

02

At thermal equilibrium condition
 Rate of Induced absorption = Rate of Spontaneous emission + Rate of Stimulated emission

Using eqn (1) (2) & (3), we have

$$B_{12} N_1 E_\nu = A_{21} N_2 + B_{21} N_2 E_\nu$$

$$E_\nu [B_{12} N_1 - B_{21} N_2] = A_{21} N_2$$

$$E_\nu = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} \quad \text{--- (4)}$$

Dividing both N_1 & N_2 by $B_{21} N_2$

$$E_\nu = \frac{A_{21} N_2 / B_{21} N_2}{\frac{B_{12} N_1}{B_{21} N_2} - 1}$$

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right] \quad \text{--- (5)}$$

According to Boltzmann's law $\frac{N_1}{N_2} = e^{\frac{h\nu}{KT}} \quad \text{--- (6)}$

Eqn (5) becomes

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{KT}} - 1} \right] \quad \text{--- (7)}$$

But, the energy density of black body radiation is given by Planck's law is

$$E_\nu = \frac{8\pi h \nu^3}{c^3} \left[\frac{1}{e^{\frac{h\nu}{KT}} - 1} \right] \quad \text{--- (8)}$$

By comparing eqn (7) & (8), we have

$$\frac{B_{12}}{B_{21}} = 1$$

$$B_{12} = B_{21}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}$$

Thus coeff of Induced absorption = coeff of Stimulated emission.

4 b. Describe attenuation & explain the various fiber losses. 03

Defn of attenuation:

The power loss of optical signals when they propagated in an optical fibers.

$$P_L = -10 \log_{10} \left(\frac{P_{out}}{P_{in}} \right) \text{ in dB.}$$

Various losses in fibers:

1. Absorption loss:

Absorption of photons by impurities like metal ions such as iron, chromium, cobalt & copper in the silica glass of which the fiber is made of. During signal processing photons interact with electrons of impurity atoms. The atoms are excited & de-excite by emitting photons of different characteristics.

The absorption of photons by fiber material itself is called intrinsic absorption.

2. Scattering loss:

When the wavelength of the photon is comparable to the size of the particle then the scattering take place. Because of the non-uniformity in manufacturing, the R.I. changes with lengths leads to a scattering. This type of scattering is called as Rayleigh scattering. It is inversely proportional to the fourth power of λ .

3. Radiation loss:

Radiation losses occur due to

a) macroscopic bending:

All optical fibers are having critical radius of curvature provided by the manufacturers. If the fiber is bent below that specification of radius of curvature, the light ray incident on the core cladding interface will not satisfy the condition of TIR. This causes loss of optical power.

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b) Microscoping loss:

Optical power loss in optical fibres is due to non-uniformity of the optical fibres when they are laid. Non-uniformity is due to manufacturing defects & also lateral pressure built up on the fiber. The defect due to micro-bendings can be overcome by introducing optical fiber inside a good strengthen polyurethane jacket.

1 c. Given the numerical aperture 0.30 & R.I of core 1.49. Calculate the critical angle for the core-cladding interface.

Given data:

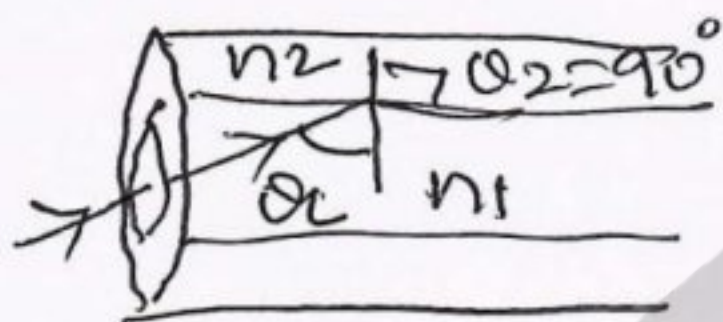
$$N.A = 0.30$$

$$n_1 = 1.49 \quad \theta_c = ? \quad n_2 = 1.$$

$$n_0 \sin \theta_0 = n_1 \sin \theta_1$$

$$n_1 \sin \theta_c = n_2 \sin \theta_2$$

$$\theta_2 = 90^\circ$$



$$\sin \theta_c = \frac{n_2 \cdot \sin \theta_2}{n_1}$$

$$= \frac{1 \times \sin 90^\circ}{1.49}$$

$$\sin \theta_c = \frac{1}{1.49} = 0.6711$$

$$\theta_c = \sin^{-1}(0.6711)$$

$$\boxed{\theta_c = 42.155^\circ}$$

2a. Discuss the applications of LASER in bar-code scanner & LASER cooling.

Ans: A barcode scanner or barcode reader, is a device with light lenses & a sensor that decodes & captures the information contained in barcodes. In the early days 1D codes, codes could only be read by lasers.

Laser light is shone on the label surface & its reflection is captured by a sensor (laser photo detector) to read a bar code. A laser beam is reflected off a mirror & swept left & right to read a bar code.

LASER COOLING:

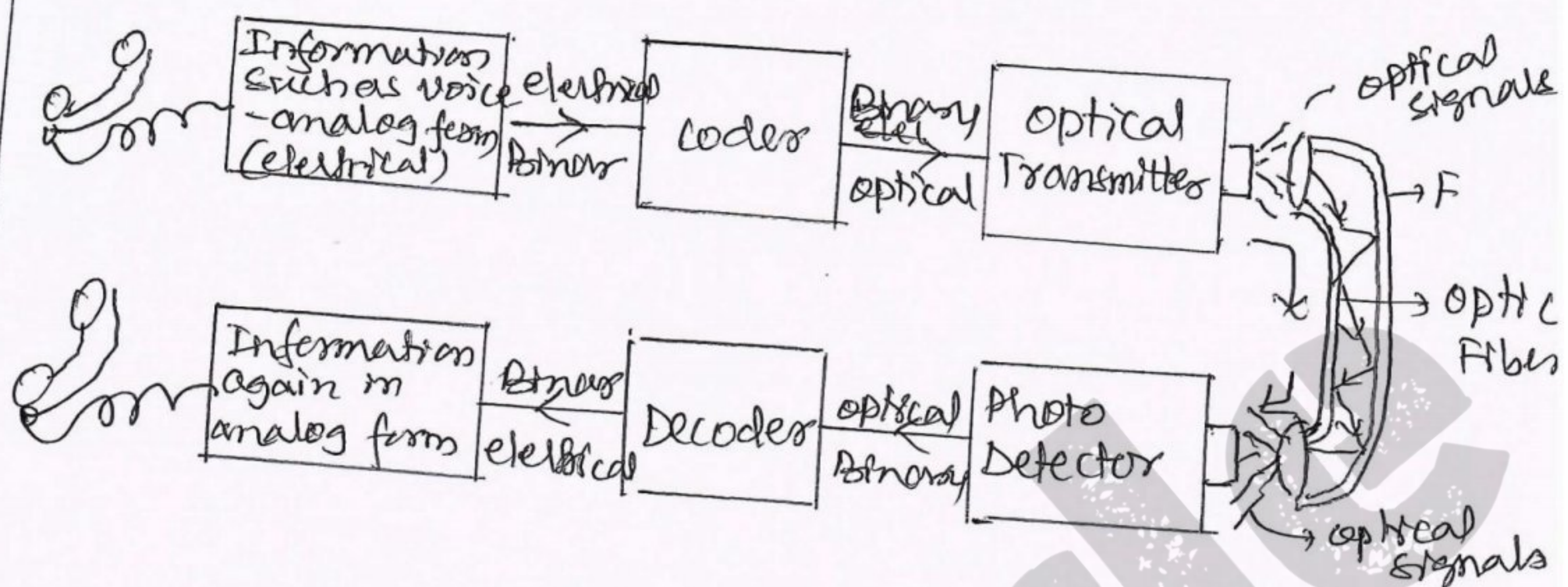
Principle of laser cooling: is the use of dissipative light forces for reducing the random motion & thus the temperature of small particles, typically atoms or ions. Depending on the mechanisms used, the temp achieved can be in milli kelvin, microkelvin or even nanokelvin regime.

If an atom is travelling towards a laser beam & absorbs a photon from the laser, it will be slowed by the fact that the photon has momentum.

$$p = \frac{h}{\lambda}$$

It would take a large number of such absorptions to cool the atoms to near 0°K.

2b. Discuss point to point communication using optical fibers.



In an optical fiber communication system, the input signals (audio, video or other digital data) are used to modulate light from a source like a LED or a semiconductor LASER & is transmitted through optical fiber. At the receiving end the signal is demodulated to reproduce the input signal. If data transfer takes place between only two devices then, it is called point to point communication.

The communication using optical fiber is as follows. First voice is converted into electrical signal using a transducer. It is digitized using a coder. The digitized signal, which carries the voice information, is fed to an optical transmitter. The light source in optical transmitter (LED or LASER Diode) emits modulated signals, which is transmitted through the fiber. At the other end the modulated light signal is detected by a photo detector & is decoded using decoder. Finally the information is converted into analog electrical signal & is fed to a loud speaker (sound).

2c. Calculate the ratio of population for a given pair of energy levels corresponding to emission of radiation 694.3 nm at a temp of 300 K

Soln

$$\frac{N_2}{N_1} = ?$$

$$\lambda = 694.3 \text{ nm} \quad T = 300 \text{ K}$$

$$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} \quad c = 3 \times 10^8 \text{ m/s}$$

$$K = 1.38 \times 10^{-23} \text{ J/K}$$

We have the Boltzmann's factor

$$\frac{N_2}{N_1} = e^{-\frac{h\nu}{kT}} = e^{-\frac{hc}{\lambda kT}}$$

$$\frac{hc}{\lambda kT} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{694.3 \times 10^{-9} \times 1.38 \times 10^{-23} \times 300}$$

$$\boxed{\frac{N_2}{N_1} = 8.874 \times 10^{-30}}$$

Q. 3a. Derive an expression for de-Broglie wavelength by analogy & hence discuss the significance of de-Broglie waves.

Ans: We know that, for an e^- accelerated under a p.d. of V , the energy acquired will be eV .

If ' m ' is the mass & ' v ' is the velocity of the e^- , then the energy equation for non-relativistic case can be written as

$$eV = \frac{1}{2} mv^2 \quad \text{--- (1)} \quad (E = eV) \quad (E = \frac{1}{2} mv^2)$$

If ' p ' is the momentum of the e^- , then

$$p = mv$$

Squaring on both sides, we have

$$p^2 = m^2 v^2$$

$$m \cdot mv^2 = p^2$$

$$mv^2 = \frac{p^2}{m}$$

Eqn (1) becomes

$$eV = \frac{p^2}{2m}$$

$$p^2 = 2meV$$

$$p = \sqrt{2meV}$$

By de-Broglie wavelength, we have

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{1}{\sqrt{V}} \left[\frac{h}{\sqrt{2me}} \right]$$

Now, h , m & e are universal physical constants

$$\lambda = \frac{1}{\sqrt{V}} \left[\frac{h}{\sqrt{2me}} \right]$$

$$= \frac{1}{\sqrt{V}} \left[\frac{6.63 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{19}} \right]$$

$$\lambda = \frac{1.226 \times 10^{-9}}{\sqrt{V}}$$

$$\boxed{\lambda = \frac{1.226}{\sqrt{V}} \text{ nm}}$$

Properties associated with the matter waves

1. Matter waves are associated only with particles in motion.
2. They are not EM in nature
3. Group velocity is associated with matter waves
4. As a result of superposition of large number of component waves which slightly differ in frequency, matter waves are localized.
5. The wavelength of matter waves is given by $\lambda = \frac{h}{mv}$

3 b. Explain the wave function with mathematical form & discuss the physical significance of a wave function.

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Wave function:

The quantity whose periodic variations make up the matter wave is called wave function.

The variable quantity that characterizes the de-Broglie wave of the particle is called a wave function, denoted by the symbol " ψ ".

The wave function eqn

$$\psi = A e^{i(kx - \omega t)}$$

$A \rightarrow$ constant, $k \rightarrow$ propagation constant
 $\omega \rightarrow$ angular frequency.

Physical significance of wave function:

① Probability density:

If ψ is the wave function associated with a particle, then $|\psi|^2$ is the probability of finding a particle in unit volume. If " τ " is the volume in which the particle is present in certain elemental volume $d\tau$ is given by $|\psi|^2 d\tau$.

Thus $|\psi|^2$ is called probability density.

The probability density is real & +ve quantity.

In the case of complex functions, the probability density is

$$|\psi|^2 = \psi^* \psi \quad \text{where } \psi^* \text{ is complex conjugate of } \psi.$$

Normalization:

The probability of ~~density~~ finding a particle having wave function " ψ " in a volume " $d\tau$ " is $|\psi|^2 d\tau$.

If it is certain that the particle is present in finite volume " τ ", then

$$\int_0^{\tau} |\psi|^2 d\tau = 1$$

If we are not certain that the particle is present in finite volume, then

$$\int_{-\infty}^{+\infty} |\psi|^2 d\tau = 1$$

The process of integrating the square of the wave function within a suitable limits & equating it to unity the value of the constant involved in the wave function is estimated. The constant value is substituted in the wave function. This process is called normalization.

The wave function with constant value included is called as the normalized wave function & the value of constant is called normalization factor.

3C. Calculate the energy of the first three states for electron in 1-dimensional potential well of width 0.1 nm.

Soln

Given data:

- | | | |
|--------------------------|-----------|---|
| (i) Ground state energy | $E_1 = ?$ | width of potential well $a = 0.1 \text{ nm}$
$= 10^{-10} \text{ m}$
$a = 1 \text{ \AA}$ |
| (ii) First excited state | $E_2 = ?$ | |
| (iii) Second " " | $E_3 = ?$ | |
| (iv) Third " " | $E_4 = ?$ | |

Ans

Energy of the electron in an 1-dimensional infinite potential well is given by.

$$E_n = \frac{n^2 h^2}{8ma^2} =$$

$$(i) \text{ If } n=1 \quad E_1 = \frac{h^2}{8ma^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$E_1 = 6.0314 \times 10^{-18} \text{ J} = 37.64 \text{ eV}$$

$$(ii) \text{ If } n=2 \quad E_2 = 4 \times E_1 = 4 \times 6.0314 \times 10^{-18} \text{ J}$$

$$E_2 = 2.4125 \times 10^{-17} \text{ J} = 150.54 \text{ eV}$$

$$(iii) \text{ If } n=3 \quad E_3 = 9 \times 6.0314 \times 10^{-18} =$$

$$5.4282 \times 10^{-17} \text{ J} = 338.7 \text{ eV}$$

$$(iv) \text{ If } n=4 \quad E_4 = 16 \times 6.0314 \times 10^{-18} =$$

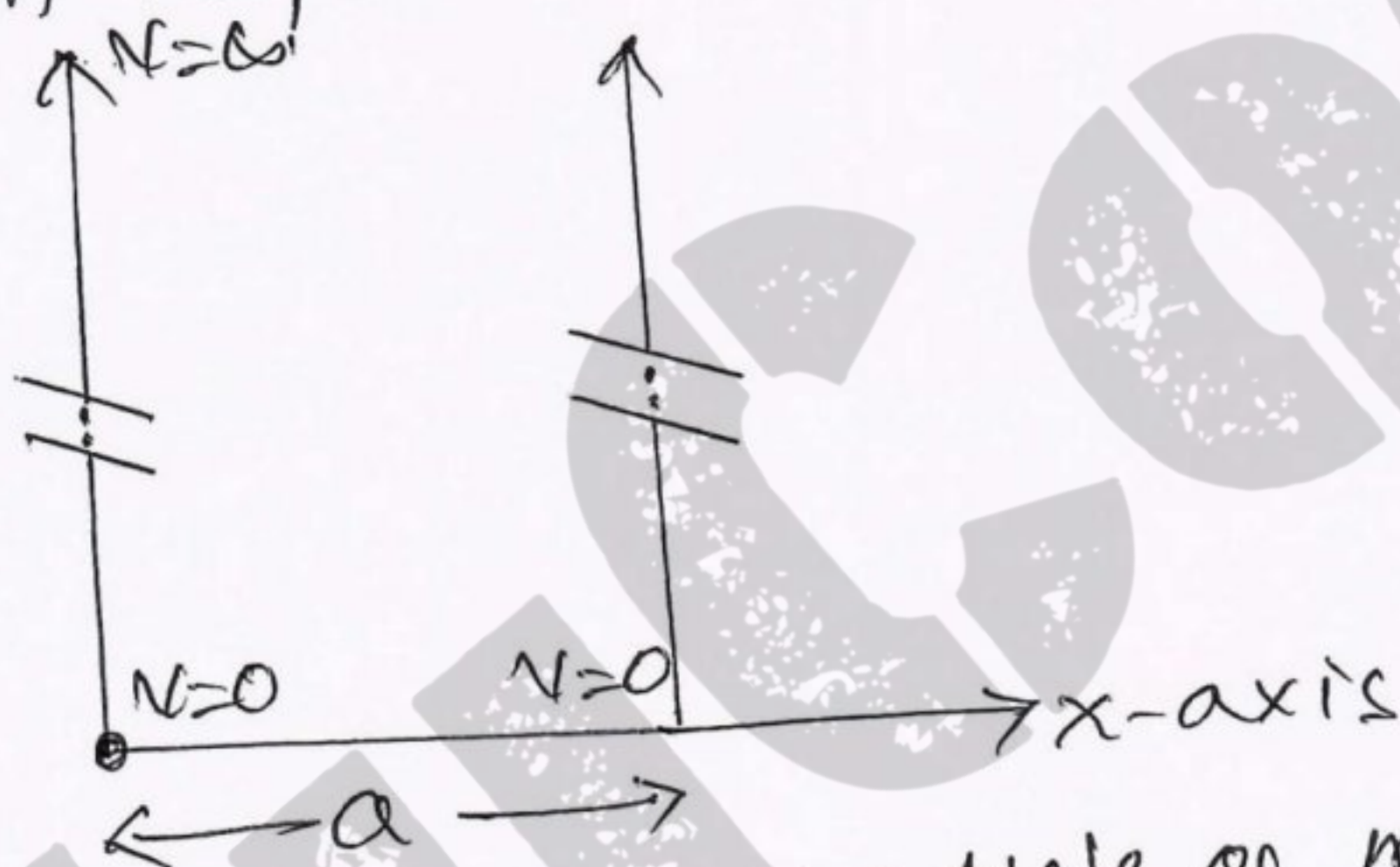
$$97.010 \times 10^{-17} \text{ J} = 606.314 \text{ eV}.$$

4a. Explain Eigen functions & Eigen values & hence derive the Eigen function of a particle inside infinite potential well of width "a" using the method of normalization.

Ans:

Eigen functions are those wave functions in Quantum mechanics which possesses the properties:

- (i) They are single valued,
- (ii) Finite value everywhere
- (iii) The wave functions & their first derivatives with respect to their variables are continuous.



Consider a particle of mass "m" free to move in 1-dimensional along +ve x-axis betn $x=0$ to $x=a$.

The potential energy outside this region is infinite & within region is zero.

The particle is in bound state.

Such a configuration of potential in space is called infinite potential well. It is also called particle in a box.

The Schrodinger equation outside the well

$$\text{is } \frac{d^2\psi}{dx^2} + \frac{8\pi^2m(E-V)}{h^2}\psi = 0 \quad \text{--- (1)}$$

If $V = 0$, outside the well

If $V = 0$ Equⁿ (1) becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2mE}{h^2}\psi = 0 \quad \text{--- (2)}$$

Putting $\frac{8\pi^2mE}{h^2} = k^2$ --- (3)

Equⁿ (2) can be written as

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{--- (4)}$$

The general solution of the quadratic equation (4) is of the form

$$\psi = C \cos kx + D \sin kx \quad \text{--- (5)}$$

Where C & D are constants determined from boundary conditions as follows.

case (i) $\psi(x) = 0$ at $x = 0$

From equⁿ (5) becomes

$$0 = C \cdot \cos k(0) + D \cdot \sin k(0)$$

$$\boxed{C = 0} \quad \text{--- (6)}$$

case (ii) $\psi(x) = 0$ at $x = a$.

$$0 = C \cdot \cos ka + D \cdot \sin ka$$

But $C = 0$

$$0 = D \cdot \sin ka$$

If $D \neq 0$ (because wave concept vanishes).

$$ka = \sin^{-1}(0)$$

$$\boxed{ka = n\pi} \quad \text{where } n = 0, 1, 2, 3, 4, \dots \text{ (Quantum numbers)}$$

$$\boxed{k = \frac{n\pi}{a}}$$

Using equⁿ (5), we have

$$\psi = D \cdot \sin\left(\frac{n\pi}{a}x\right)$$

$$\psi = D \cdot \sin\left(\frac{n\pi}{a}\right)x \quad \text{--- (7)}$$

which gives permitted wave function.

To find out the value of D , normalization of the wave function is to be done.

$$\int_0^a |\psi_n|^2 dx = 1 \quad \text{--- (8)}$$

Using eqn (7)

$$\int_0^a D^2 \cdot \sin^2\left(\frac{n\pi}{a}\right)x dx = 1$$

$$D^2 \int_0^a \sin^2\left(\frac{n\pi}{a}\right)x dx = 1 \quad \left(\sin^2\theta = \frac{1 - \cos 2\theta}{2}\right)$$

$$\frac{D^2}{2} \int_0^a \sin^2\left(\frac{n\pi}{a}\right)x dx = 1$$

$$\frac{D^2}{2} \int_0^a \left[\frac{1 - \cos\left(\frac{2n\pi}{a}\right)x}{2} \right] dx = 1$$

$$\frac{D^2}{2} \left[\int_0^a dx - \int_0^a \cos\left(\frac{2n\pi}{a}\right)x dx \right] = 1$$

$$\frac{D^2}{2} \left[x - \frac{a}{2n\pi} \cdot \sin\left(\frac{2n\pi}{a}\right)x \right]_0^a = 1$$

$$\frac{D^2}{2} [a - 0] = 1$$

$$\frac{D^2}{2} a = 1 \quad D = \sqrt{\frac{2}{a}}$$

Hence the normalized

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}\right)x$$

Q. b. Show that electron does not exist inside the nucleus using Heisenberg's uncertainty principle.

Ans: Electron to be present in the nucleus, maximum uncertainty in position 10^{-14} m (diameter). According to HUP,

The minimum uncertainty in momentum ΔP_x

$$\Delta P_x \cdot \Delta x \geq \frac{h}{4\pi}$$

$$\Delta P_x \geq \frac{h}{4\pi \cdot \Delta x} \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}}$$

$$\Delta P_x \geq 5.275 \times 10^{-21} \text{ kg}\cdot\text{m/s} = p (\text{say}).$$

The minimum energy of the e^- in the nucleus is given by 1. $E = \frac{p^2}{2m} = \frac{(5.275 \times 10^{-21})^2}{2 \times 9.01 \times 10^{-31}}$

$$E \geq 1.527 \times 10^{-11} \text{ J}$$

$$E \geq \frac{1.527 \times 10^{-11}}{1.6 \times 10^{-19}} \geq 95.45 \text{ MeV}$$

$$E \geq 95 \text{ MeV}$$

But the maximum K.E of the electron (β -particle) emitted from the nucleus does not exceed 4 MeV. Hence the electron does not exist inside the nucleus.

4c. An electron is associated with a de-Broglie wavelength of 1nm . calculate the energy & the corresponding momentum of the e^- .

Soln

Given data:

$$\lambda = 1\text{nm} = 10^{-9}\text{m} \quad E = ? \quad p = ?$$

Using de-Broglie wavelength equation

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Square on both sides, we get

$$\lambda^2 = \frac{h^2}{2mE}$$

$$E = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (10^{-9})^2}$$

$$E =$$

de-Broglie wavelength

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-9}}$$

$$\boxed{p = 6.63 \times 10^{-25}} \text{ Kg} \cdot \text{m/s.}$$

5 a. Discuss the working of phase gate mentioning its matrix representation & truth table.

Ans:

Phase gate or S-gate:

The phase gate turns a $|0\rangle$ into $|0\rangle$ and a $|1\rangle$ to $i|1\rangle$

The matrix representation of the S gate is given by

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

The effect of S gate on input $|0\rangle$ is given by

$$S|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

Similarly the effect of S gate on input $|1\rangle$ is given by the effect of S gate on input $|0\rangle$ is given by

$$S|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

The transformation of state $|\psi\rangle$ is given by

$$\begin{aligned} S|\psi\rangle &= S(\alpha|0\rangle + \beta|1\rangle) \\ &= \alpha S|0\rangle + \beta S|1\rangle \end{aligned}$$

$$S|\psi\rangle = \alpha|0\rangle + \beta i|1\rangle$$

The S gate & the Truth table are given by for S-gate

Input	output
$ 0\rangle$	$ 0\rangle$
$ 1\rangle$	$i 1\rangle$
$\alpha 0\rangle + \beta 1\rangle$	$\alpha 0\rangle + i\beta 1\rangle$

$$\alpha|0\rangle + \beta|1\rangle \longrightarrow \boxed{S} \longrightarrow \alpha|0\rangle + i\beta|1\rangle.$$

5b. Explain orthogonality & orthonormality with an example for each.

Ans: Orthogonality:

Two states $|\psi\rangle$ and $|\phi\rangle$ are said to be orthogonal if their inner product is zero. Mathematically

$$\langle\phi|\psi\rangle = 0$$

The two states are orthogonal means they are mutually exclusive.

Like spin up & spin down of an e^- consider $\langle 0|1\rangle$

$$\begin{aligned}\langle 0|1\rangle &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \times 0 + 0 \times 1 \end{bmatrix} \\ &= \langle 0 + 0 \rangle \\ &= \langle 0 \rangle.\end{aligned}$$

Orthonormality:

The states $|\psi\rangle$ & $|\phi\rangle$ are said to be orthonormal if

- ① $|\psi\rangle$ and $|\phi\rangle$ are normalized
- ② $|\psi\rangle$ and $|\phi\rangle$ are orthogonal to each other.

5C. Given $|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$

Prove that $\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$

Ans: $|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$

$$|\psi\rangle\langle\phi| = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix}$$

$$|\psi\rangle\langle\phi| = \alpha_1\beta_1 + \alpha_2\beta_2 \quad \text{--- (1)}$$

and $\langle\phi|\psi\rangle^* = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix}$

$$\langle\phi|\psi\rangle^* = \beta_1\alpha_1 + \beta_2\alpha_2 \quad \text{--- (2)}$$

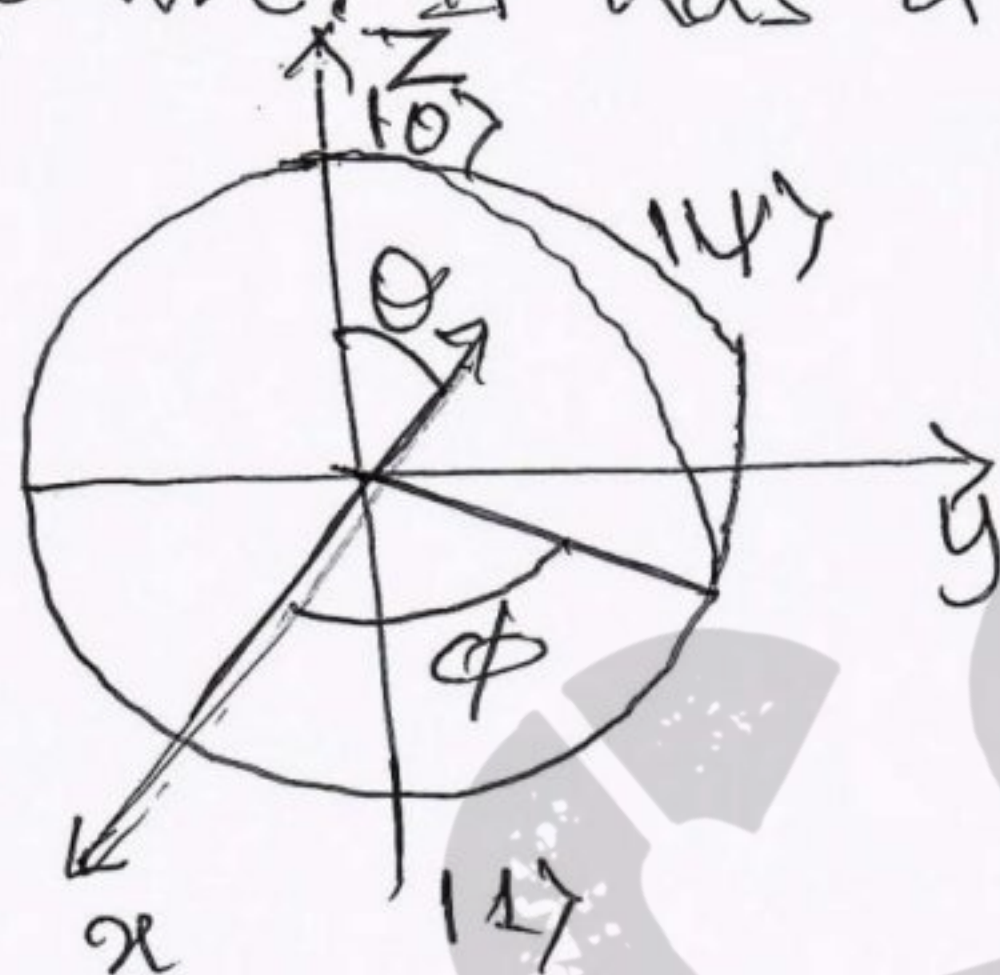
Hence

$$\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$$

6a. Explain the representation of qubit using Bloch sphere.

Ans: Representation of Qubits by Bloch Sphere

The pure state space qubits (Two level Quantum Mechanical Systems) can be visualized using an imaginary sphere called Bloch sphere. It has a unit radius.



The arrow on the sphere represents the state of the qubits. The north and south poles are used to represent the basis states $|0\rangle$ and $|1\rangle$ respectively.

The other locations are superpositions of $|0\rangle$ and $|1\rangle$ states & represented by $\alpha|0\rangle + \beta|1\rangle$ with $\alpha^2 + \beta^2 = 1$

Thus a qubit can be any point on the Bloch sphere.

The Bloch sphere allows the state of the Qubit to be represented using spherical co-ordinates

They are the polar angle θ & the azimuthal angle ϕ .

The Bloch sphere is represented by the equation.

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

Here $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$

The normalization constraint is given by

$$\left|\cos\frac{\theta}{2}\right|^2 + \left|\sin\frac{\theta}{2}\right|^2 = 1$$

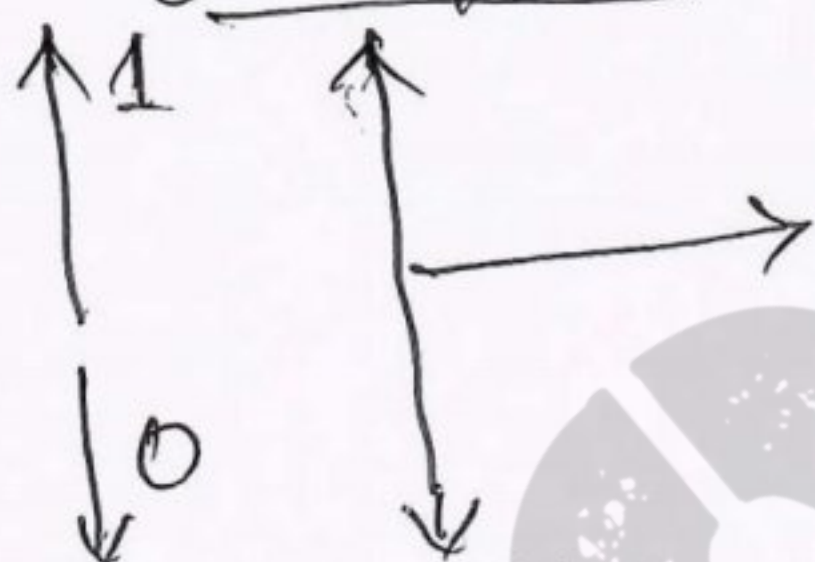
6 b. Explain single Qubit & multiple qubit gate with an example for each.

Ans: Single Qubit:

A single Qubit has two computational basis states $|0\rangle$ and $|1\rangle$.

The pictorial representation of the single Qubit is as follows.

$\alpha|0\rangle + \beta|1\rangle$
one qubit



Multiple Qubit:

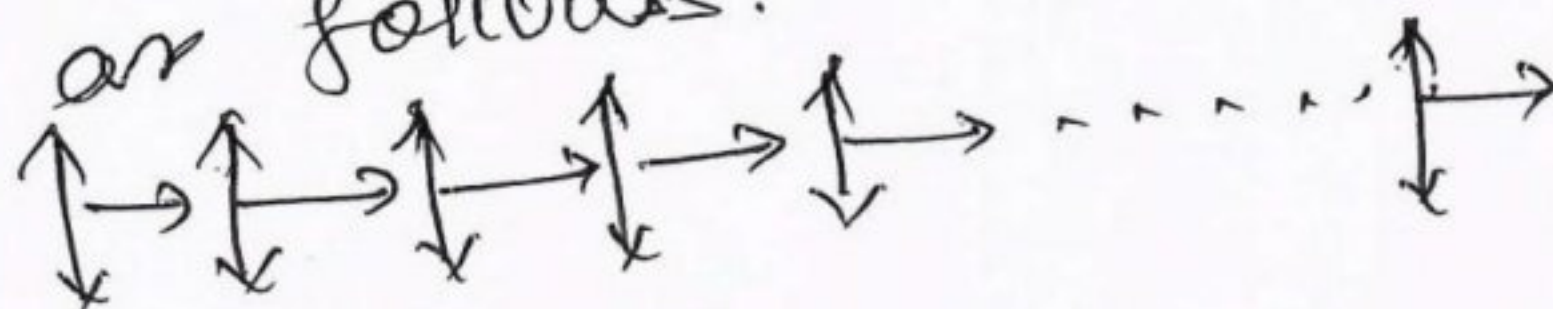
A multiple qubit system of N -qubits has 2^N computational basis states.

For example a state with 3 qubits has 2^3 computational basis states.

Thus for N qubits, the the combinational basis n -states are denoted as.

$100 \dots 00, 100 \dots 01, 100 \dots 10,$
 $100 \dots 11, \dots, 11 \dots 11.$

The block diagram of representation of N -qubit is as follows.



6C. Explain the matrix representation of 0 & 1 states & apply identity operator I to $|0\rangle$ & $|1\rangle$ states.

Ans: Matrix representation of 0 & 1 states:
The wave function could be expressed in ket notation as

$|\psi\rangle$ (ket vector), ψ is the wave function,

$$\text{the } |\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

The matrix for of the states $|0\rangle$ & $|1\rangle$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Identify operator:

The operator of type $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is called identity operator.

When an identify operator acts on a state vector it keeps the state intact.

By analogy we study identify operator as an identify matrix.

Let us consider the operation of identify operator on $|0\rangle$ and $|1\rangle$ states.

As per the principle of identity operation

$$I|0\rangle = |0\rangle \text{ and } I|1\rangle = |1\rangle$$

$$I|0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$I|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Thus the operation of identify matrix (operator) on $|0\rangle$ and $|1\rangle$ leaves the states unchanged.

7a. Enumerate the failures of classical free electron theory & assumption of a Quantum free electron theory of metals.

Ans: Failures of CFET:

(i) Electrical & thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temp. dependence of conductivity & dependence of electrical conductivity on electron concentration.

(ii) Specific heat:

The molar sp. heat of a gas at constant volume is

$$C_v = \frac{3}{2} R$$

As per the CFET, free e^- in a metal are expected to behave just as gas molecules.

Thus the above eqn holds good equally well for the free e^- s also.

But experimentally it was found that the contribution to the specific heat of a metal by its conduction e^- was $C_v = 10^{-4} RT$ which is far lower than expected value.

i

(i) Temperature dependence of electrical conductivity
Experimentally, electrical conductivity σ is inversely proportional to the temperature T .

$$\sigma_{\text{expt}} \propto \frac{1}{T} \quad \text{--- (1)}$$

According to assumption of CFET

$$\sigma \propto \frac{1}{\sqrt{T}} \quad \text{--- (2)}$$

From eqn (1) & (2), it is clear that the experimental value is not agreeing with the theory.

Quantum Free Electron Theory:

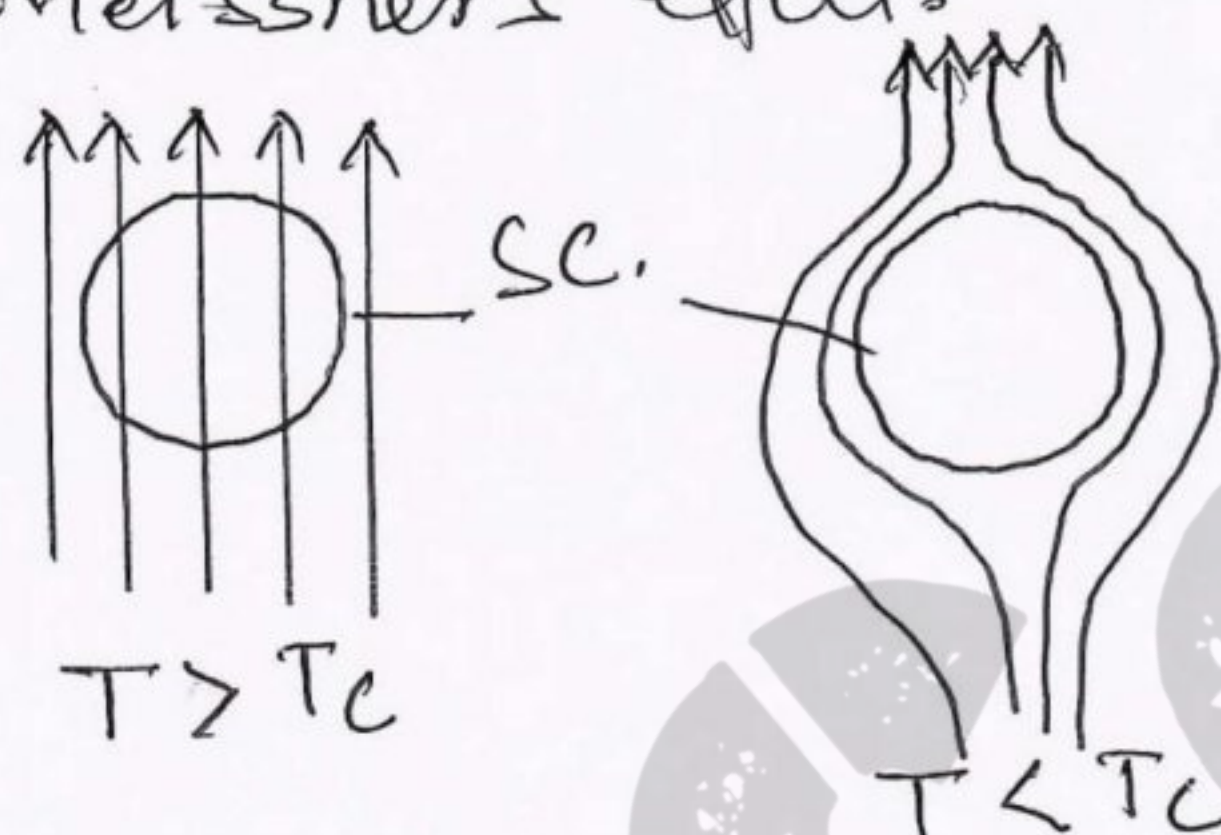
Assumptions:

1. The energy values of the conduction e^- are quantized. The allowed energy values are realized in terms of a set of energy values.
2. The distribution of e^- in the various allowed energy levels occurs per per Pauli's exclusion principle.
3. The electrons travel with a constant potential inside the metal but confined within its boundaries.
4. The attraction between the e^- & lattice ions & the repulsion between the e^- themselves are ignored.

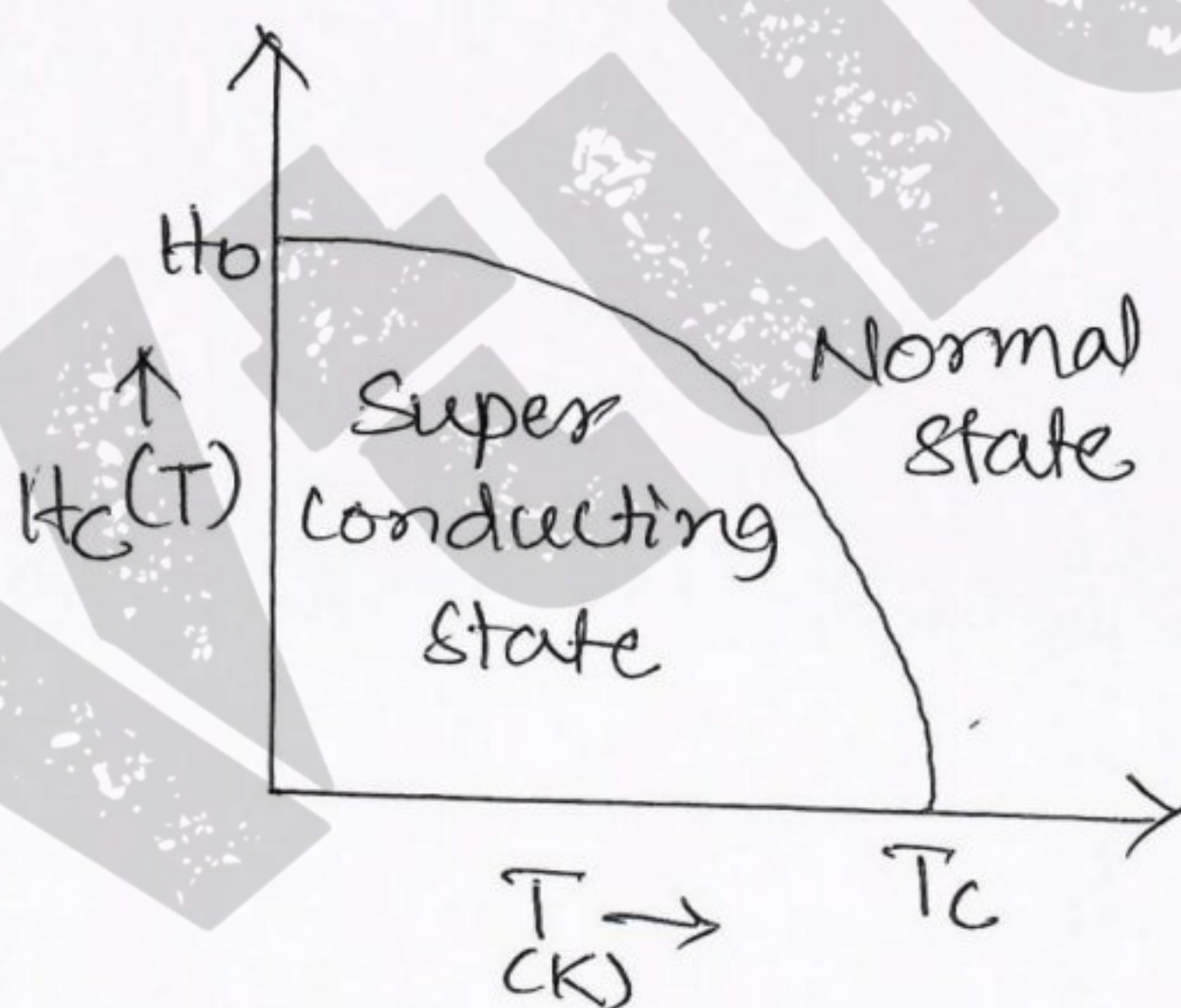
7b. Explain Meissner's effect & the variation of critical field with temp.

Ans:

Statement: A superconducting material kept in a magnetic field expels the magnetic field flux out of its body when it is cooled below the critical temperature & thus becomes perfect diamagnetic. This effect is called as Meissner's effect.



Effect of temp critical field with temp:



The superconducting state of a metal exists only in a particular range of temp & field strength. The condition for the superconducting state to exist in the metal is that some combination of temp & field strength should be less than a critical value.

29

Superconductivity will disappear, if the temp of the specimen is raised above T_c or if sufficiently strong magnetic field is employed.

"The minimum magnetic field required to destroy the superconductivity in the material is called critical field H_c ".

The critical field as a function of temp is nearly parabolic & can be reasonably represented by

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right]$$

$H_c \rightarrow$ critical field at 0°K

$T_c \rightarrow$ critical temp

$H \rightarrow$ Applied or external magnetic field.

7C. A superconducting tin has a critical temp of 3.7K at zero magnetic field & a critical field of 0.0306 Tesla at 0°K. Find the critical field at 2K

Soln

Given data:

$$T_c = 3.7K, \quad H_0 = 0.0306T$$

$$H_c = ?, \quad T = 2K$$

Relation betⁿ H_c & H_0 is

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right]$$

$$H_c = 0.0306 \left[1 - \frac{2^2}{3.7^2} \right]$$

$$= 0.0306 [1 - 0.292]$$

$$= 0.0306 \times 0.7078$$

$$\boxed{H_c = 0.02165 T}$$

3a. Explain the phenomenon of superconductivity & discuss qualitatively the BCS theory of superconductivity for negligible resistance of metal at temp close to absolute zero.

Ans:

Superconductivity:

Superconductivity is the phenomenon observed in some metals & materials. K. Onnes in 1911 observed that the electrical ~~conductivity~~ resistivity of pure mercury drops abruptly to zero at about 4.2K. This state is called superconducting state. The material is the superconductor.

The temperature at which superconductivity is attained is called the critical temp T_c .

BCS Theory:

Bardeen, Cooper & Schrieffer (BCS) in 1957 explained the phenomenon of superconductivity based on the formation of Cooper pairs. It is called BCS theory. It is a quantum mechanical concept.

During the flow of current in a superconductor, when an electron approaches a +ve ion lattice of the metal there is a Coulomb force of attraction between the electron & the lattice ion & thus ion core is set in motion causing lattice distortion. Smaller the mass of the +ve core, larger will be distortion. The lattice vibrations are quantized in terms of phonons.

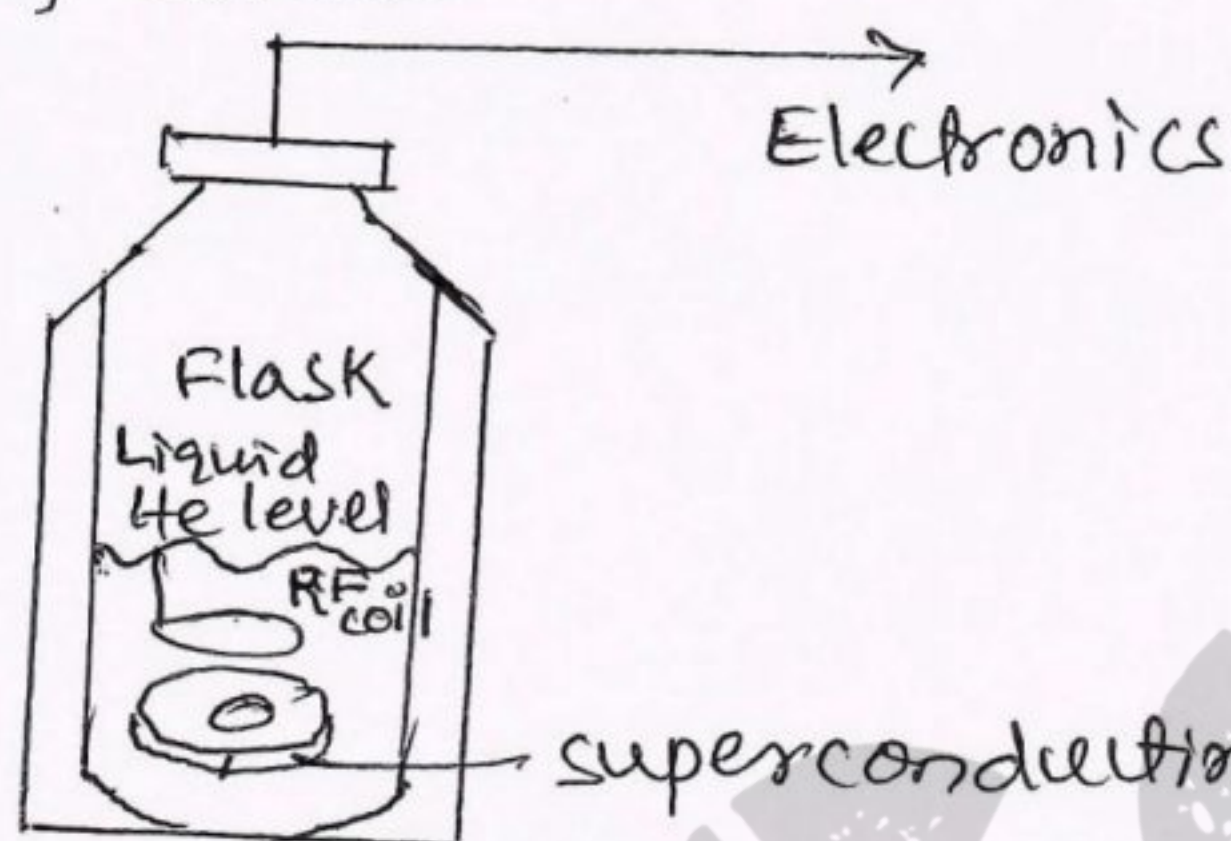
Now another e^- passing by this distorted lattice will interact with it & thus the energy of this electron is also reduced. This interaction is looked upon as if the two electrons interact via phonon field, resulting in lowering of energy for the electrons. Due to this interaction an apparent force of attraction develops between the electrons & they tend to move in pairs. This interaction is strongest when the two electrons have equal & opposite spins & momenta. This leads to the formation of Cooper pairs.

"Cooper pair are a bound pair of electron formed by the interaction between the electrons with opposite spin & momenta in a phonon field."

3b. Give the qualitative explanation of RF SQUID with the help of a neat sketch.

Ans: RF SQUIDS:

The RF (Radio Frequency) SQUID is a one-junction SQUID loop that can be used as magnetic field detector. Although it is less sensitive than the DC SQUID, it is cheaper & easier to manufacture & is therefore more commonly used.



In RF SQUID the flux is coupled into a loop containing a single JJ ~~met~~ through an input coil & an RF coil. RF coil is part of a high-Q-resonant circuit to read out current changes due to induced flux in the SQUID loop. The tuned circuit is driven by a constant RF oscillator which is weakly coupled to the loop.

8C. Find the temperature at which there is 1% probability that a state with an energy 0.5 eV above Fermi energy is occupied.

Soln

Given data:

Probability $f(E) = 1\% = 0.01$

Energy above E_F is

$$E - E_F = 0.5 \text{ eV} = 0.5 \times 1.6 \times 10^{-19} \text{ J}$$

To find: Temp $T = ?$

We have the eqn for the probability of occupation that a given energy state is occupied as

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} \quad \text{--- (1)}$$

$$\frac{E-E_F}{kT} = \frac{0.5 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times T} = \frac{5797}{T}$$

Substituting the above in eqn (1)

$$0.01 = \frac{1}{e^{\frac{5797}{T}} + 1}$$

$$e^{\frac{5797}{T}} = \frac{1}{0.01} - 1 = 100 - 1 = 99$$

$$e^{\frac{5797}{T}} = 99$$

taking natural logarithms on both sides, we have

$$\frac{5797}{T} = \ln 99 = 4.595$$

$$\frac{5797}{T} = 4.595$$

$$\boxed{T = 1261.1 \text{ K}}$$

1a. Elucidate the importance of size & scale and weight & strength in animations.

ms. Size and Scale:

The size & scale of characters often play a central role in a story's plot. What would Superman be without this height & bulging biceps?

We often equate large characters with height & strength, and smaller characters with agility & speed. There is a reason for this. In real life, larger people & animals do have a larger capacity for strength, while smaller critters can move & move over faster than their large counterparts. When designing characters, you can run into different situations having to do with size & scale such as:

1. Human or animal based characters those are much ~~smaller~~ larger than, we see in our everyday experience. Superheroes, Greek gods, monsters.
2. characters that need to be noticeably larger, smaller, older, heavier, lighter or more energetic than other characters.

Weight and Strength:

Body weight is proportional to volume. The abilities of your muscles & bones, however, increase by area because their abilities depend more on cross sectional area than volume. To increase a muscle or bone's strength, you need to increase its cross-sectional area.

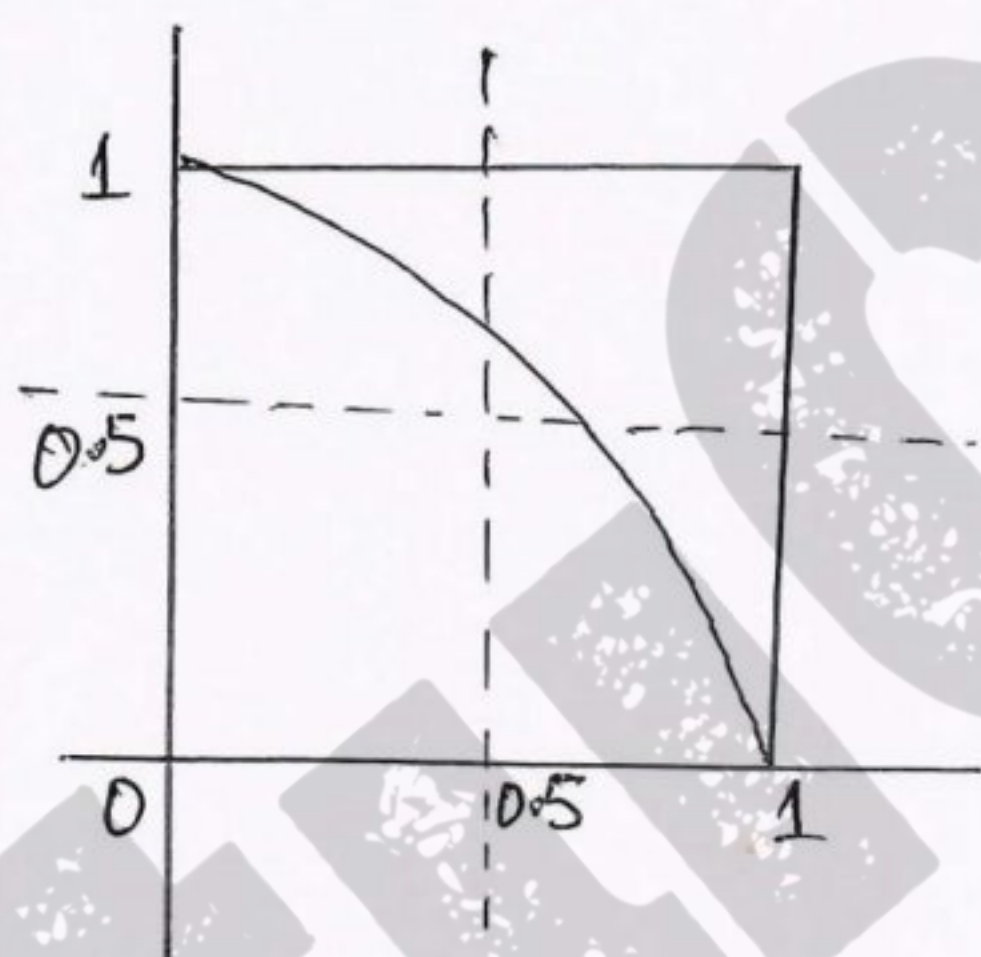
9b. Mention the general pattern of Monte-Carlo method & hence determine the value of π

Ans. Monte-Carlo Method:

Monte-Carlo method vary, but tend to follow a particular pattern.

1. Define a domain of possible inputs.
2. Generate inputs randomly from a probability distribution over the domain.
3. Perform a deterministic computation on the inputs.
4. Aggregate the results.

Determination of value of π :



Consider a quadrant inscribed in a unit square. Given that the ratio of their areas is $\pi/4$, the value of π can be approximated using a Monte-Carlo method.

1. Draw a ^{square} ~~circle~~, then inscribe a quadrant within it.
2. Uniformly scatter a given number of points over the square.
3. Count the number of points inside the quadrant, i.e. having a distance from the origin ≤ 1 .
4. The ratio of the inside count & the total sample count is an estimate of the ratio of the two areas $\pi/4$.
Multiply the result to estimate π .

9c. Describe the calculation of push time & stop time with example.

Ans: Push time:

The ratio of jump time to push time.
 $JM = \frac{\text{Jump time}}{\text{Push time}} \Rightarrow \text{Push time} = \frac{\text{Jump time}}{JM}$

Example: $JM = 3$

Jump time = 15 frames.

Push time = $\frac{\text{Jump time}}{JM} = \frac{15}{3} = 5$ Frames.

Stop time:

The stop height is often a bit larger than the push height, but the timing of the push & stop are the same in the scene that the CG moves the same distance per frame in push & stop.

$\frac{\text{Push Height}}{\text{Push Time}} = \frac{\text{Stop Height}}{\text{Stop Time}}$

$\frac{\text{Push Time}}{\text{Push Height}} = \frac{\text{Stop Time}}{\text{Stop distance}}$

$\text{Stop Time} = \frac{\text{Push Time} \times \text{Stop Distance}}{\text{Push Height}}$

Example:

Push Time = 5 Frames, Push Height = 0.4 m

Stop Height = 0.5 m

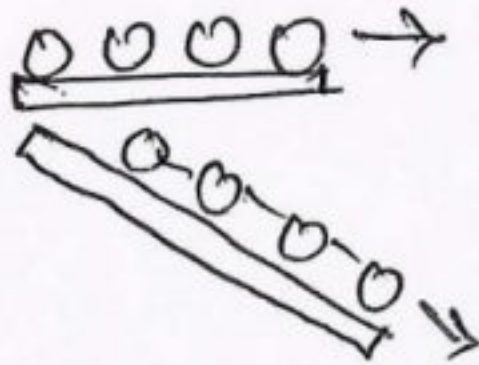
$\text{Stop Time} = \left[\frac{5 \times 0.5}{0.4} \right]$

Stop Time = 6 Frames

10 a. Sketch & explain the motion graph for linear, easy ease, easy ease in & easy ease out cases of animation.

Ans: Linear Motion:

Linear motion refers to motion in a straight line, always in the same direction. An object moving with linear motion might ~~depend~~^{speed} up or speed down as it follows a linear path.



When motion is accelerating or decelerating (etc), we refer to this type of motion as a slow in or slow out. These types of motion are sometimes called ease in or ease out.

1. Slow in - ease in - The object is slowing down, often in preparation for stopping.
2. Slow out, ease out - The object is speeding up, often from a still position.

The term slow out can be confusing, since it essentially means "speed up". One can think of slow out as the same as ease out, as in easing out of a still position & speeding up to full speed.

For example:

A ball ~~moving~~ rolling down an incline or dropping straight down is slowing out, as it goes from a still position or slow down speed to a fast speed. A ball rolling up an incline is slowing up.

10 b. Discuss modelling the probability for proton decay.

Ans: Modelling the probability for proton decay:

The experimental search for Proton decay was undertaken because of the implications of the Grand unification theories. The lower bound for the life time is now projected to be on the order of $\tau = 10^{33}$ years. The probability for observing proton decay can be estimated from the nature of particle decay & the application of Poisson statistics.

The number of protons N can be modelled by the decay equation. $N = N_0 e^{-\lambda t}$

Here $\lambda \approx 1/\tau = 10^{-33}/\text{year}$ is the probability that any given proton will decay in a year. Since the decay constant λ is so small, the exponential can be represented by the first two terms of the exponential series.

$$e^{-\lambda t} = 1 - \lambda t, \text{ Thus } N = N_0(1 - \lambda t)$$

For a small sample, the observation of a proton decay is infinitesimal, but suppose, we consider the volume of proton represented by the Super Kameokande neutrino detector in Japan. The number of protons in the detector volume is reported by Ed Kearns of Boston University to be 7.5×10^{33} protons.

For one year of observation, the number of expected proton decays is then

$$N - N_0 = N_0 \lambda t = (7.5 \times 10^{33} \text{ protons})(10^{-33}/\text{year})(1 \text{ year}) = 7.5$$

Poisson statistics provides a convenient means for assessing the implications of the absence of these observations. If we presume that $\lambda = 3$ observed decays per year is the mean, then the Poisson distribution function tells us that the probability for zero observations of a decay is

$$p(k) = \frac{\lambda^k e^{-\lambda}}{k!} \Rightarrow p(k) = \frac{3^0 e^{-3}}{0!} = 0.05$$

This low probability for a small null results suggests that the proposed life time of 10^{33} years is too short.

10 C A slowing-in object in an animation has a first frame distance 0.5m & the first ~~time~~ slow in frame 0.35m. Calculate the base distance & the number of frames in sequence.

Soln

Given data:

Frame distance = 0.5m

Slow in frame = 0.35m

Base distance = ?

Number of frames in ^{second} sequence = ?

$$\text{Base distance} = \frac{\text{Total distance}}{(\text{Last frame number} - 1)^2}$$

To find the base distance,

$$= \frac{0.5 - 0.35}{2} = \frac{0.15}{2} = 0.07$$

To figure out, how many frames are in the Slow-in, divide the first distance by base distance to find out which odd number it corresponds to

$$\frac{0.5}{0.07} = 7.$$

This means the first distance corresponds to 7. In 7, 5, 3, 1 sequence, making the sequence four frames long.

(Dr. Mahesh Bannir).



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