MODULE 2

ELECTRICAL PROPERTIES OF SOLIDS

Classical free Electron Theory

Introduction

In classical mechanics, the future history of a particle is completely determined by its initial position and momentum together with the forces that act upon it. In everyday world, these quantities can all be determined well enough for the predictions of Newtonian mechanics to agree with what we find. Quantum mechanics also arrives at relationships between observable quantities, but the uncertainty principle suggests that the nature of an observable quantity is different in the atomic realm.The quantities whose relationships quantum mechanics explores are probabilities instead of asserting for exactness.
However classical mechanics turns out to be just an approximate version of quantum mechanics, the certainties of classical mechanics are illusory and their apparent agreement with experiment occurs because ordinary objects consists of so many individual atoms that departs from average behavior which are unnoticeable.
The application of quantum mechanics to problems involving nuclei, atoms, molecules, and matter in the solid state made it possible to understand a vast body of data which is vital for any theory leading to predictions of remarkable accuracy. Quantum mechanics has survived every experimental test thus far of even its most unexpected conclusions.

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Assumptions of Quantum Free Electron Theory

1. The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy levels.
2. The distribution of electrons in various energy levels occurs as per Pauli's exclusion principle.
3. The free electrons travel in a constant potential inside the metal but stay confined within its boundaries.
4. The attraction between the free electrons and the lattice ions, and the repulsion between the electrons themselves are ignored.

Fermi Energy:The energy of an electron in a highest occupied energy level of an atom is called as Fermi energy.

Fermi Level:The highest occupied level in an atom is called as Fermi level.

Fermi-Dirac statistics:Fermi-Dirac statistics describes the distribution of electrons among the various permitted energy levels of a material under thermal equillibrium .It provides the evaluation of the probability of finding electrons occupying energy levels in a certain range.

Fermi factor

Fermi factor is defined as the probability of occupation of electrons in a given energy state for a material at thermal equillibrium .
Then the probability of a given energy state with energy E being occupied at a steady temperature T is given by,

$$\boxed{f(E) = \frac{1}{e^{\frac{(E-E_F)}{KT}} + 1}} \tag{1}$$

Dependence of Temperature on Probability of Occupation of Electrons

1. Probability of occupation for $E < E_F$ at $T = 0K$
Equation(1) becomes, $f(E) = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$

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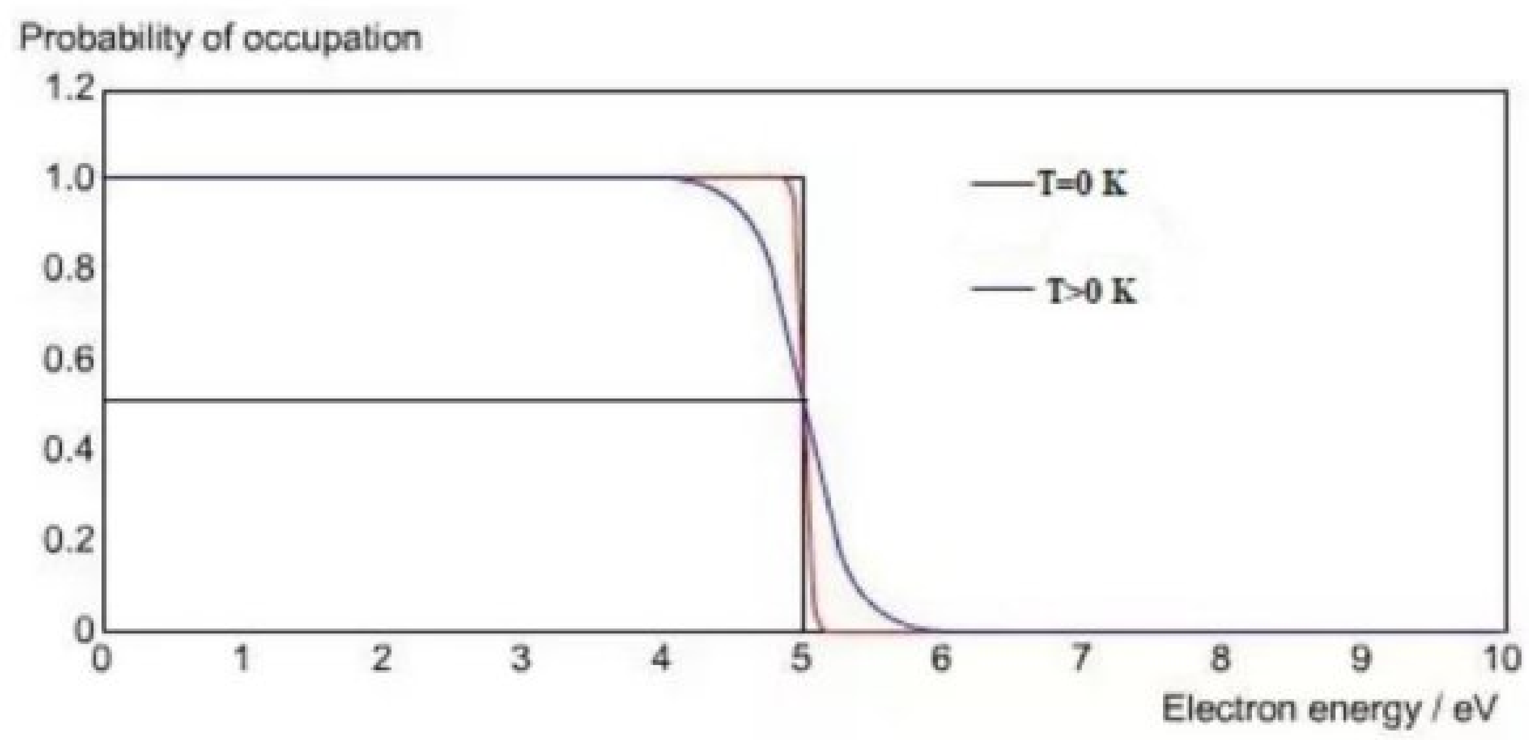
$\Longrightarrow f(E) = 1.$
$i.e,$ All the energy levels below Fermi level are occupied by electrons.

2. Probability of occupation for $E > E_F$ at $T = 0K$

Equation(1) becomes, $f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty}$
$i.e,\ f(E) = 0, \quad for\ E > E_F$
$\therefore$ All the energy levels above Fermi level are unoccupied.

3.Probability of occupation for $E = E_F$ at $T > 0K$

Equation(1) becomes, $f(E) = \frac{1}{e^{0}+1} = \frac{1}{1+1} = \frac{1}{2}$
$i.e.,\ f(E) = \frac{1}{2}$. $\therefore$ At ordinary temperature the probable value starts decreasing from 1 as the values of E become closer to E_F .



Fermi-Dirac distribution for several temperatures

Fermi velocity

The velocity acquired by the electrons at fermi level is called as Fermi velocity.

Fermi Temperature

The temperature at which the average thermal energy of the free electron in a solid becomes equal to the Fermi energy at 0 K is called as Fermi temperature.

Mean Free Path:

The distance travelled by the electron between two successive collisions is called as mean free path.

Mean collision time : The average time elapsed by the electron between two

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consecutive collisions is called as mean collision time.

Expression for Electrical conductivity based on classical free electron theo

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$

where 'n' is the electron concentration.
'm' is the mass of the electron.
'e' is the charge on an electron.
'τ' is the mean collision time.

Expression for Electrical conductivity based on quantum free electron the

$$\boxed{\sigma = \frac{ne^2\tau_F}{m^*}}$$

where 'n' is the electron concentration.
'm^*' is the effective mass of the electron.
'e' is the charge on an electron.
'τ_F' is the mean collision time.

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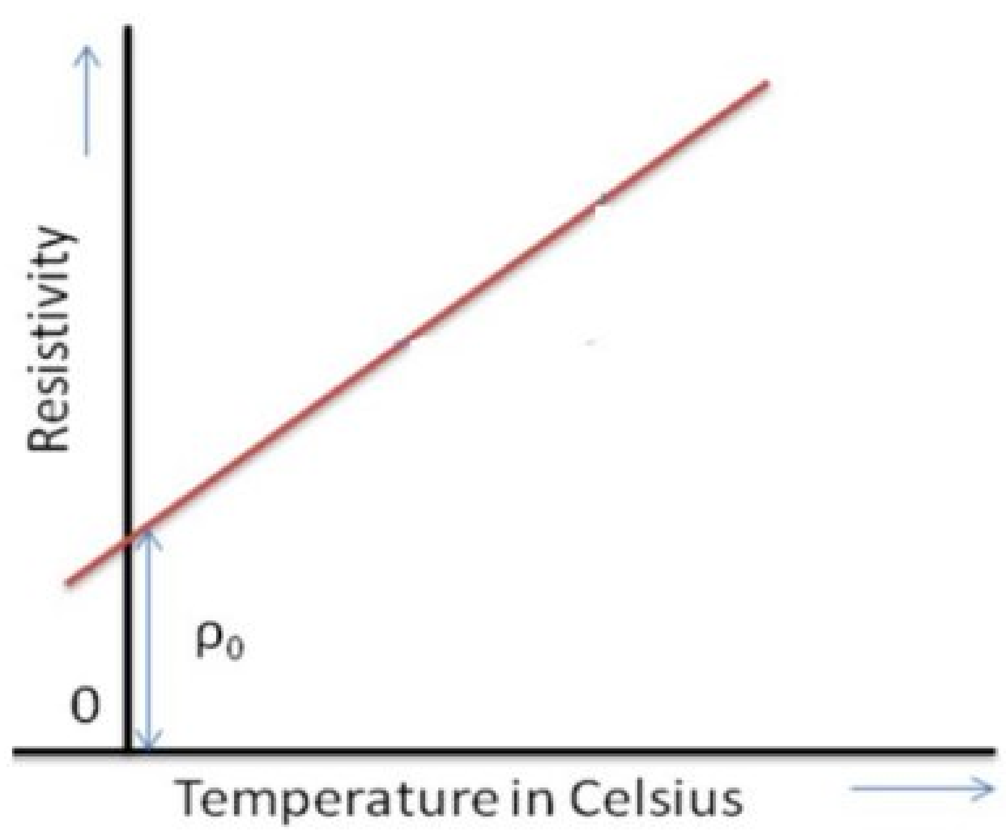
SUPERCONDUCTORS

The phenomenon in which the resistance offered by certain metals to the flow of electric current abruptly drops to zero below a critical temperature (threshold temperature) is called as superconductivity and the materials exhibiting such phenomenon are called as superconductors.

Temperature Dependence of Resistivity in metals



In normal metals the resistivity ‘ρ‘ decreases with decrease in temperature and at zero kelvin the resistivity ρ accquires a value of ρ_o which is a residual resistivity, the residual resistivity is due to impurities present in the metal. This variation is expressed as Matheissen’s rule by the expression,
$\rho = \rho_o + \rho(T)$

Temperature Dependence of Resistivity in superconductors

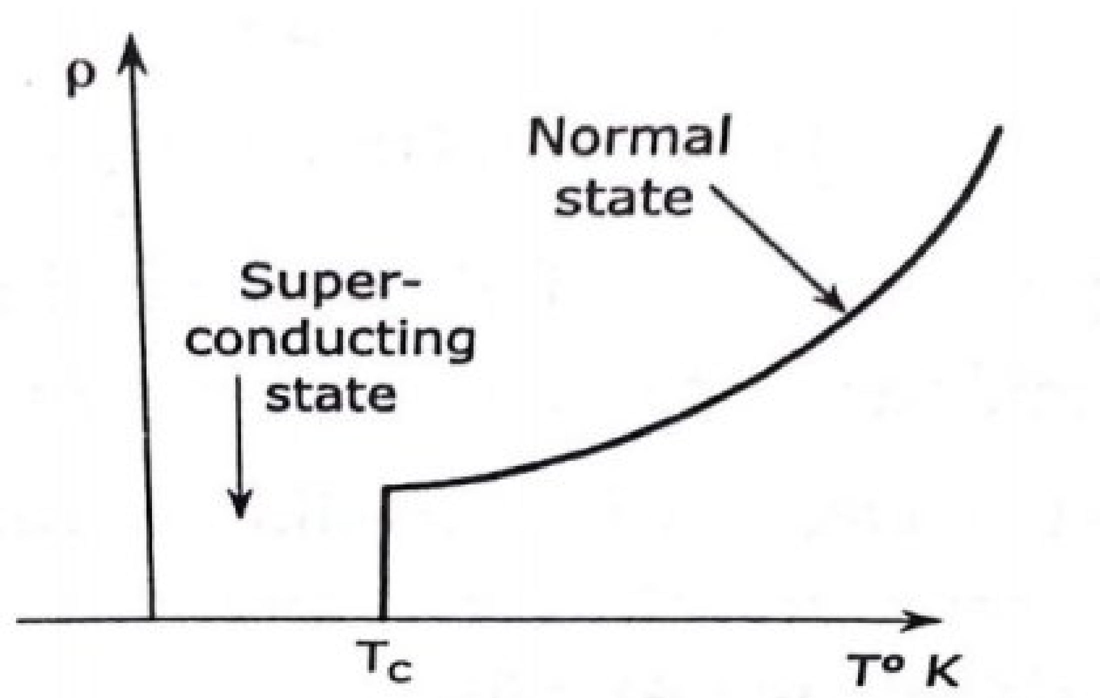


Figure 4.2 : Resistivity of a *Superconductor*

The dependence of resistivity 'ρ' decreases with decrease in temperature as in the case of normal metals upto a particular temperature 'T_c'.

At 'T_c' the 'ρ' abruptly drops to zero and the temperature 'T_c' is called as critical temperature. And it signifies the transition from normal state to superconducting state of the material under study. Critical temperature is different for different superconductors.

BCS THEORY

In 1957 Bardeen, Cooper and Schreiffer gave a theory to explain the phenomenon of superconductivity which is known as BCS theory.

As per the theory during the flow of current in a superconductor, when an electron comes near a positive ion core of the lattice, electron and ion experiences a force of attraction.Because of the opposite charge of e^-, the ion core will be displaced from the lattice which is called as lattice distortion.Then the another electron which comes near the same position, will start interacting with the distorted lattice.

Overall the process is looked upon as equivalent to interaction between the two electrons via the lattice ion. This interaction occurs with the help of a phonon field (quantized lattice vibrations). At this point the Coulomb repulsive force of e^--e^- is overcome by Coulomb attractive force at temperatures below the critical temperature leading to the formation of Cooper pairs (e^--e^- pair).

When the electrons flow as cooper pairs in materials, they do not encounter any kind of scattering, because of which resistance factor vanishes or in other words conductivity becomes infinite. This is the essence of BCS theory.

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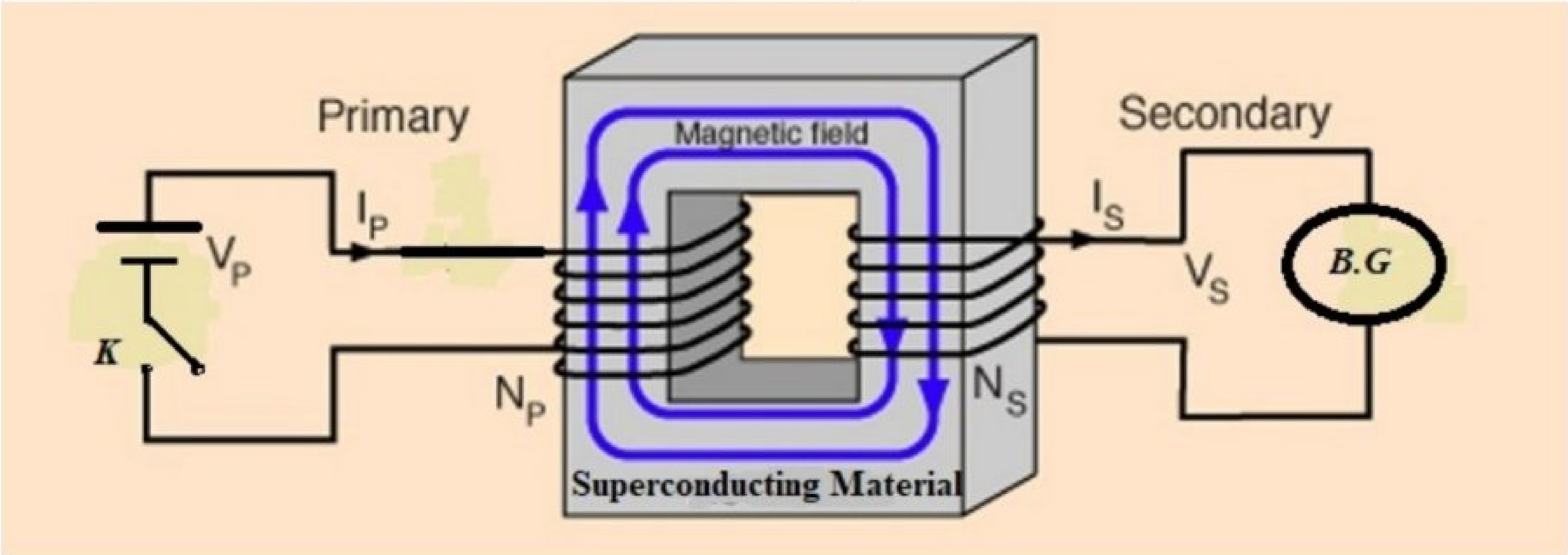
Meissner Effect

A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus it becomes a perfect diamagnet. This effect is called as Meissner's effect.

Explanation of Meissner Effect



Consider a superconducting material above its critical temperature. A primary coil and a secondary coil, are wound on the material, the primary coil is connected to a battery and a key K. The secondary coil is connected to a ballistic galvanometer.
When the key K is pressed, the primary circuit is closed and a current flows through the primary coil which sets up a magnetic field in it. The magnetic flux instantly links with the secondary coil, and hence a momentary current is driven through the B.G which shows a deflection.
Since the primary current is steady, the magnetic flux will also become steady and the flux linkage with the secondary circuit becomes unchanging. Now the temperature of the superconducting material is decreased gradually. As soon as the temperature crosses down the critical temperature the ballistic galvanometer suddenly changes the deflection, indicating that the flux linkage with the secondary coil has changed.The change in flux linkage is attributed to the expulsion of the magnetic flux from the body of the superconducting material.

Critical field

The strength of minimum magnetic field required to just switch a material from superconducting state to normal state is called as "Critical Field". For a superconducting material in the superconducting state, the critical field will be higher when the temperature is lower. If 'T' is the temperature of the

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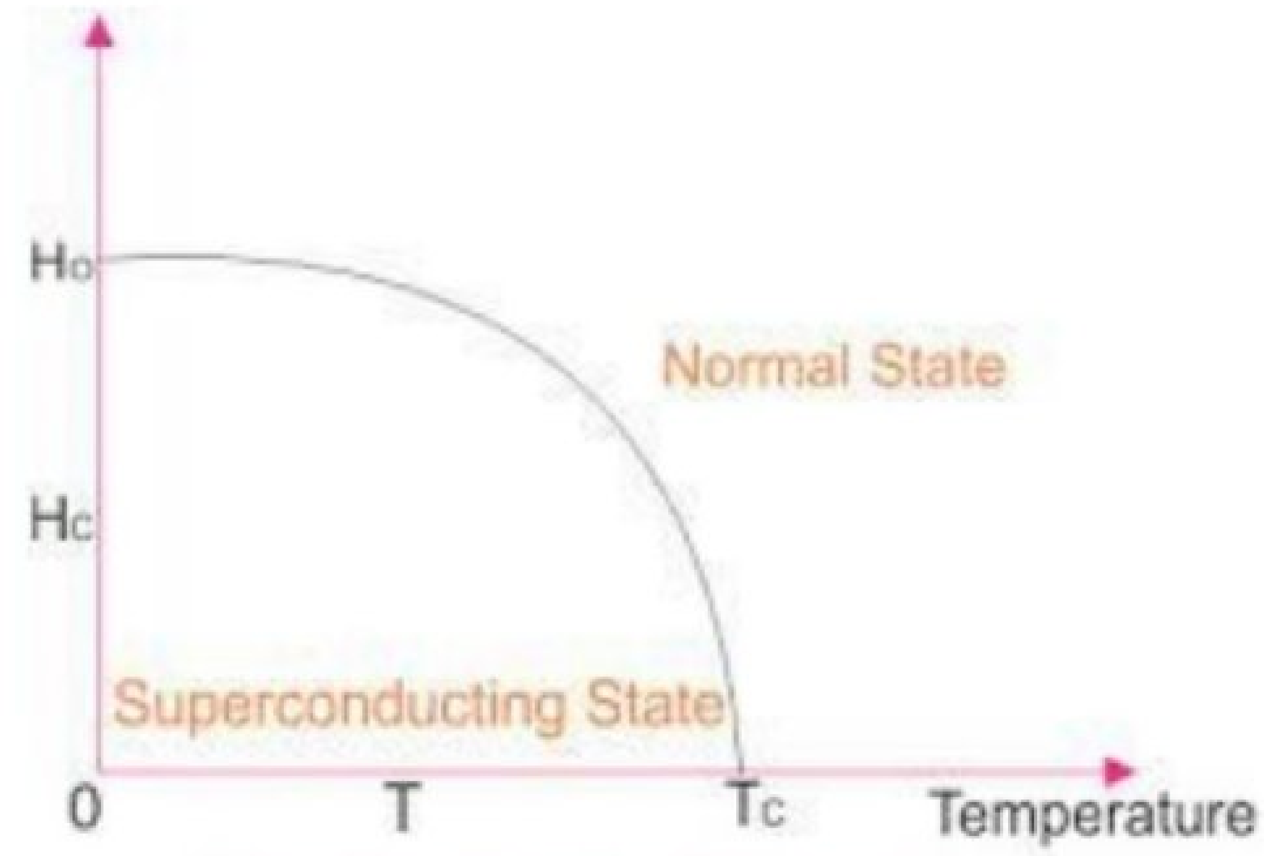


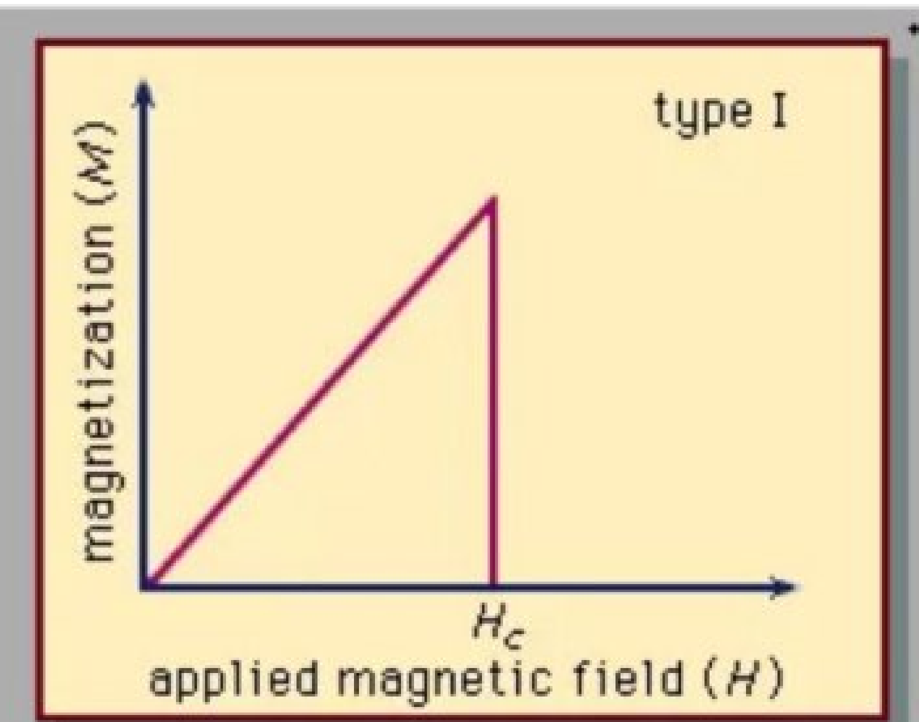
Variation In Critical Magnetic Field with The Temparature

superconducting material, H_c is the critical field then $H_{c(o)}$ the field required to turn the superconductor to a normal conductor at 0^0K. Then the relation for critical field is given by,

$$H_c = H_{c(0)} \left[1 - \frac{T^2}{T_{c^2}}\right]$$

Types of Superconductors

Type 1-Superconductors



For a type I superconductor, magnetic flux is expelled, producing a magnetization (M) that increases with magnetic field (H) until a critical field (H_c) is reached, at which it falls to zero as with a normal conductor.

Type I superconductors exhibit complete Meissner effect. As the applied field 'H' exceeds H_c, the entire material becomes normal by loosing its diamagnetic property completely. Then the magnetic flux penetrates throughout the body of the material. As the critical field value for type 1 superconductors is very low, they are not used for the construction of superconducting magnets wherein they are exposed to extremely high magnetic fields.Type I superconductors are also called as soft superconductors.

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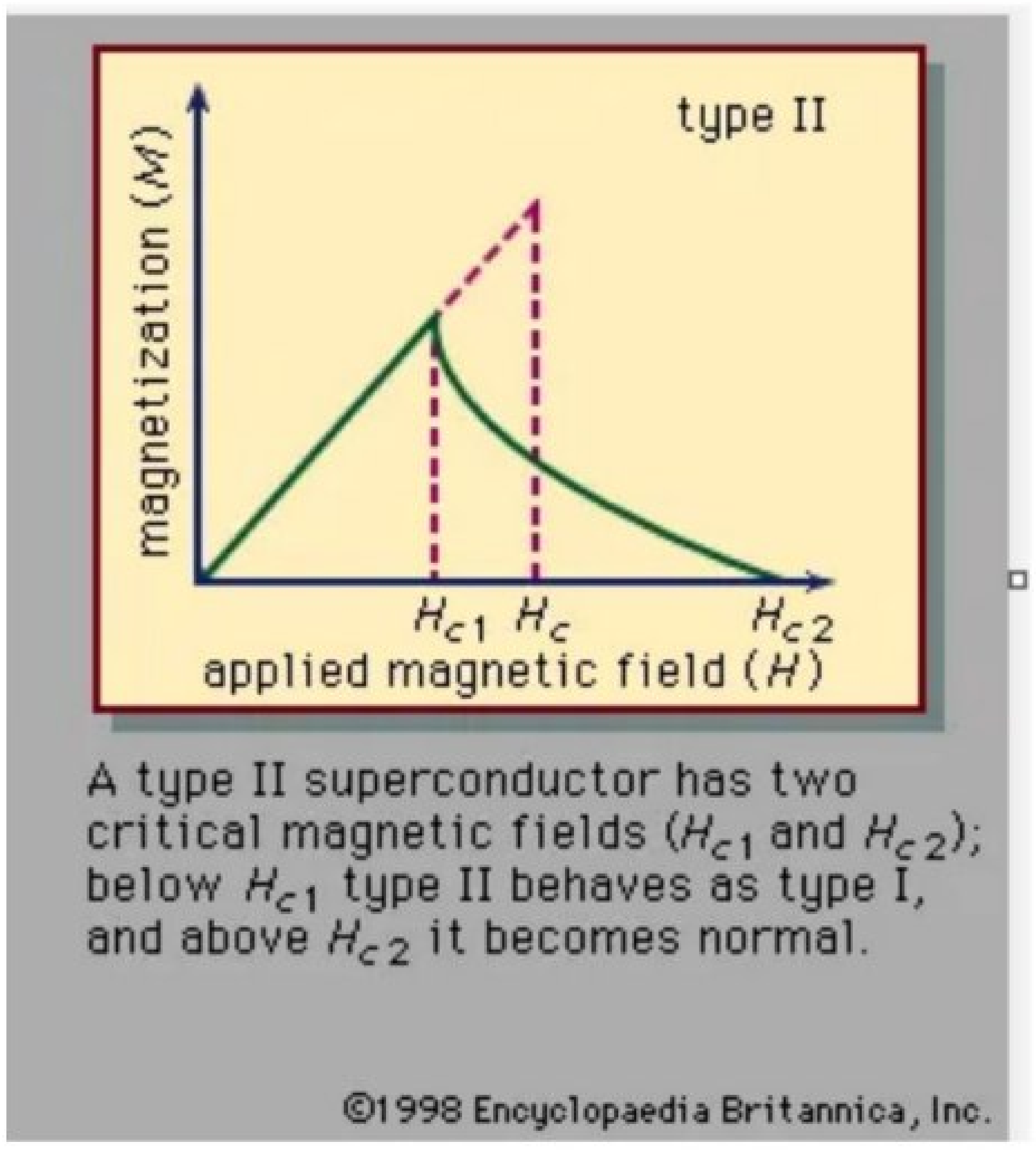
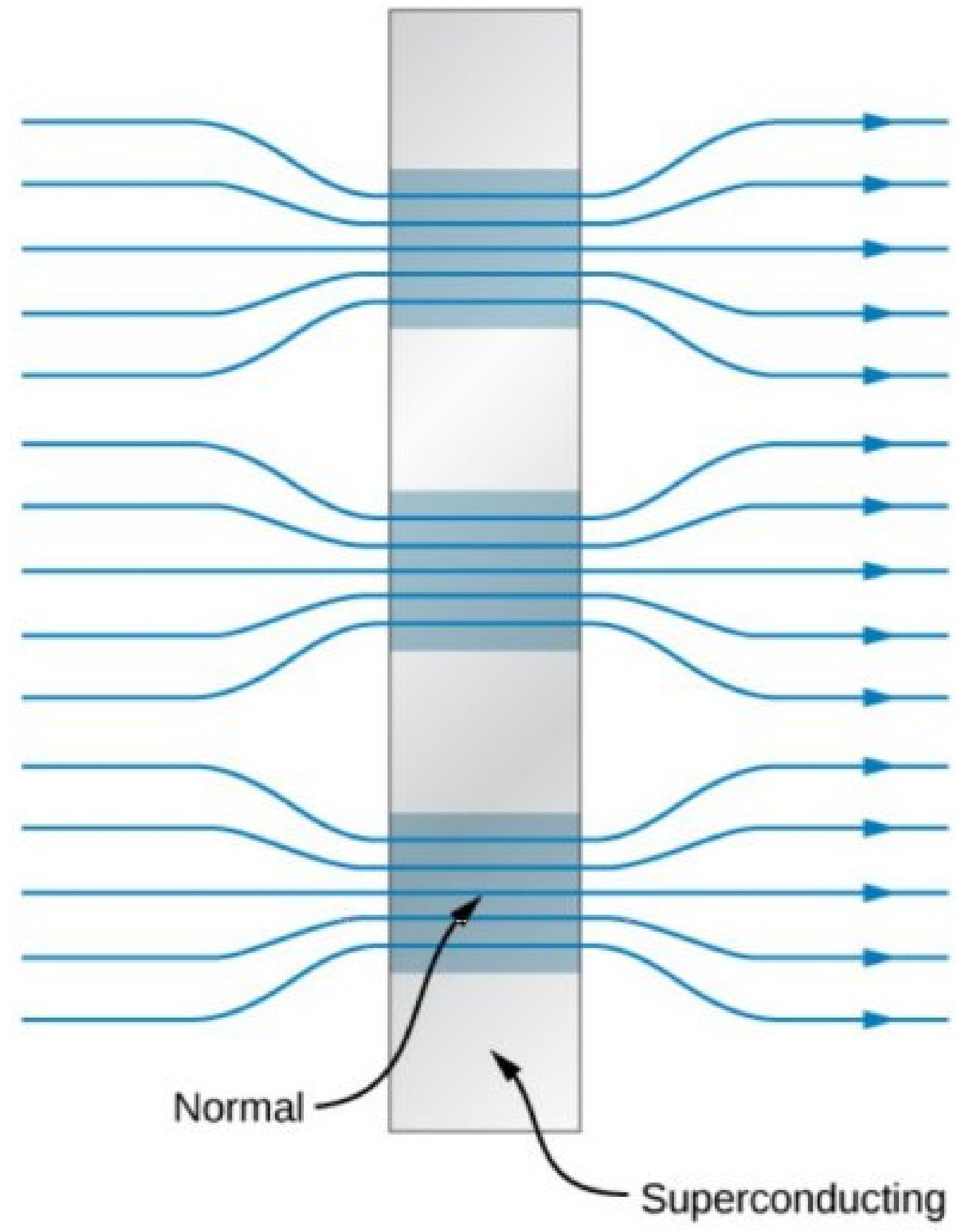
Type 2-Superconductors





Type 2 superconductors are characterized by two critical magnetic fields H_{c1} (lower critical field) and H_{c2} (upper critical field). For the applied field strength, less than the critical value H_{c1}, material expels the magnetic field from its body completely and behaves as a perfect diamagnet from end to end. When 'H' exceeds H_{c1}, the flux penetrates the body and fills in partially, this state between H_{c1} and H_{c2} is called as vortex state or mixed state. The flux penetration occurs through small channelized parts of the body called filaments. Though there is flux penetration the material retains its zero resistance property.

With further increase in magnetic field, the flux line penetration also increases thereby decreasing the diamagnetic part of the material and covers the entire body of the material. When $H \geq H_{c2}$, the material turns into a normal conductor and resistance changes from zero to its normal value.Type 2 superconductors have a very high value of H_{c2} at the given temperature. Thus they are used to build the devices which work in high magnetic fields.They are also called as hard superconductors.

High Temperature superconductors

High temperature superconductors are addressed as high critical temperature (T_c) materials.The phenomenon was first discovered in ceramic materials. All high temperature superconductors are different types of oxides of copper and

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DI-ELECTRIC MATERIALS

A dielectric is an electrical insulating material that can be polarized by an applied electric field (unit:V/m). When a dielectric is placed in an electric field, electric charges do not flow through the material as they do in a conductor, but only slightly shift from their average equilibrium positions causing dielectric polarization.

Polar and Non-polar Dielectric Materials

1. Polar dielectrics: are those which have molecules with permanent dipole but randomly oriented. Under the influence of applied electric field the dipole moment of these dipoles are inclined at an angle to the field direction. Example: NH3, water, HCl

2. Non-polar dielectrics: non polar dielectrics the centers of both positive as well as negative charges coincide. Dipole moment of each molecule in non polar dielectric is zero. All those molecules which belong to this category are symmetric in nature. Examples of non polar dielectrics are: methane , benzene etc.

Dielectric Constant

The electric field strength 'E' and the flux density 'D' for isotropic materials are related by the equation,

$$D = \epsilon_0 \epsilon_r E$$

Relation between Polarization and Dielectric Constant

The relation between polarization $\vec{P}$ and dielectric constant 'ϵ_r' is given by,

$$\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$$

Types of Polarization

1. Electronic Polarization

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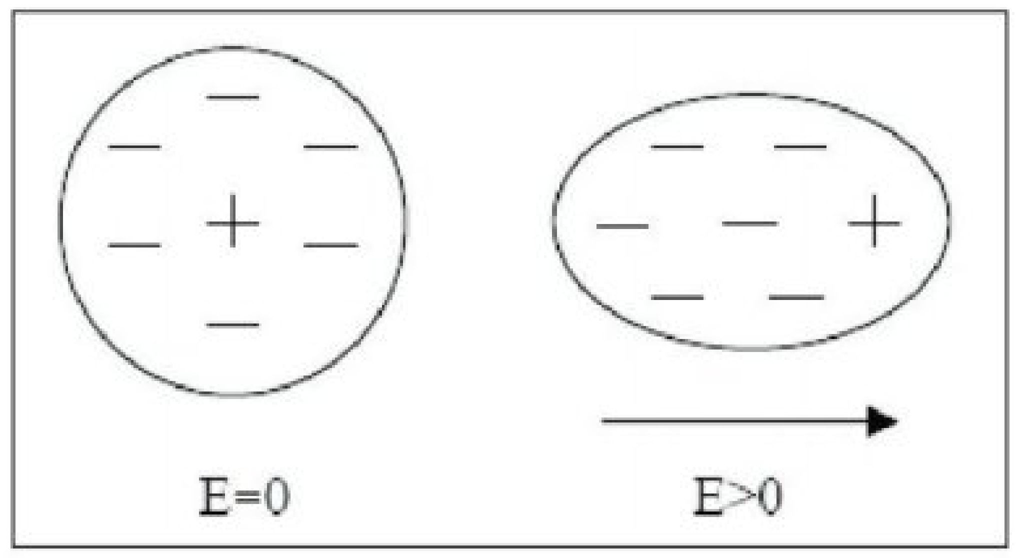
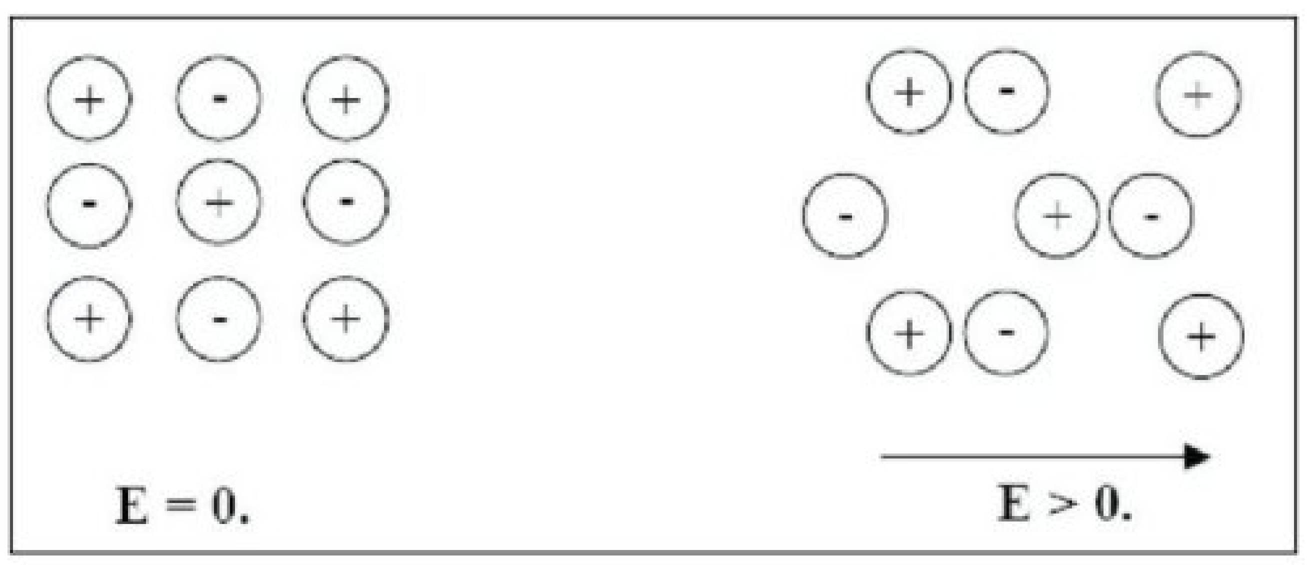


The polarization occurred due to displacement of positive charge and negative charge in dielectric material is called electronic polarization. Figure shows the charge distribution of an atom in absence of electric field and the charge distribution in the presence of external electric field. This process occurs throughout the material and as a result whole material is polarized.

2. Ionic Polarization



There is no net polarization inside these materials in the absence of an external electric field because the dipole moments of the negative ions are canceled out with the positive ions. However, when an external field is applied, the ions get displaced, which leads to an induced polarization. The mechanism of Ionic polarization contributes to the relative permittivity of a material. This type of polarization typically occurs in ionic crystal elements such as NaCl, KCl, and LiBr.

3. Orientational Polarization

Orientational polarization arises when there is a permanent dipole moment in the material. Materials such as HCl and H_2O will have a net permanent dipole moment because the charge distributions of these molecules are skewed. For example, in a HCl molecule , the chlorine atom will be negatively charged and the hydrogen atoms will be positively charged causing the

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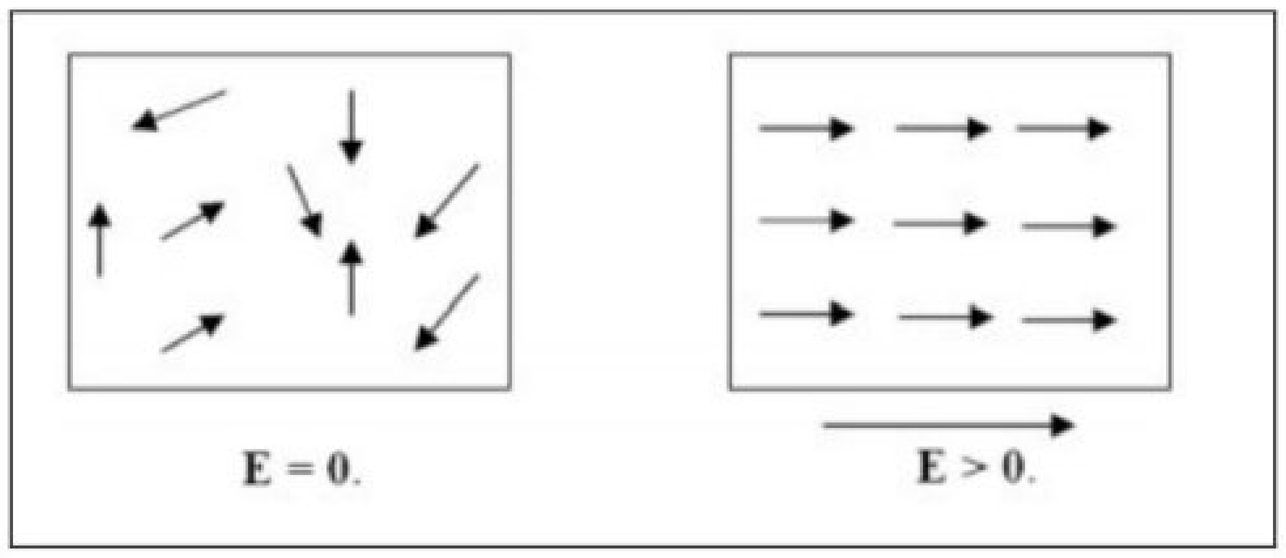

molecule to be dipolar.
However in the absence of an electric field, the dipole moment is canceled out by thermal agitation resulting in a net zero dipole moment per molecule. When an electric field is applied however, the molecule will begin to rotate to align the molecule with the field, causing a net average dipole moment per molecule.

Expression for the internal field in case of liquids and solids.

consider an array of equidistant atomic dipoles arranged parallel to the direction of the electric field, if the interatomic distance is 'd' and the electronic polarizibility is α_e, then the internal field equation E_i is given by,

$$E_i = \frac{E}{1 - \frac{1.2\,\alpha_e}{\pi\epsilon_0 d^3}}$$

Expression for Internal field and Lorentz relation

$$\boxed{E_i = E + \left(\frac{\gamma}{\epsilon_0}\right) P}$$

Where E_i is the internal field
'P' is the polarization,
γ is the internal field constant
For 3-dimensional case $\gamma = \frac{1}{3}$ for cubic lattice.

$$\boxed{E_{Lorentz} = E + \frac{P}{3\epsilon_0}}$$

The above equation is called as Lorentz relation.

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Expression for Clausius-Mossotti Equation

Consider an elemental dielectric material whose dipole moment/volume ‘p’ is given by the equation,

$$P = N\alpha_e E_i$$
$$\implies E_i = \frac{P}{N\alpha_e} \tag{1}$$

where ‘P’ is the polarization of a dielectric material.
The relation between the polarization and the dielectric constant is given by,

$$P = \epsilon_O(\epsilon_r - 1)E$$
$$\therefore E = \frac{P}{\epsilon_O(\epsilon_r - 1)} \tag{2}$$

The expression for the binternal field of the dielectric material is given by,

$$E_i = E + \frac{\gamma P}{\epsilon_O} \tag{3}$$

where γ is the internal field constant.
substituting for ‘E_i and ‘E’ in equation (3) from equation (1) and (2), we get

$$\frac{P}{N\alpha_e} = \frac{P}{\epsilon_O(\epsilon_r - 1)} + \frac{\gamma P}{\epsilon_O}$$
$$P\left(\frac{1}{N\alpha_e}\right) = P\left(\frac{1}{\epsilon_O(\epsilon_r - 1)} + \frac{\gamma}{\epsilon_O}\right)$$
$$\implies \frac{1}{N\alpha_e} = \frac{1}{\epsilon_O(\epsilon_r - 1)} + \frac{\gamma}{\epsilon_O}$$

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Considering the field to be an Lorentz field, $\gamma = \frac{1}{3}$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_O(\epsilon_r - 1)} + \frac{1}{3\epsilon_O}$$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_O}\left\{\frac{1}{(\epsilon_r - 1)} + \frac{1}{3}\right\}$$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_O}\left\{\frac{\epsilon_r + 2}{3(\epsilon_r - 1)}\right\}$$

$$\frac{3\epsilon_O}{N\alpha_e} = \frac{\epsilon_r + 2}{(\epsilon_r - 1)}$$

$$\boxed{\frac{\epsilon_r - 1}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_O}} \tag{4}$$

Equation (4) represents the expression for Clausius-Mossotti relation.

Solid, Liquid and Gaseous Dielectric materials

Depending on the application dielectrics are classified into solid, liquid and gaseous type. Since their main purpose is to provide electrical insulation and for charge storage purpose, they are generally addressed as insulators. Examples of solid dielectric materials are mica, porcelain, glass or synthetic materials. While silicon fluids, viscous vaseline , fluoro-organic fluids form liquid dielectrics and nitrogen, air, inert gases, carbon-dioxide are gaseous dielectric materials. Dielectric materials like mica, paper and cloth are used in high voltage transformers to provide insulation between individual windings in the coils.

APPLICATIONS

Dielectrics as capacitors, transformersand electric insulation

Increasing the capacitance of a capacitor:

When an dielectrics is between the two parallel plate capacitors, they fully or partially occupy the region between the plates and the applied electric field polarises it.The electric field between the plates of parallel plate capacitor is directly proportional to capacitance C of the capacitor.

The strength of the electric field is reduced due to the presence of dielectric, as dielectrics do not conduct. If the total charge on the plates is kept constant, then the potential difference is reduced across the capacitor plates. In this way, dielectric increases the capacitance of the capacitor.

Dielectrics as Transformers and Electric Insulation:

The turn-to-turn and layer-to-layer insulating materials of the transformer winding can be referred to as "Dielectric Material". This typically consists of fiberglass, mica, aramid fiber paper etc., its function is to provide electrical insulation, suppress corona (hissing noise in transmission lines) and electric arcing, and to serve as a coolant.

Dielectric, insulating material are a very poor conductor of electric current. The positive charges within the dielectric are displaced minutely in the direction of the electric field, and the negative charges are displaced minutely in the direction opposite to the electric field. This slight separation of charge, or polarization, reduces the electric field within the dielectrics and provides electrical insulation.

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