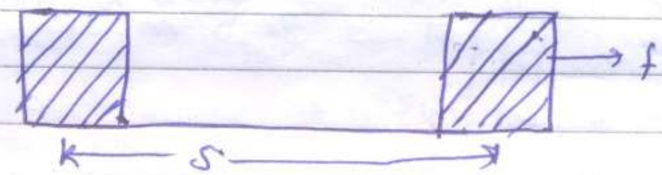
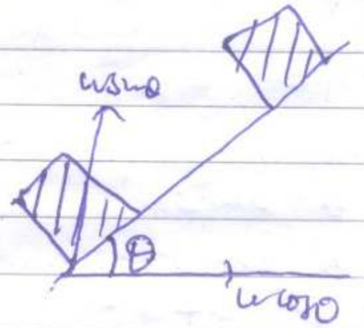
UNIT-5

Work :-

When a force act on a body and body undergoes some displacement then some work is said to be done by force and It is equal to the force multiplied by distance travelled in the direction of force.



$W = F \cdot S.$



$W = w \sin\theta \cdot s.$

Energy :-

Energy of a body is the capacity of doing work.

we always says a man has heigher capacity for doing work then a child.

The grater energy passessed by the machi It will be able to perform more work.

for ex :-

A Body possessing more heat energy is capable to produce more work.

Therefor the amount of energy possessed by the body is an indicator of capacity to producer.

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Potential Energy:- The energy which a body possesses by virtue of a position or configuration is called potential energy.

If a stone of mass m is lifted through a height H above ground, then the work done on the stone is mgh. The work remain stored in the stone. It can perform the same amount of work again in coming down to its original position.

The potential energy of a mass m is given by P.E. = mgh (N/m)

Kinetic Energy:- The energy which is a body possesses by virtue of its motion is called kinetic energy. It is measured by the amount of work required to bring the body to rest.

Assume a body of mass m is moving with a velocity u and it is brought to rest by applying a force f. which causes retardation a and stop the work in the distance s.

The kinetic energy of the body

= work done by force for stop body

$K.E. = mas$ — (I)

In this eqn we have assume the force f is constant. from IIIrd eqn of motion

$v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$as = \frac{1}{2}u^2$ — (II)

Substituting this value in eqn (I) then Teacher's Signature

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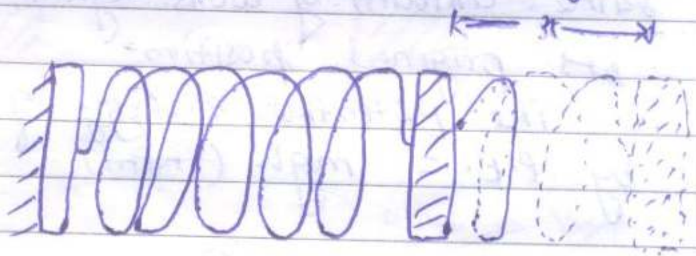
$$\boxed{K.E. = \frac{1}{2} m u^2}$$

Spring Energy :-

When a spring is compressed some work is done to compress it. This work is stored in the form of spring energy.



Consider a mass connected to spring as shown in figure equilibrium position $x=0$. Now spring is stretched by distance "x". A force "F" act on mass due to stiffness k of spring that try to bring in the original position. And this force is equal to

$$\boxed{F_x = -kx}$$

Negative sign shows that this is force always in opposite direction of the displacement.
Now consider a small displacement dx is given to mass m then some work done due to stretched the spring by dx

$$dw = -F_x\,dx$$
$$= -(-kx)dx$$
$$dw = kx\,dx$$

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$\int dw = k\int x\,dx$

$$\boxed{w = \frac{kx^2}{2} + c}$$

chance

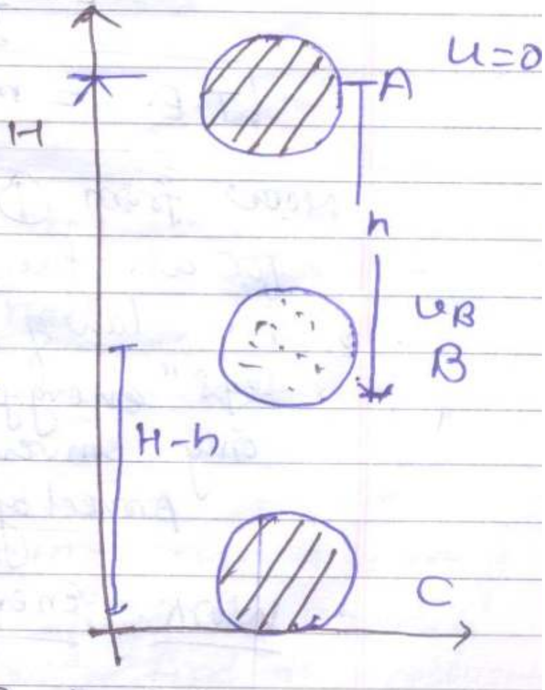
Law of conservating Energy:-

This law state that energy can neither be created nor destroyed. It can be transfer from one form of energy into any other form.

Assume a ball at position A. Its is potential energy

$$\boxed{P.E. = mgH} \quad —(1)$$

Let the ball falls through a height h, and its velocity at B point



By the eqn motion

$V_B^2 = u^2 + 2gh$

$$\boxed{v_B^2 = 2gh} \quad (\because u = 0)$$

Total energy at point B is potential energy and kinetic energy

$$T.E. = P.E. + K.E.$$
$$= mg(H-h) + \frac{1}{2}mv_B^2$$
$$= mgH - mgh + \frac{1}{2}m(2gh)$$

$$\boxed{T.E. = mgH} \quad —(1)$$

Consider now the motion of ball from A to C.

at point c

$$V_c^2 = u^2 + 2gH$$
$$V_c^2 = 0 + 2gH$$
$$\boxed{V_c^2 = 2gH}$$

Total energy at point c

$$= P.E. + K.E.$$
$$= 0 + \frac{1}{2} m v_c^2$$
$$= \frac{1}{2} m (2gH)$$
$$\boxed{T.E. = mgH} \quad -③$$

Now from ①, ② & ③ the total is same for all the points and we understand the law of conservation of energy that says "energy cann't be created or destroyed only converted". So this law is further proved by this example.

<u>Work</u> Energy <u>Relation</u>:-

According to the work. Energy theorem the work done by an external force on a moving body is equal to the change in kinetic energy of the body.

suppose an external constant force F is applied on a body of mass 'm' and due to this force velocity of mass changes from u to v.

$u^2 = -2gh$

mathematically

$$F = ma$$
$$= m\frac{dv}{dt}$$

work done for small displacement dx

$$dw = f \cdot dx$$
$$= m\frac{dv}{dt} \cdot dx$$
$$= m\,dv\,\frac{dx}{dt}$$
$$\int dw = \int_u^v mv\,dv$$
$$w = \left[\frac{mv^2}{2}\right]_u^v$$
$$= \frac{1}{2}\left[mv^2 - mu^2\right]$$
$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

work done $\boxed{W = \Delta K.E.}$

Q. A aeroplane of moving with a speed of 720 Km/hour reduce it speed to 90 km/h when approch the airport. find the reduction of its kinetic energy if the plane mass is 6000 kg.

Sol^n Given $u = 720\ km/h = 720 \times \frac{5}{18} = 200\ m/s$

$v = 90\ km/h = 90 \times \frac{5}{18} = 25\ m/s$

$m = 6000\ kg$

$$K.E._1 = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 6000 \times 200 \times 200$$
$$= 12 \times 10^8\ J.$$

$$K.E._2 = \frac{1}{2} \times 6000 \times 25 \times 25$$

$$= 1.8 \times 10^6 J.$$

$$K.E. = K.E_1 - K.E_2$$

$$= 1.2 \times 10^8 - 1.8 \times 10^6$$

$$= 10^6 [1.2 \times 10^2 - 1.8]$$

$$= 10^6 [120 - 1.8]$$

$$K.E. = 1.1\cancel{82} \times 10^7 \text{ Joule}$$

must maybe in midterm imp

Q. The position of vector of particle is given by $\vec{r} = 8t\hat{i} + 1.2t^2\hat{j} ~~- 0.5t\hat{k}~~ \; 0.5(t^3-1)\hat{k}$ where t is the time in second from the start of the motion and r is expressed in meter. for the condition when t is 4s determine the power developed by the force $\vec{F} = 40\hat{i} - 20\hat{j} - 36\hat{k}$ (kN) which act on the particle.

Soln

$$\text{Power} = \vec{f} \cdot \vec{v}$$

$$\therefore \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[8t\hat{i} + 1.2t^2\hat{j} - (0.5t^3 + 0.5)\right]$$

$$\vec{v} = 8\hat{i} + 2.4t\hat{j} + 1.5t^2\hat{k}$$

$$\text{Power} = (40\hat{i} + 20\hat{j} - 36\hat{k}) \cdot (8\hat{i} + 2.4t\hat{j} + 1.5t^2\hat{k})$$

$$\text{Power} = 320 + 48t + 54t^2$$

when $t = 4$ sec $P = 320 + 192 + 864$

$$\boxed{P = 992 \text{ Jm}}$$

Impulse & Momentum :-

Momentum :- Momentum can be defined as the quantity of motion contained in a body. It is measured by the force required to stop body in a unit time.

The product of mass of body and its velocity is called momentum and it is denoted by p.

$$\boxed{P = mv}$$

Impulse :- When a large force is applied on the body for a very short time then total effect of force is considerable and such types of force is called Impulsive force

The total effect of Impulsive force on the motion of body is called Impulse. And it is denoted by J.

And Impulse is a vector quantity.

If a force F is act on the body of a short time dt then impulse is

$$d\vec{J} = \vec{f} \cdot dt$$

If the force act from t_1 to t_2 time then

$$\int dJ = \vec{f} \int_{t_1}^{t_2} dt$$

$$J = \vec{f} [t]_{t_1}^{t_2}$$

$$\vec{J} = \vec{f} [t_2 - t_1]$$

$$\boxed{\vec{J} = f \Delta t}$$

Impulse - momentum Theorem:- If p is momentum of the body at time t and force f is applied on the body then Newton's second law

$$F = \frac{dP}{dt}$$

$$dP = F \cdot dt$$

If the momentum of a body at time t_1 is P_1 & time t_2 is P_2 then

Integrating both sides

$$\int_{P_1}^{P_2} dP = \int_{t_1}^{t_2} f\,dt$$

$$[P]_{P_1}^{P_2} = f[t]_{t_1}^{t_2}$$

$$P_2 - P_1 = f[t_2 - t_1]$$

$$P_2 - P_1 = f\Delta t \quad -(2)$$

$\because$ Impulse is $J = f\Delta t$ -(3) from (2) & (3)

$$\boxed{P_2 - P_1 = J}$$

The Impulse of the force is equal to the change in momentum due to the force applied on the body. This is called Impulse momentum Theorem

ELASTIC & INELASTIC IMPACT:-

When balls of different materials are allow to fall on a floor, they rebound to different height. The property of

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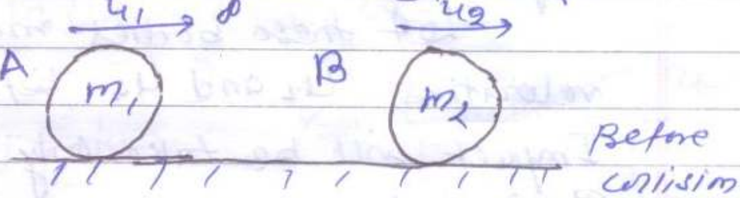
bodies which leads to rebound after impact is called ELASTICITY.

The impact is elastic, if the body rebounds after impact. Grater the elasticity of the body, grater will be Rebound to the different height.

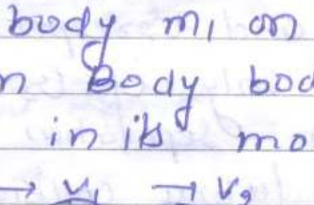
The property of bodies that after impact it doesn't rebound, this type of body is called Inelastic Body.

Conservation of momentum:-

Cosider two bodies of mass m_1 and m_2 moving with respective velocities u_1 and u_2.



After collision the velocities v_1 and v_2 respectively.
During collision an impulse $f \cdot t$ is exerted by body m_1 on body m_2. This impulse on Body body B is measured by change in its momentum



$$\boxed{f \times t = m_2 v_2 - m_2 u_2} \quad (1)$$

According to the newton's third low Action and reaction b/w the colliding body is equal in the magnitude and opposite direction.

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and it acts for same time
then. Impulse on body $m_1 = -f \times t$
$$-f \times t = m_1 v_1 - m_1 u_1$$
$$\boxed{f \times t = m_1 u_1 - m_1 v_1} \quad \text{--- (2)}$$
The mom. of the ... before collision is equal to the after collision.
Now from the Eq^n (1) & (2)
$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Coefficient of Restitution:-

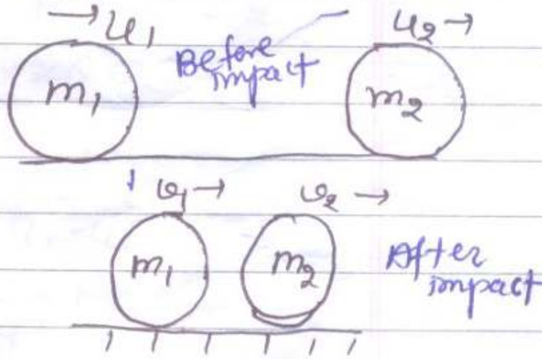
Consider two bodies of mass m_1 and m_2 respectively.
Let these bodies moving with respective velocities u_1 and u_2 before impact. The Impact will be take only if $u_1 > u_2$.
Thus velocity of approach is $u_1 - u_2$.
After a short period of contact the bodies will seperate and will start moving with velocities v_1 and v_2 respectively. The seperation will be occur only if v_2 is grater then v_1 (v_2 ... and velocity of seperation is $v_2 - v_1$.
Newton's law of collision for elastic bodies state that when two moving bodies collide with each other their velocity of seperation bears constant ratio to their velocity of approach. Then
$$\frac{(v_2 - v_1)}{u_1 - u_2} = e$$
$$\boxed{v_2 - v_1 = e(u_1 - u_2)}$$

Where e is a constant of preportionality and called the coefficient of restitution.
The value of e lies b/w 0 to 1.
If $e=0$, the bodies are inelastic. If $e=1$, the bodies are perfectly elastic.

* Loss of kinetic energy for impact ⇒

consider two bodies of mass m_1 and m_2 moving with respective velocities u_1 and u_2 before impact and v_1 and v_2 after impact.
kinetic energy of two bodies before impact

$$E_1 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$



kinetic energy of two bodies after impact

$$E_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Loss of kinetic Energy

$$E_L = E_1 - E_2 = \frac{1}{2}\left[m_1 u_1^2 + m_2 u_2^2 - m_1 v_1^2 - m_2 v_2^2\right] \quad —(1)$$

from the law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad —(2)$$

The Relation for coefficient of restitution.

$$v_2 - v_1 = e(u_1 - u_2) \quad —(3)$$

solving eqn (1), (2) and (3) gives

$$\boxed{E_L = \frac{m_1 m_2}{2(m_1+m_2)}(u_1-u_2)^2(1-e^2)} \quad \text{--- (9)}$$

Case Ist :

When the colliding bodies are in-elastic or $e=0$

$$E_1 = \frac{m_1 m_2}{2(m_1+m_2)}(u_1-u_2)^2$$

Case IInd

When the colliding bodies are perfectly elastic then $e=1$

$$\boxed{E_L = 0}$$

Q1. A ball of mass 2kg moving with a velocity of 2m/s & directly a ball of mass 4kg at rest. The first ball after impacting comes to rest find the velocity of second ball after the impa[ct] and the coefficient of restitution.

Sol. Given

$m_1 = 2\,kg$, $m_2 = 4\,kg$

$u_1 = 2\,m/s$, $u_2 = 0\,m/s$

Now from the conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$4 + 0 = 2v_1 + 4v_2$

$v_1 + 2v_2 = 2$ —① $\Rightarrow \boxed{v_2 = 1\,m/s}$

Now from the conservation of energy

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\frac{1}{\cancel{2}} \times \cancel{2} \times 4 + 0 = \frac{1}{\cancel{2}} \times \cancel{2} \times v_1^2 + \frac{1}{2} \times 4 \times v_2^2$

$4 = v_1^2 + 2v_2^2$

$\because (v_2 - v_1) = e(u_1 - u_2)$

$1 - 0 = e(2 - 0)$

$\boxed{e = 1/2 = 0.5}$

Q2. A Ball of 8kg moving with a velocity of 10m/s collides with another ball of 3kg moving with 5m/s in the same direction. and then they form a single body and move determine the velocity with which the single body will move. If ball were moving in opposite direction then what will be the velocity of single body

Now As per given data

$m_1 = 8kg$ $m_2 = 3kg$

$u_1 = 10m/s$ $u_2 = 5m/s$

Let the velocity v of single body after collision. then

If direction is same then –

By the conservation of momentum

$$m_1u_1 + m_2u_2 = (m_1+m_2) v$$

$$v = \frac{m_1u_1 + m_2u_2}{m_1+m_2}$$

$$= \frac{8\times10 + 3\times5}{8+3}$$

$$= \frac{95}{11} = 8.6\ m/s$$

If they are in opposite direction then $u_2 = -u_2$

Now By the conservation of momentum

$$m_1u_1 - m_2u_2 = (m_1+m_2) v$$

$$v = \frac{m_1u_1 - m_2u_2}{m_1+m_2}$$

$$= \frac{8\times10 - 3\times5}{8+3}$$

$$= \frac{65}{11}$$

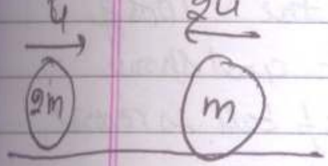
$$= 5.9\ m/s$$

Q. The masses of two balls are in the ratio of 2 : 1 and their velocities are the ratio of 1 : 2 but in the opposite diretion before impact. If the coefficient of restitution be 5/6. The prove that after the impact each ball will move back with 5/6th of its original velocity.

Sol^n

Let m_1 & m_2 are the mass and u_1 & u_2 are the velocities before impact respective

As given



$$\frac{m_1}{m_2} = \frac{2}{1} \Rightarrow \boxed{m_1 = 2m_2}$$

$$\frac{u_1}{u_2} = \frac{1}{2} \Rightarrow \boxed{u_1 = \tfrac{1}{2}u_2}$$

$$\because \quad (v_2 - v_1) = e(u_1 - u_2)$$

$$v_2 - v_1 = \frac{5}{6}(u_1 - u_2)$$

Now from the conservation of momentum

$$m_1 u_1 - m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$0 = 2m_2 v_1 + m_2 v_2$$

$$m_2(2v_1 + v_2) = 0$$

$$\boxed{v_1 = -\frac{1}{2}v_2} \quad -(1)$$

Newton's Law of collision for elastic bodies, coefficient of restitution.

$$(v_2 - v_1) = e(u_1 - u_2)$$

$$= \frac{5}{6}[u - (-2u)]$$

$$v_2 - v_1 = \frac{5}{2}u \quad -(2)$$

Now from (1)

$$v_2 - \left(-\frac{1}{2} v_2\right) = \frac{5}{2} u$$

$$\frac{3}{2} v_2 = \frac{5}{2} u$$

$$\boxed{v_2 = \frac{5}{3} u}$$

Q. A Ball of 1kg moving at 3m/sec impacting on another ball of 5kg moving in the same line at 0.6m/s. find the loss of kinetic energy during impact and show that the direction motion of first ball is reversed. the coefficient of restitution is 0.75.

Sol As per given data

$m_1 = 1kg$ $\quad$ $m_2 = 5kg$

$u_1 = 3m/s$ $\quad$ $u_2 = 0.6 m/s$

Loss of Energy $= \frac{m_1 m_2}{2(m_1+m_2)} (u_1-u_2)^2 (1-e^2)$

$= \frac{1 \times 5}{2(1+5)} (3-0.6)^2 [1-(0.75)^2]$

$= \frac{5}{12} [2.4]^2 (1-.5625)$

$= 1.05$ Joule

Now from the conservation of momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$1 \times 3 + 5 \times 0.6 = 1 \times v_1 + 5 \times v_2$

$v_1 + 5v_2 = 6$ — (1)

Now the coefficient of restitution.

$$v_2 - v_1 = e(u_1 - u_2)$$

$$v_2 - v_1 = .75[3 - .6]$$

$$v_2 - v_1 = 1.8 + 0$$

$$v_1 - v_2 = -1.8 \quad -(2)$$

(1) - (2)

$$5v_2 + v_2 = 7.8$$

$$v_2 = \frac{7.8}{6}$$

$$= 1.3 \text{ m/s}$$

$$v_1 = v_2 - 1.8$$

$$= 1.3 - 1.8$$

$$\boxed{v_1 = -0.5 \text{ m/s}}$$

Q. A Bullet of 30 gm fired through a gun towards a suspended ball of 10kg the string which is attach to the suspended is 1m long due to impact of bullet. The bullet penetrate into the ball and the ball with bullet swing through an angle of 30° with the verticle. find the velocity of bullet when fire.

Solⁿ

After the impact combind ball and bullet (m+M) is Lifted through a height h and its all kinetic energy is transferred to the potential energy to lift the height h.

$$\frac{1}{2}(m+M)v^2 = (m+M)gh$$

$$\boxed{h = \frac{v^2}{2g}} \Rightarrow v = \sqrt{2gh}$$

we can write

$$\cos 30 = \frac{L-h}{L}$$

$$\cos 30 = \frac{1-h}{1}$$

$$h = 1-\cos 30 = 1-\frac{\sqrt{3}}{2} = 0.13$$

$$v = \sqrt{2\times 9.81 \times 0.13}$$

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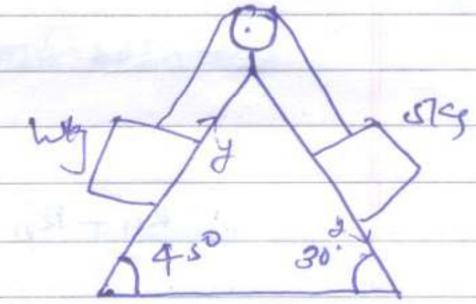
Q A weight of 5KN resting on a smooth surface inclined 30° to the horizontal is supported by a load W resting on another surface inclined to horizontal 45° as shown in figure.



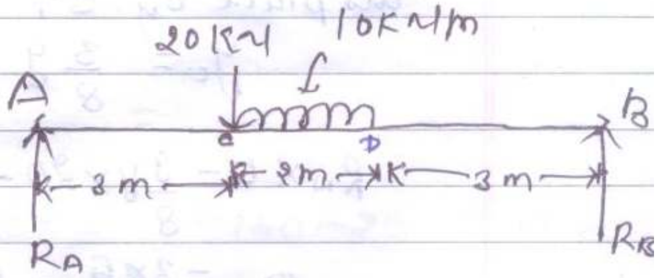
Let body move y displacement then

$Wy\sin 45 = 5y\sin 30$

$W = \frac{\sin 30° \times 5}{\sin 45}$

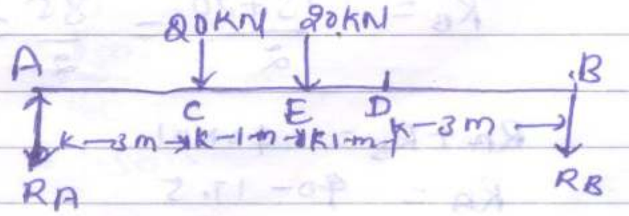
$\boxed{W = 3.5KN}$

Q A Beam AB is simply supported at point point load 20KN act and UDL 10KN/m from C to D.



Sol

R_1 from moment A to B



point load b/w the centre of CD

$E = 10 \times 2\ KN = 20KN$

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Taking moment from A to B

$$R_A \times 0 - 20\times 3 - 20\times 4 + R_B \times 8 = 0$$

$$-60 - 80 + 8R_B = 0$$

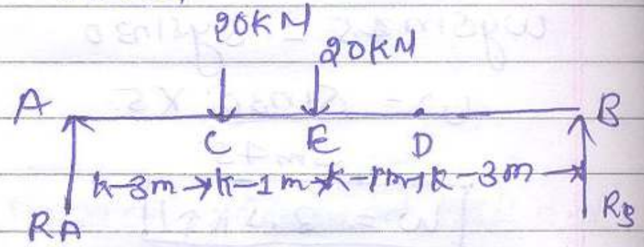
$$8R_B = 140$$

$$R_B = \frac{140}{8}$$

$$R_B = 17.5\,KN$$

$$\because R_A + R_B = 20 + 20$$

$$= 40\,KN$$



Let virtual displacement is y then

displace on C,

$$y_c = \frac{3}{8}y,\ y_E = \frac{4}{8}y,\ y_B = y$$

$$R_A \times 0 - \frac{3}{8}y \times 20 - \frac{4}{8}\times 20 \times y + R_B \times y = 0$$

$$0 - 3\times\frac{5}{2} - 4\times\frac{5}{2} + R_B$$

$$R_B = \frac{15+20}{2} = \frac{35}{2} = 17.5\,KN$$

$$R_A + R_B = 40\,KN$$

$$R_A = 40 - 17.5$$

$$= 22.5\,kN$$

Q A m/c Lift a load of 250N an effort of 160N and a load of 375 N by an effort of 170N. If the velocity Ratio of m/c is 20
(I) law of m/c
(II) effeciency of m/c at 250N and 375N load
(III) effort loss in friction in both case

Sol^n

As per given data

$P_1 = 160N$ $\quad$ $P_2 = 170N$

$W_1 = 250N$ $\quad$ $W_2 = 375N$

By the law of m/c

$P = Wm + C$

for (I)

$160 = 250m + C$ —(I)

for (II)^nd

$170 = 375m + C$ —(II)

(II) − (I)

$$\begin{aligned} 170 &= 375m + C \\ -160 &= 250m + C \\ \hline 10 &= 125m \end{aligned}$$

$m = \frac{10}{125} = \frac{2}{25} =$

$C = 160 - 250 \times \frac{2}{25} = 160 - 20 = 140$

(II) $M.A_1 = \frac{W_1}{P_1}$

$= \frac{250}{160}$

$= 1.5625$

$M.A_2 = \frac{W_2}{P_2}$

$= \frac{375}{170}$

$= 2.2058$

$\eta_1 = \frac{M.A_1}{V.R}$

$= \frac{1.5625}{20} = 0.078$

$= 7.8\%$

$\eta_2 = 0.11$

$= 11\%$

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$$P\text{-}Pi = Pi - \boxed{\frac{W}{V.R}}$$

(III) effort loss

for (I) $fp_1 = P_1 - \frac{W_1}{VR}$

$= 160 - \frac{250}{20}$

$fp_1 = 147.5$ ~~k~~N

for IInd

$fp_2 = P_2 - \frac{W_2}{VR}$

$fp_2 = 170 - \frac{375}{20}$

$fp_2 = 151.25$ N

Max. ... ...