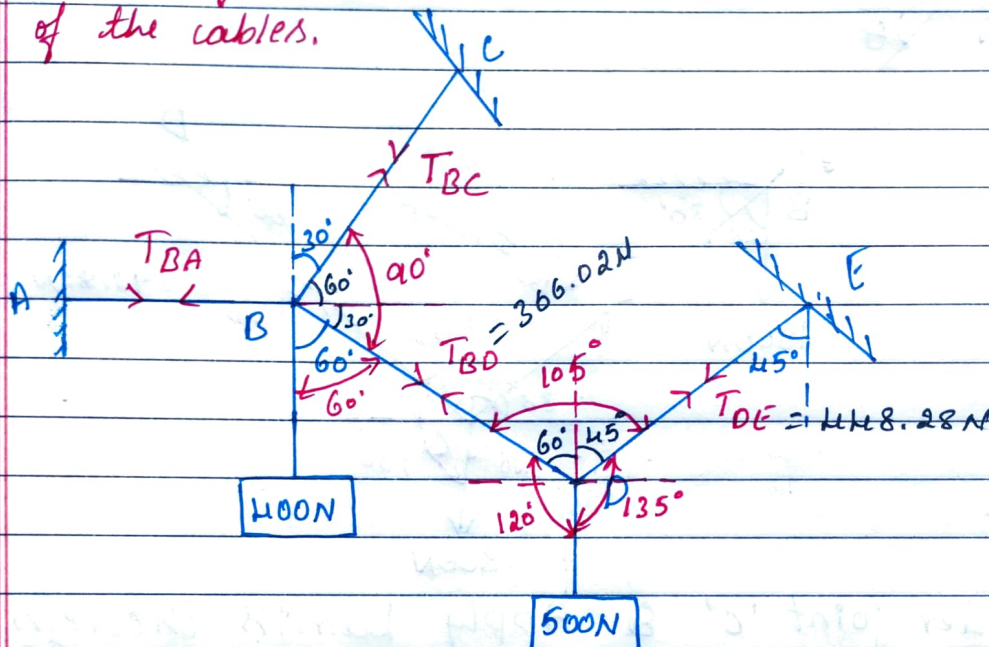


Lami's theorem is applicable only for
Three concurrent force system

PAGE 003

DATE: / /

- ⑦ The system of connected flexible cables shown in figure is supporting 2 loads of 400 N & 500 N at points B & D, respectively. Determine the tensions in the various segments of the cables.

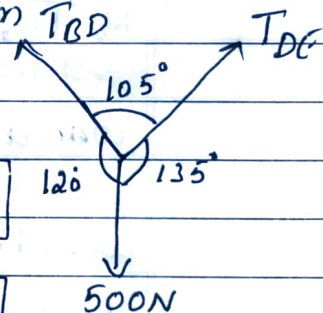


consider joint 'D' & apply Lami's theorem T_{BD}

$$\text{WKT, } \frac{T_{BD}}{\sin 135} = \frac{500}{\sin 105} = \frac{T_{DE}}{\sin 120}$$

$$\text{consider, } \frac{T_{BD}}{\sin 135} = \frac{500}{\sin 105} \Rightarrow \boxed{T_{BD} = 366.02 \text{ N}}$$

$$\text{consider, } \frac{T_{DE}}{\sin 120} = \frac{500}{\sin 105} \Rightarrow \boxed{T_{DE} = 448.28 \text{ N}}$$



consider joint 'B'

Applying eq^m condition

$$\text{i.e., } \sum F_x = 0$$

$$-T_{BA} + T_{BC} \cos 60 + 366.02 \cos 30 = 0 \rightarrow \textcircled{1}$$

$$\text{Applying } \sum F_y = 0$$

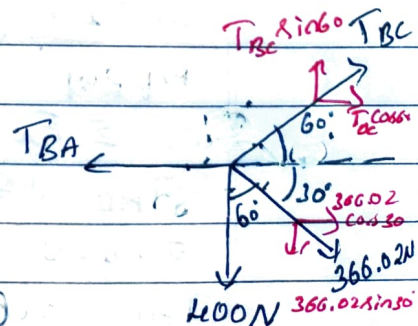
$$T_{BC} \sin 60 - 366.02 \sin 30 - 400 = 0 \rightarrow \textcircled{2}$$

$$\boxed{T_{BC} = 673.2 \text{ N}}$$

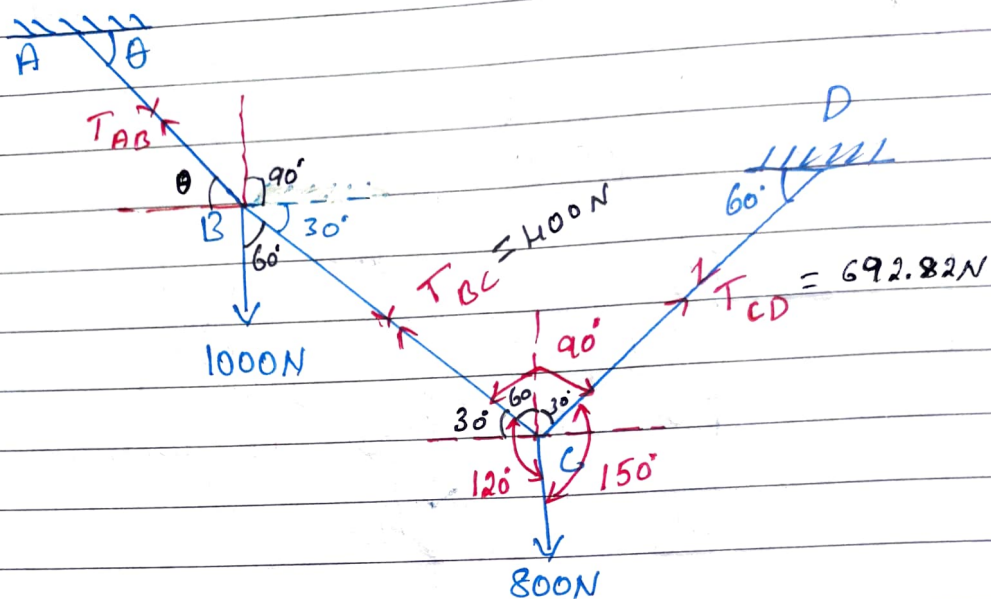
Substitute the value of T_{BC} in eqⁿ ①

$$-T_{BA} + 673.2 \cos 60 + 366.02 \cos 30 = 0$$

$$\boxed{T_{BA} = 653.58 \text{ N}}$$



⑧ Compute the tensions in the strings AB, BC and CD as shown in figure.

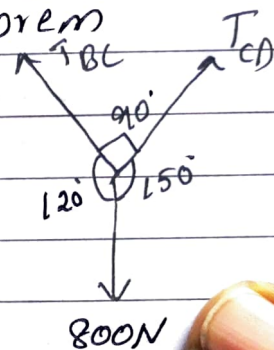


Solⁿ:- Consider joint 'C' & Apply Lami's theorem

$$\frac{T_{BC}}{\sin 150^\circ} = \frac{800}{\sin 90^\circ} = \frac{T_{CD}}{\sin 120^\circ}$$

consider, $\frac{T_{BC}}{\sin 150^\circ} = \frac{800}{\sin 90^\circ} \Rightarrow T_{BC} = 400 \text{ N}$

consider, $\frac{T_{CD}}{\sin 120^\circ} = \frac{800}{\sin 90^\circ} \Rightarrow T_{CD} = 692.82 \text{ N}$



consider joint 'B' & Apply Lami's theorem

Apply eqⁿ conditions

ie, $\Sigma F_x = 0$

$$-T_{AB} \cos \theta + 400 \cos 30^\circ = 0 \rightarrow \textcircled{1}$$

$$\Sigma F_y = 0$$

$$\therefore T_{AB} \cos \theta = 346.41 \text{ N} \rightarrow \textcircled{1}$$

$$T_{AB} \sin \theta - 400 \sin 30^\circ - 1000 = 0$$

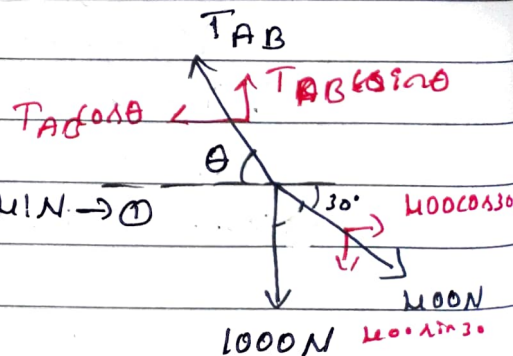
$$\therefore T_{AB} \sin \theta = 1200 \text{ N} \rightarrow \textcircled{2}$$

Squaring & adding eqⁿ ① & ②

$$T_{AB}^2 \sin^2 \theta + T_{AB}^2 \cos^2 \theta = 1200^2 + 346.41^2$$

$$T_{AB}^2 (\sin^2 \theta + \cos^2 \theta) = 1200^2 + 346.41^2$$

$$\therefore T_{AB} = 1249 \text{ N}$$



eqn ② $\div$ eqn ① gives

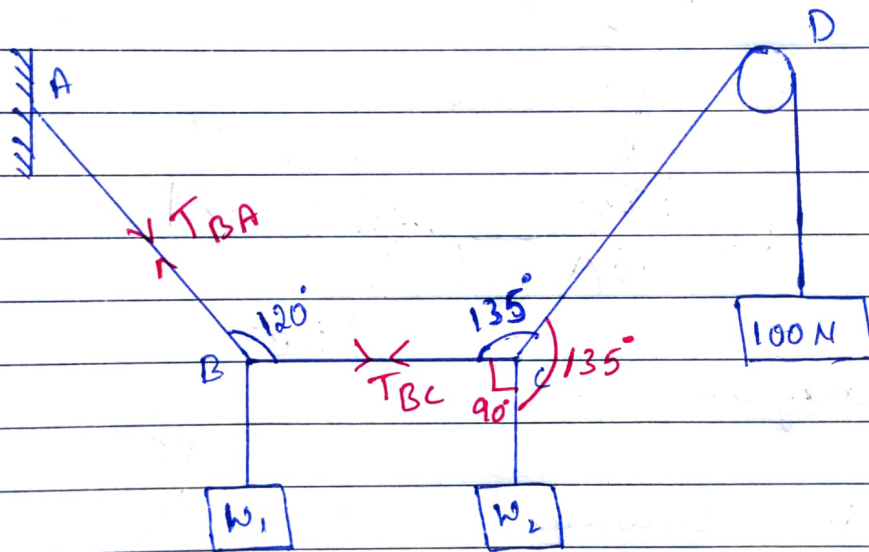
$$\frac{T_{AB} \sin \theta}{T_{AB} \cos \theta} = \frac{1200}{346.41}$$

$$\tan \theta = \frac{1200}{346.41}$$

$$\theta = \tan^{-1} \left(\frac{1200}{346.41} \right)$$

$$\therefore \theta = 73.9^\circ$$

- ⑨ Determine the tension in different parts of the string shown in figure. Also, find the value of w_1 & w_2 , if the portion BC is horizontal.

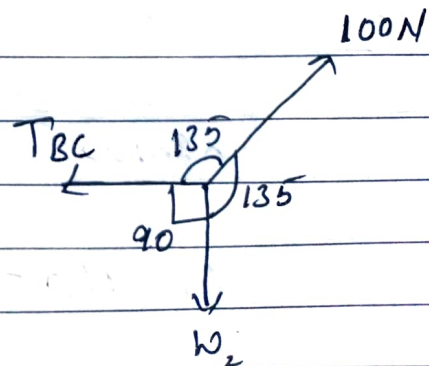


Sol:- Joint 'C' & Apply Lami's theorem

$$\frac{T_{BC}}{\sin 135} = \frac{w_2}{\sin 135} = \frac{100}{\sin 90}$$

$$T_{BC} = \frac{100 \sin 135}{\sin 90} = 70.71 \text{ N}$$

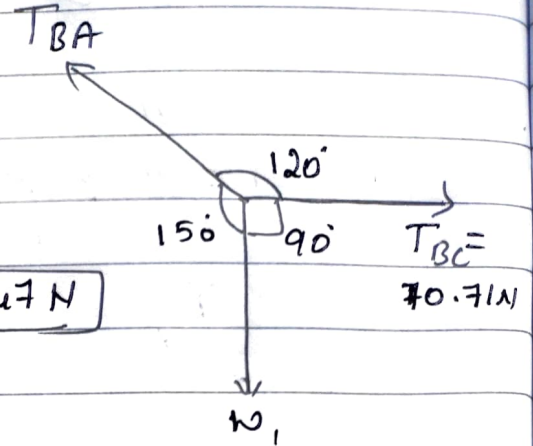
$$w_2 = \frac{100 \sin 135}{\sin 90} = 70.71 \text{ N}$$



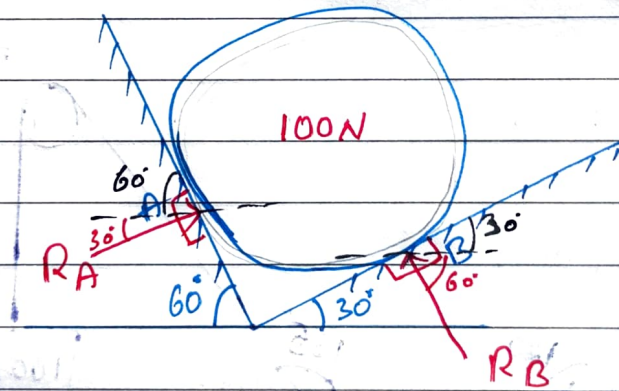
Joint 'B' & Apply Lami's theorem

$$\frac{T_{BA}}{\sin 90^\circ} = \frac{W_1}{\sin 120^\circ} = \frac{70.71}{\sin 150^\circ}$$

$$\therefore T_{BA} = 141.42 \text{ N} \quad \& \quad W_1 = 122.47 \text{ N}$$



- (10) A sphere weighing 100N is fitted in a right-angled notch as shown in figure. If all contact surfaces are smooth, determine the reaction at contact surfaces.



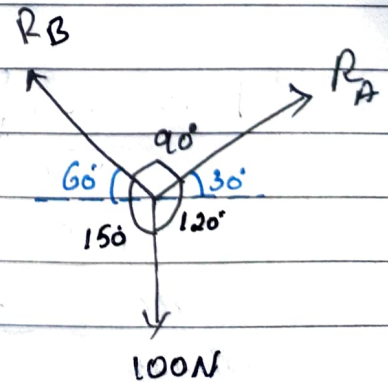
Solⁿ: Free body diagram

Apply Lami's theorem

$$\frac{R_B}{\sin 120^\circ} = \frac{R_A}{\sin 150^\circ} = \frac{100}{\sin 90^\circ}$$

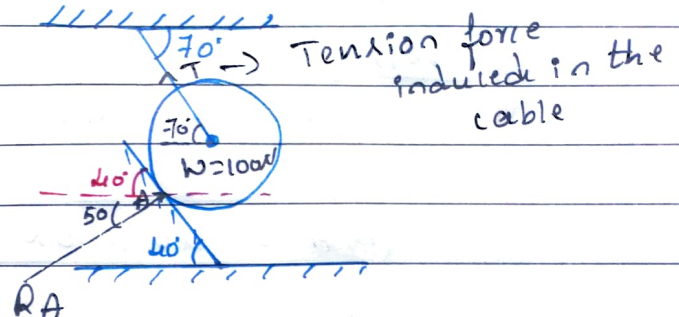
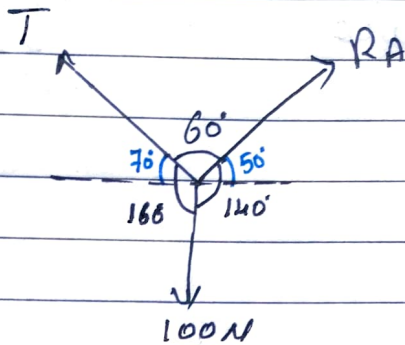
$$\therefore R_A = 50 \text{ N}$$

$$R_B = 86.6 \text{ N}$$



- (11) A sphere weighing 100 N is fitted in a right angled notch as shown in figure. If all contact surfaces are smooth, determine the reaction at contact surfaces.

Solⁿ:- Free body diagram



Apply Lami's theorem

$$\frac{T}{\sin 140} = \frac{R_A}{\sin 160} = \frac{100}{\sin 60}$$

$$74.22\text{ N}$$

$$\therefore R_A = 177.7\text{ N}$$

$$39.49$$

$$T = 334\text{ N}$$

- (12) A horizontal shaft with inner clearance of 1000 mm carries two spheres of radius 350 mm & 250 mm as shown in figure. The weights are 600 N & 500 N respectively. Find the reactions at all the points of contact.

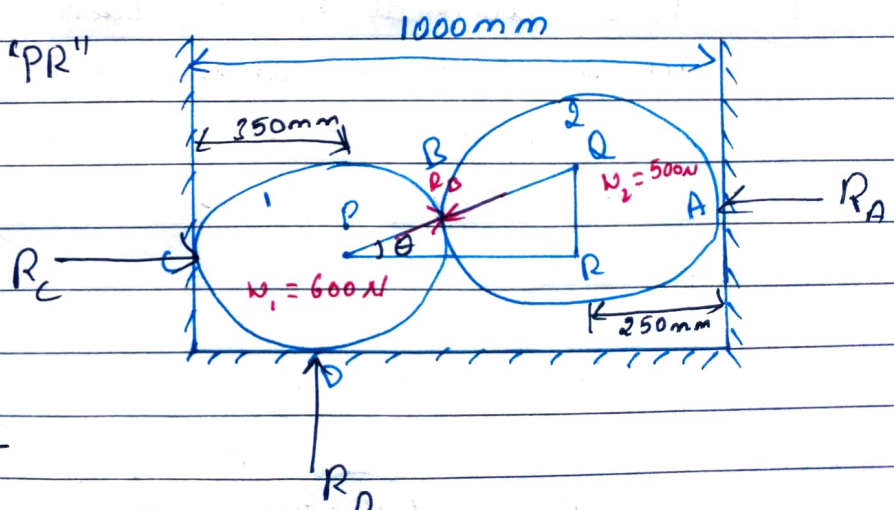
Solⁿ:- From figure length "PR"
 $= 1000 - 350 - 250$
 $= 400\text{ mm}$

$$PQ = 350 + 250$$

$$PQ = 600\text{ mm}$$

$$\cos \theta = \frac{PR}{PQ} = \frac{400}{600}$$

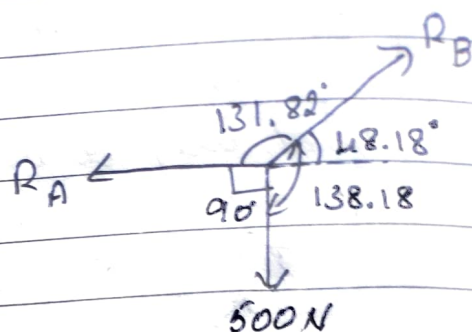
$$\theta = 48.18^\circ$$



consider sphere - 2

Apply Lamis theorem

$$\frac{R_A}{\sin 138.18} = \frac{R_B}{\sin 90} = \frac{500}{\sin 131.82}$$



$$\therefore R_A = 447.36 \text{ N}$$

$$R_B = 670.92 \text{ N}$$

consider sphere - 1

Apply $\Sigma F_x = 0$

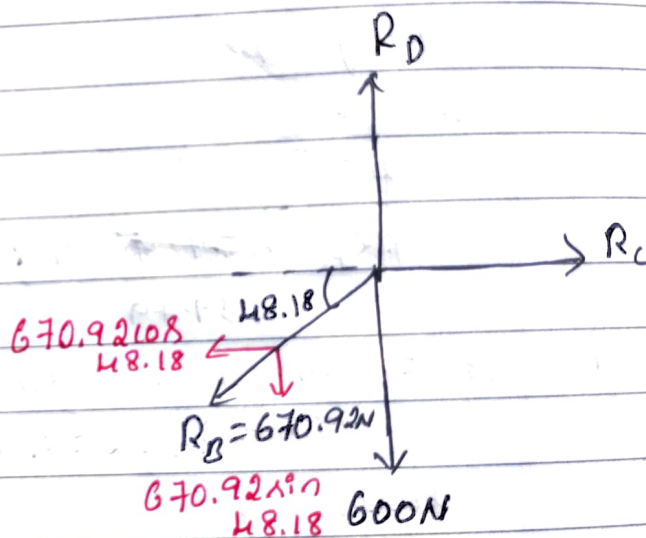
$$-670.92 \sin 48.18 + R_C = 0$$

$$R_C = 447.36 \text{ N}$$

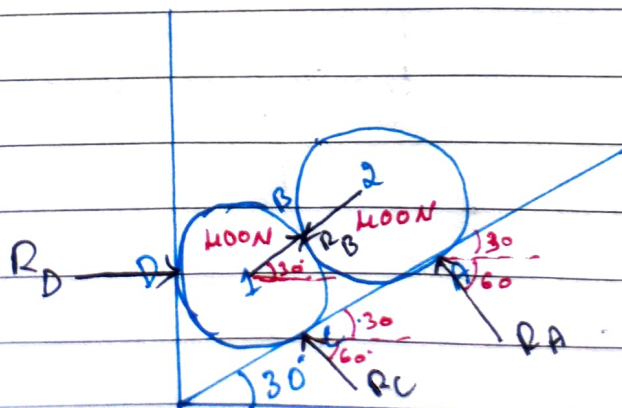
$$\Sigma F_y = 0$$

$$R_D - 600 - 670.92 \sin 48.18 = 0$$

$$R_D = 1100 \text{ N}$$



- (13) Two identical rollers, each weighing 1000 N, are placed in a trench as shown in figure. Assuming that all contact surfaces are smooth, determine the reactions at contact points A, B, C and D.



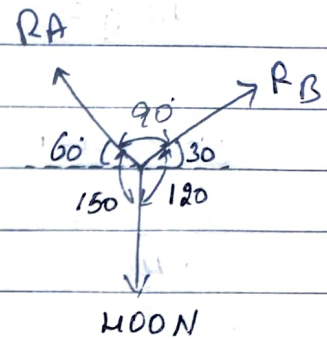
Solⁿ?- consider Sphere-2

Apply Lami's theorem

$$\frac{R_A}{\sin 120} = \frac{R_B}{\sin 150} = \frac{400}{\sin 90}$$

$$\therefore R_A = 346.41 \text{ N}$$

$$R_B = 200 \text{ N}$$



consider Sphere-1

$$\Sigma F_x = 0$$

$$-R_C \cos 60 - 200 \cos 30 + R_D = 0 \rightarrow (1)$$

$$\Sigma F_y = 0$$

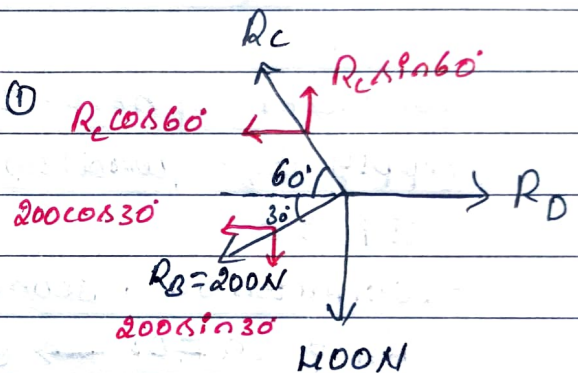
$$R_C \sin 60 - 200 \sin 30 - 400 = 0$$

$$\therefore R_C = 577.35 \text{ N}$$

Substitute the value of R_C in eqⁿ (1)

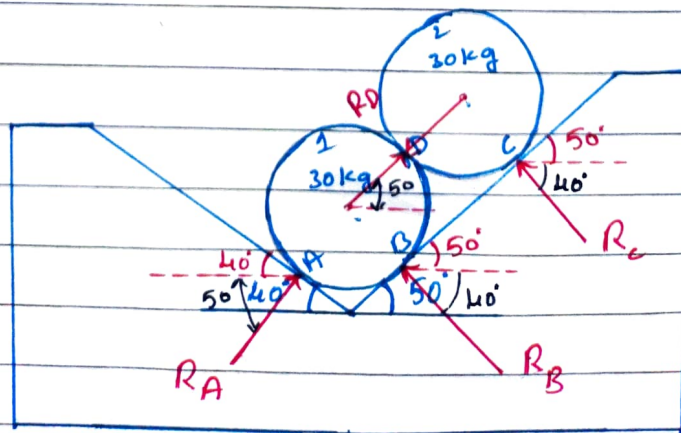
$$-200 \cos 30 - 577.35 \cos 60 + R_D = 0$$

$$R_D = 461.88 \text{ N}$$



- (14) Calculate the reaction exerted at the contact points A, B, C and D as shown in figure.

Solⁿ?-

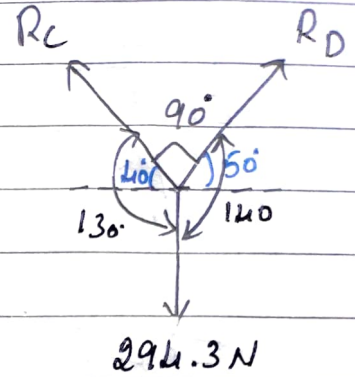


$$1 \text{ kg} = 9.81 \text{ N} \Rightarrow 30 \text{ kg} = 30 \times 9.81 = \underline{294.3 \text{ N}}$$

consider Sphere-2

$$\frac{R_C}{\sin 140} = \frac{294.3}{\sin 90} = \frac{R_D}{\sin 130}$$

$$\therefore \begin{cases} R_C = 189.17 \text{ N} \\ R_D = 225.44 \text{ N} \end{cases}$$



consider Sphere-1

Apply Eqm condition

$$\Sigma F_x = 0$$

$$-225.44 \cos 50 - R_B \cos 40$$

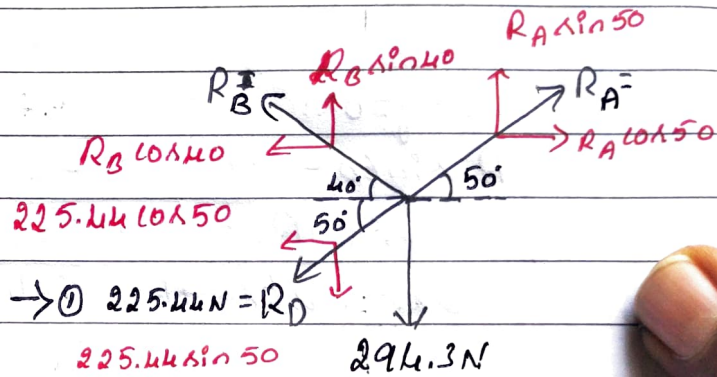
$$+ R_A \cos 50 = 0 \rightarrow \textcircled{1}$$

$$0.642 R_A - 0.766 R_B = 144.91 \rightarrow \textcircled{1} \quad 225.44 \text{ N} = R_D$$

$$\Sigma F_y = 0$$

$$-225.44 \sin 50 + R_B \sin 40 + R_A \sin 50 - 294.3 = 0$$

$$0.766 R_A + 0.642 R_B = 467 \rightarrow \textcircled{2}$$



$$R_A = 451.3 \text{ N}$$

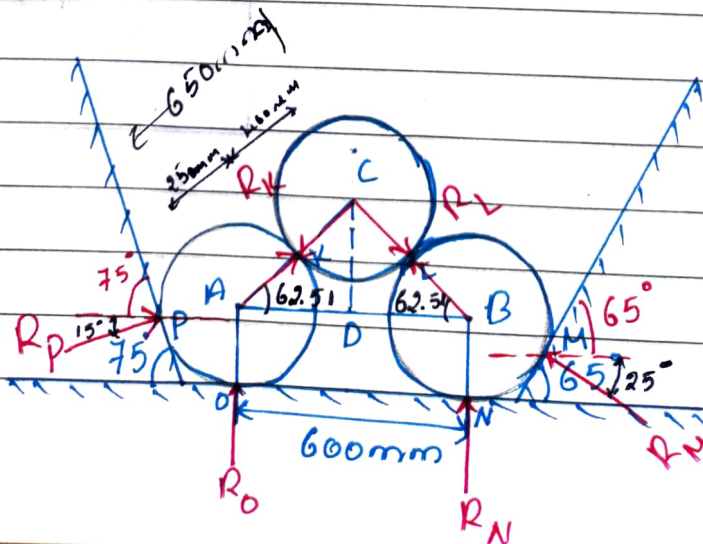
$$R_B = 189 \text{ N}$$

- (15) Determine the reactions at contact points for spheres A, B and C as shown in figure. It is given that: $W_A = W_B = W_C = 6 \text{ kN}$, $d_A = d_B = 500 \text{ mm}$, $d_C = 800 \text{ mm}$.

$$AC = 650 \text{ mm}$$

$$AB = 600 \text{ mm}$$

$$CB = 650 \text{ mm}$$



From $\triangle^{le} ADC$

$$\cos \theta = \frac{300}{650} \Rightarrow \theta = \cos^{-1} \left(\frac{300}{650} \right)$$

$$\theta = 62.51^\circ$$

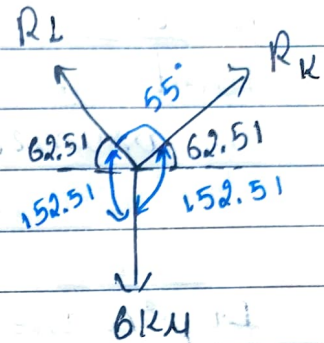
consider sphere - (C)

Apply Lami's theorem

$$\frac{6}{\sin 55} = \frac{R_K}{\sin 152.51} = \frac{R_L}{\sin 152.51}$$

$$\therefore R_K = 3.38 \text{ kN}$$

$$R_L = 3.38 \text{ kN}$$



consider Sphere - (A)

$$\sum F_x = 0$$

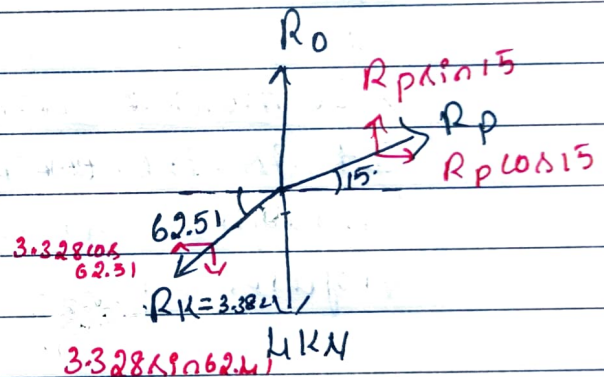
$$-3.382 \cos 62.51 + R_P \sin 15 = 0$$

$$R_P = 1.723 \text{ N}$$

$$\sum F_y = 0$$

$$-3.382 \sin 62.51 + R_0 + R_P \sin 15 - 4 = 0$$

$$R_0 = 6.27 \text{ kN}$$



consider Sphere - (B)

$$\sum F_x = 0$$

$$-R_M \cos 25 + 3.382 \cos 62.51 = 0$$

$$R_M = 1.722 \text{ kN}$$

$$\sum F_y = 0$$

$$R_M \sin 25 + R_N - 3.382 \sin 62.51 - 4 = 0$$

$$R_N = 6.272 \text{ kN}$$

