MODULE 4

Electrical Properties of Materials and Applications

Assumptions of classical free electron theory:

- A metal is imagined as the structure of 3-dimensional array of ions in between which, there are free moving valence electrons confined to the body of the material. Such freely moving electrons cause electrical conduction under an applied field and hence referred to as conduction electrons
- The free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature T is given by 3/2 kT, where k is a Boltzmann constant.
  It is related to the kinetic energy.
  $3/2\ kT = \frac{1}{2} m v_{th}^2$
  Where v_{th} is the thermal velocity same as root mean square velocity.
- The electric potential due to the ionic cores is taken to be essentially constant throughout the body of the metal and the effect of repulsion between the electrons is considered insignificant.
- The electric current in a metal due to an applied field is a consequence of the drift velocity in a direction opposite to the direction of the field.

Drift velocity (v_d):

The average velocity of occupied by the electrons in the steady state in an applied electric field is called drift velocity.

The drift velocity $v_d = \frac{eE\tau}{m}$

Thermal velocity(V_{th}):

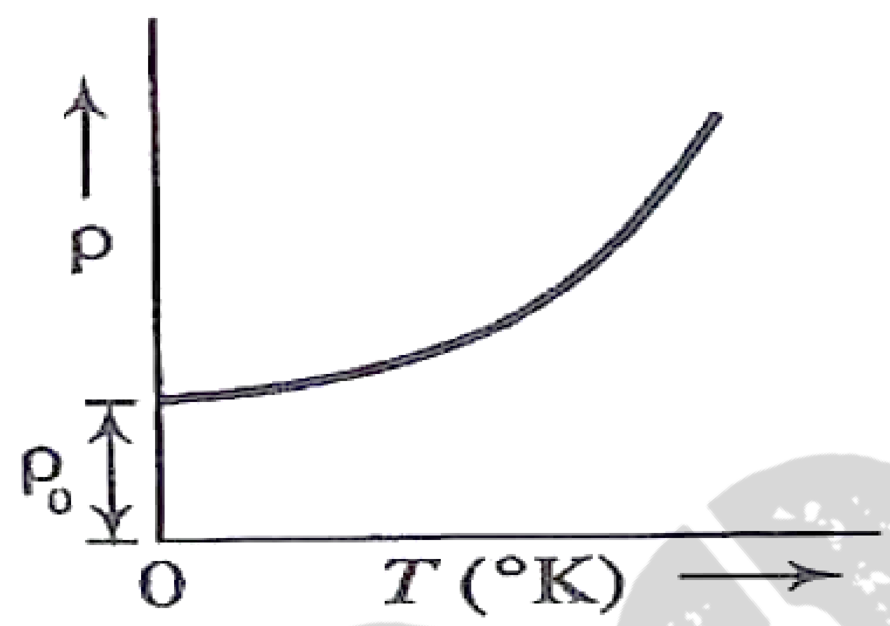
The velocity of electrons in random motion due to thermal agitation called thermal velocity.

Mean free path (λ):

The average distance travelled by the conduction electrons between any two successive collisions with lattice ions.

Temperature dependence of resistivity of a metal:

All metals are good conductors of electricity. The electrical conductivity of metal varies with the temperature. The electrical resistance of a metal, to the flow of current, is due to scattering of conduction electrons by lattice vibrations. When the temperature increases the amplitude of lattice vibrations also increases, thereby increasing the resistance. The dependence of resistance of metal (non-superconducting state) is shown in figure. The resistance decreases with temperature and reaches a minimum value at T = 0K. The residual resistance at T = 0K is due to impurities in the metal.



DEPENDENCE OF ρ ON T

By Matthiessen's rule

$\rho = \rho_0 + \rho(T)$

Where 'ρ' is the resistivity of the given material, 'ρ_0' is the residual resistivity and 'ρ (T)' is the temperature dependent part of resistivity.

"The total resistivity of a metal is the sum of the resistivity due to phonon scattering which is temperature dependent and the resistivity due to scattering by impurities which is temperature dependent"

Expression for electrical conductivity of conductor according to classical free electron theory

According to classical free electron theory the expression for electrical conductivity is given by

$$\sigma_{CFET} = \frac{ne^2\tau}{m}$$

Where σ - Electrical conductivity
n- Electron density
$\tau - mean\ collision\ time$
m- mass of electron

Failures of classical free electron theory:

Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

i) Specific heat: The molar specific heat of a gas at constant volume is

$$C_v = \frac{3}{2} R$$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also.

But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was

$$C_V = 10^{-4} RT$$

which is for lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

ii)Temperature dependence of electrical conductivity:

Experimentally, electrical conductivity σ is inversely proportional to the temperature T.

i.e. $\sigma_{exp} \; \alpha \; \frac{1}{T} \rightarrow (1)$

According to the assumptions of classical free electron theory

Since Vth $\alpha \sqrt{T}$

But $\tau \alpha \frac{1}{Vth}$, $\tau \alpha \frac{1}{\sqrt{T}}$,

substituting in conductivity equation we get

$$\sigma_{CFET} = \frac{ne^2 \tau}{m} = \frac{ne^2}{m\sqrt{T}}$$

Or $\sigma_{CFET} \alpha \frac{1}{\sqrt{T}}$ $\rightarrow (2)$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

iii) Dependence of electrical conductivity on electron concentration:

According to classical free electron the theory

$\sigma = \frac{ne^2\tau}{m}$ i.e., $\sigma \alpha n$, where n is the electron concentration,

Consider copper and aluminum. Their electrical conductivities are $5.88\times10^7/\Omega m$ and $3.65x10^7/\Omega m$. The electron concentrations for copper and aluminum are $8.45\times10^{28}/m^3$ and $18.06x10^{28}/m^3$. Hence the classical free electron theory fails to explain the dependence of σ on electron concentration.

Experimental results:

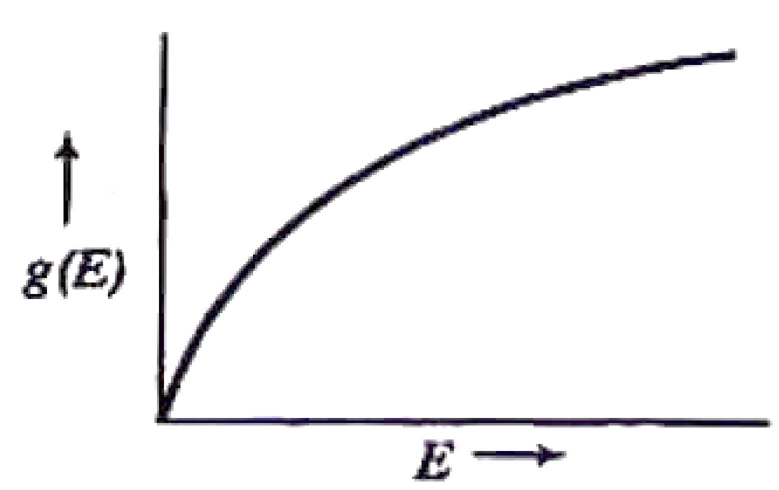
Metals	Electron concentration(n)	conductivity (σ)
Copper	$8.45\times10^{28}/m^3$	$5.88x10^7/\Omega m$
Aluminium	$18.06x10^{28}/m^3$	$3.65x10^7/ \Omega m$

Quantum free electron theory:

Assumptions of quantum free electron theory:

- The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
- The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
- The electrons travel with a constant potential inside the metal but confined within its boundaries.
- The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

Density of states g(E):

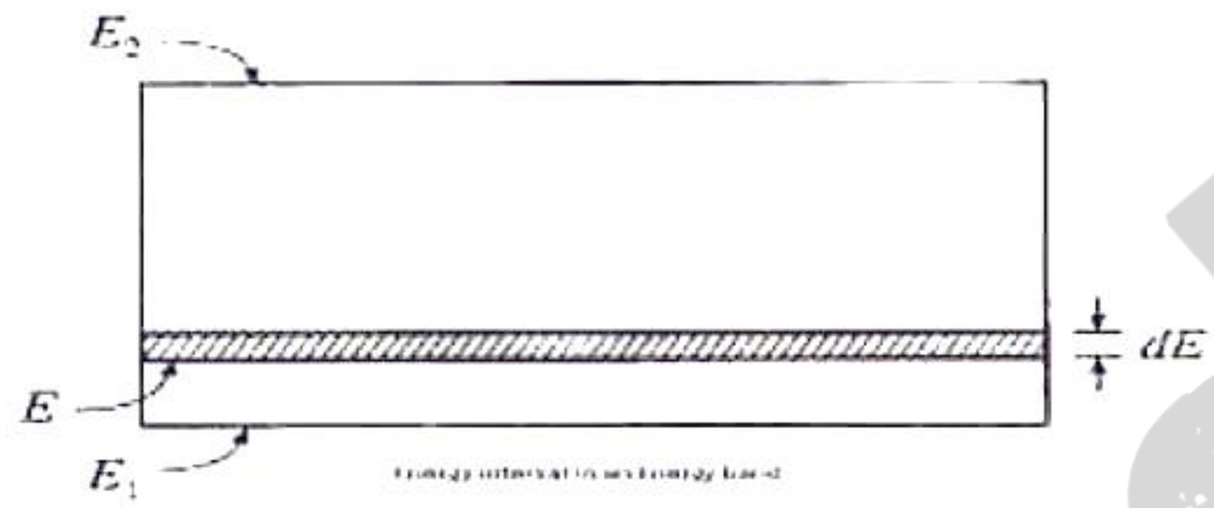


DENSITY OF STATES

Density of states is defined as the number of allowed energy states per unit energy range per unit volume in the valance band of a material. It is denoted as g(E).

A graph of g(E) verses E is shown below.

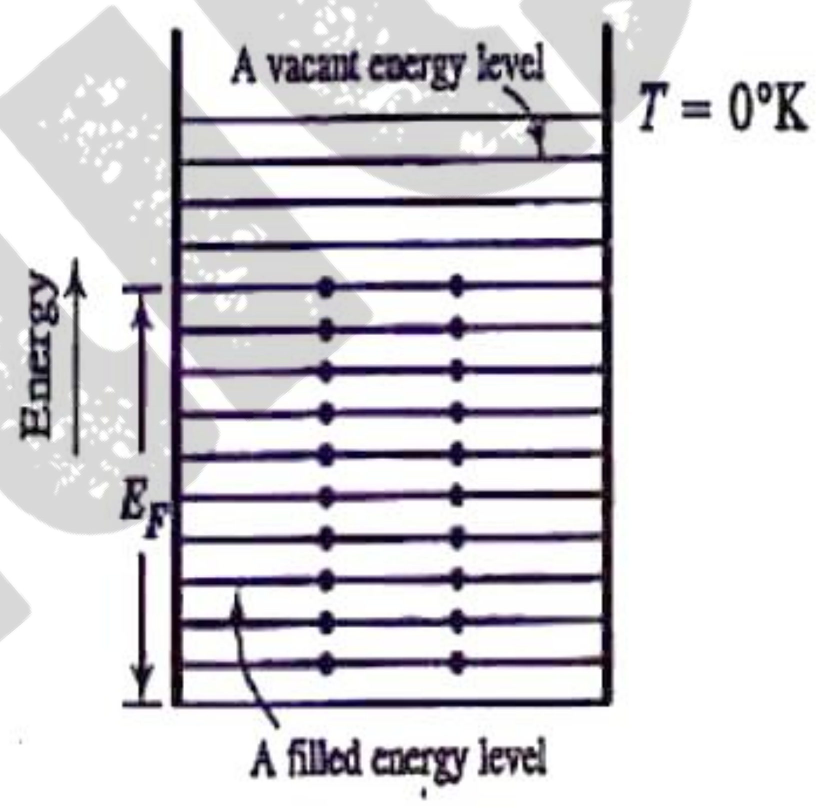
Consider an energy band spread in an energy interval between E_1 and E_2. Below E_1 and above E_2 there are energy gaps. g(E) represents the density of states at E. As dE is small, it is assumed that g(E) is constant between E and E+dE. The density of states in range E and (E+dE) is denoted by g(E)dE.



i.e. $$g(E)dE = \left[\frac{8\sqrt{2\pi m^{\frac{3}{2}}}}{h^3}\right] E^{\frac{1}{2}}\, dE$$

$$\left[\frac{8\sqrt{2}\pi m^{\frac{3}{2}}}{h^3}\right] E^{\frac{1}{2}} dE$$

It is clear g(E) is proportional to $\sqrt{E}$ in the interval dE



FERMI ENERGY

Fermi energy and Fermi level:

The energy of electrons corresponding to the highest occupied energy level at absolute $0°K$ is called Fermi energy and the energy level is called Fermi level.

Fermi factor:

Fermi factor is the probability of occupation of a given energy state by the electrons in a material at thermal equilibrium.

The probability f(E) that a given energy state with energy E is occupied by the electrons at a steady temperature T is given by

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

$$f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1}$$

f(E) is called the Fermi factor.

Dependence of Fermi factor with temperature and energy:

The dependence of Fermi factor on temperature and energy is as shown in the figure.

i) Probability of occupation for $E<E_F$ at T=0K:

When T=0K and $E<E_F$

$$f(E) = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$$

The probability of occupation of energy state is 100%

f(E)=1 for $E<E_F$.

f(E)=1 means the energy level is certainly occupied and $E<E_F$ applies to all energy levels below E_F. Therefore at T=0K all the energy levels below the Fermi level are occupied.

ii) Probability of occupation for $E>E_F$ at T=0K:

When T=0K and $E>E_F$

$$f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty} = 0$$

The probability of occupation of energy state is 0%

f(E)=0 for $E>E_F$

∴ At T=0K, all the energy levels above Fermi levels are unoccupied. Hence at T=0K the variation of f(E) for different energy values, becomes a step function as shown in the above figure.

iii) The probability of occupation at ordinary temperature(for $E\approx E_F$ at T>0K)

At ordinary temperatures though the value of probability remains 1, for $E<E_F$ it starts reducing from 1 for values of E close to but lesser than E_F as in the figure.

The values of f(E)becomes ½ at $E=E_F$

This is because for $E=E_F$

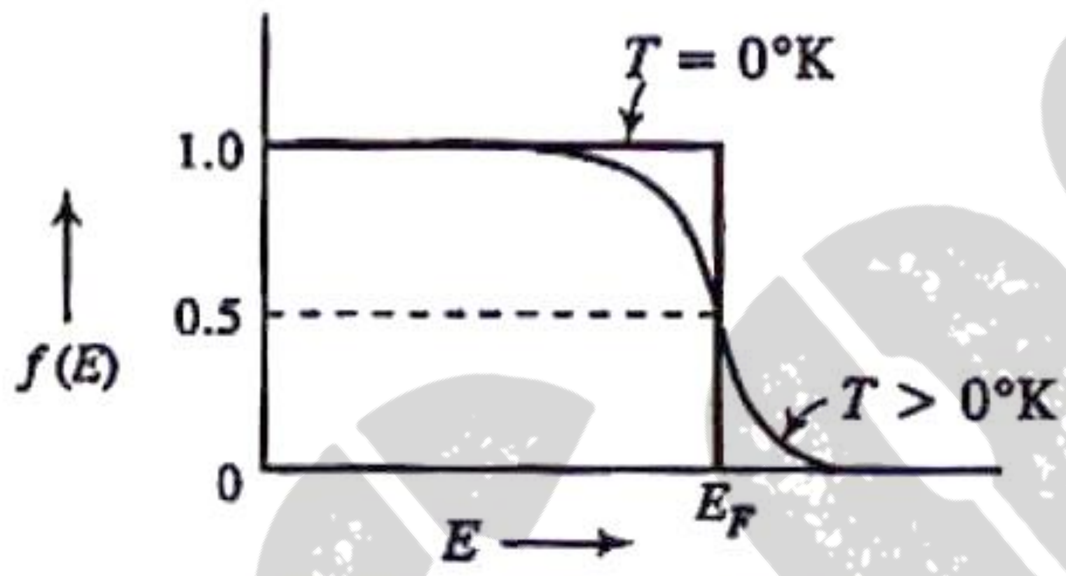
$$e^{\frac{(E-E_f)}{kT}} = e^0 = 1$$

$$\therefore f(E) = \frac{1}{e^{\frac{(E-E_f)}{kT}}+1} = \frac{1}{1+1} = \frac{1}{2}$$

The probability of occupation of energy state is 50%

Further for $E>E_F$ the probability value falls off to zero rapidly.

Hence, the Fermi energy is the most probable or the average energy of the electrons across which the energy transitions occur at temperature above zero degree absolute.

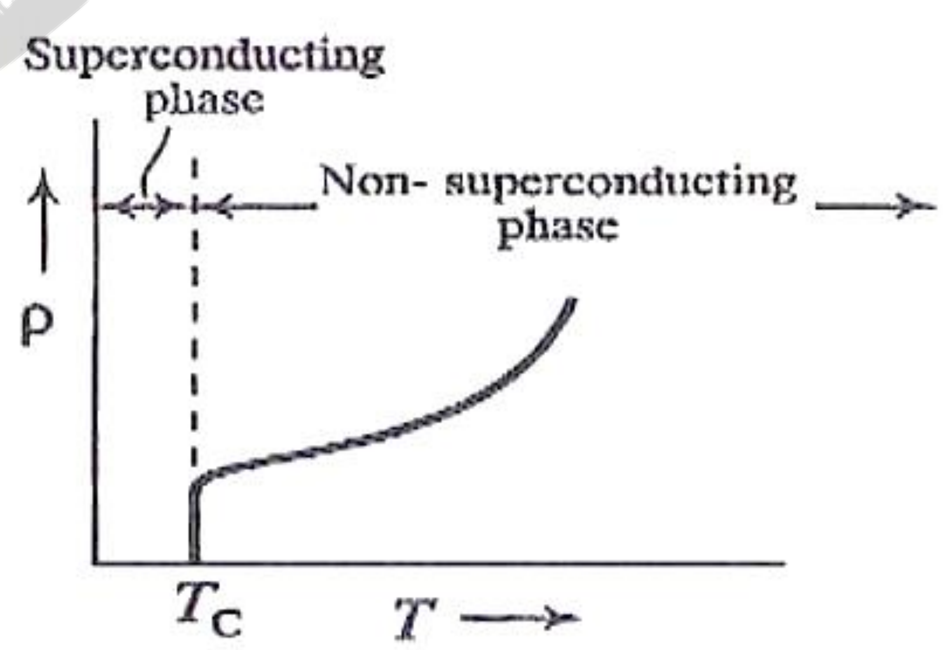


VARIATION OF f(E) WITH E

Super Conductivity:

Super conductivity is the phenomenon observed in some metals and materials. Kammerlingh Onnes in 1911 observed that the electrical resistivity of pure mercury drops abruptly to zero at about 4.2K .This state is called super conducting state. The material is called superconductor .The temperature at which they attain superconductivity is called critical temperature T_c.

Temperature dependence of resistivity of a superconductor:

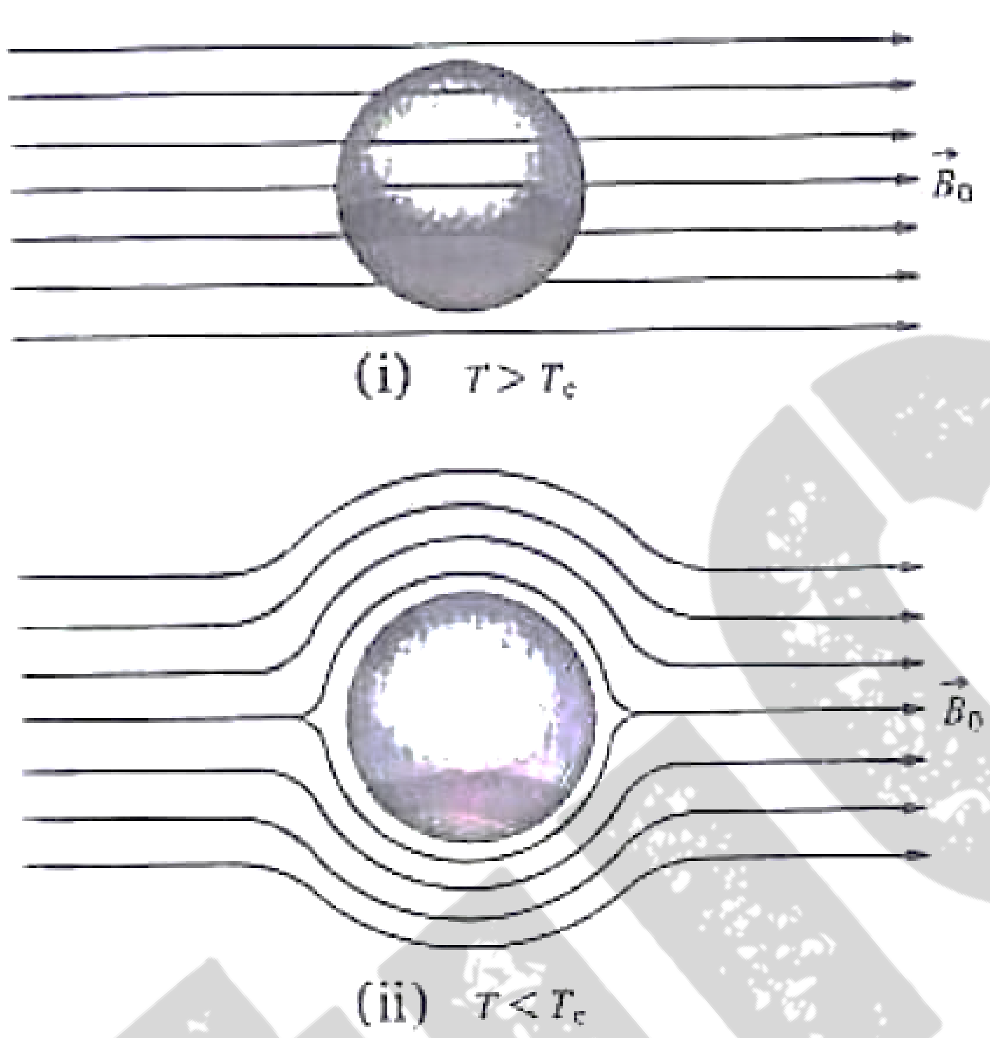


DEPENDENCE OF ρ ON T

One of the most interesting properties of solid at low temperature is that electrical resistivity of metals and alloys vanish entirely below a certain temperature. This zero resistivity or infinite conductivity is known as superconductivity. Temperature at which transition takes place is known as transition temperature or critical temperature (T_c). Above the transition temperature, the substance is in the normal state and below it will be in superconducting state. T_c value is different for different materials.

"The resistance offered by certain materials to the flow of electric current abruptly drop to zero below a threshold temperature. This phenomenon is called superconductivity and threshold temperature is called "critical temperature".

Meissner effect:



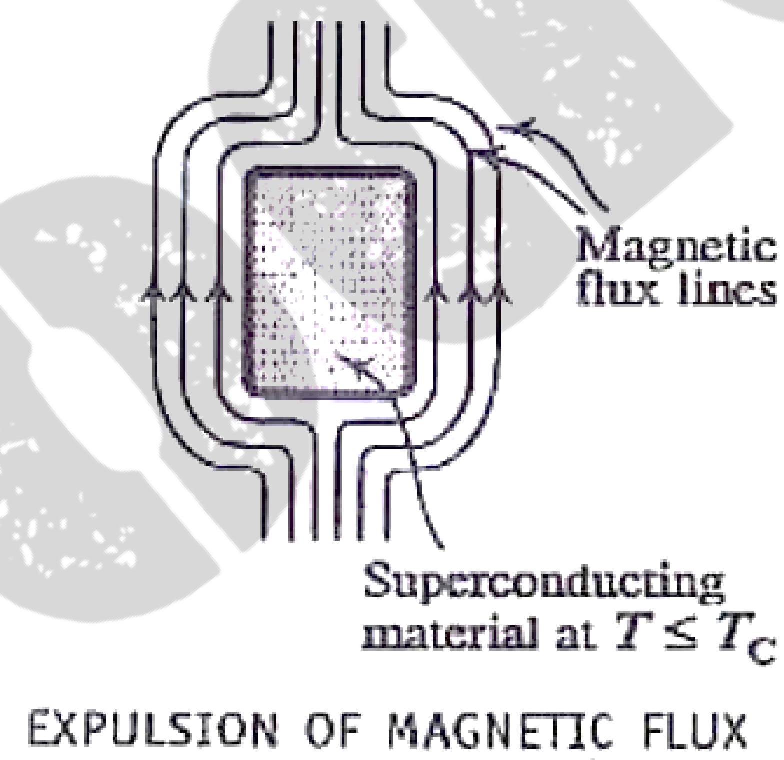


Fig: Superconductor sample subjected to an applied magnetic field with temperature (i) above and (ii) below T_C. The flux expulsion below Tc is called Meissner effect

A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus becomes perfect diamagnet. This effect is called Meissner effect.

When the temperature is lowered to T_c, the flux is suddenly and completely expelled, as the specimen becomes superconducting. The Meissner effect is reversible. When the temperature is raised the flux penetrates the material, after it reaches T_c. Then the substance will be in the normal state.

The magnetic induction inside the specimen

$$B = \mu_o (H + M)$$

Where 'H' is the intensity of the magnetizing field and 'M' is the magnetization produced within the material.

For $T < T_c$, $B = 0$

$$\mu_0 (H + M) = 0$$

$$M = -H$$

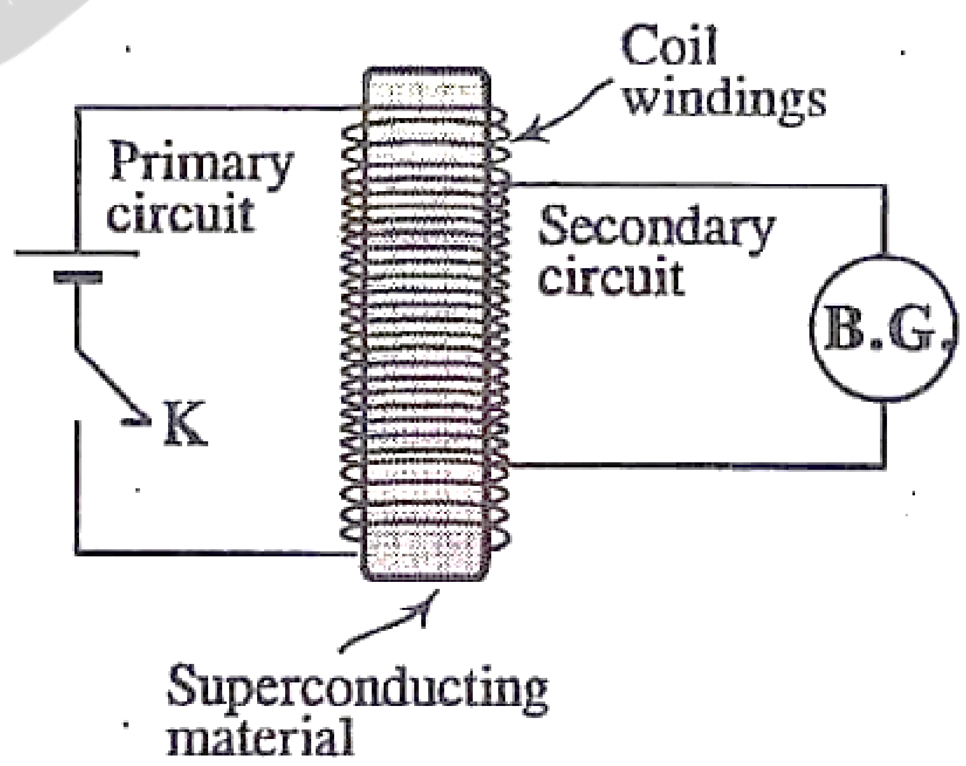
$$M/H = -1 = \chi$$

Susceptibility is -1 i.e. it is perfect diamagnetism.

Hence superconducting material do not allow the magnetic flux to exist inside the material.

Experimental demonstration of Meissner effect:

Consider a primary coil and a secondary coil, wound on a superconducting material. The primary coil is connected to a battery and a key. The secondary coil is connected to ballistic galvanometer (BG). When the key is closed the current flows through the primary coil and the magnetic field is produced. This flux is linked with the secondary coil and the current flows through the secondary coil which makes a deflection in the galvanometer. If the primary current is steady the magnetic flux and the flux linked with the coil will become steady. As the temperature of the specimen is decreased below the critical temperature, BG suddenly shows a deflection indicating that the flux linked with the secondary coil is changed. This is due to the expulsion of the magnetic flux from the specimen.



MEISSNER EFFECT

Effect of magnetic field:

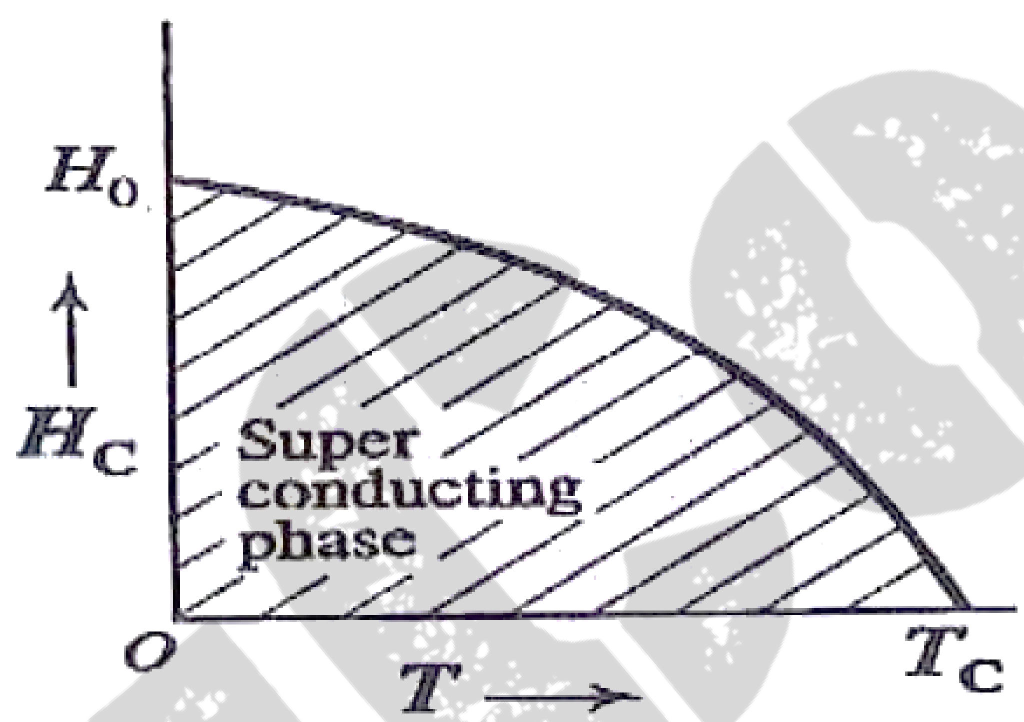
Superconductivity can be destroyed by applying magnetic field. The strength of the magnetic field required to destroy the superconductivity below the T_c is called critical field. It is denoted by $H_c(T)$.

If 'T' is the temperature of the superconducting material, 'T_c' is the critical temperature, 'H_c' is the critical field and 'H_o' is the critical field at 0°K.

They are related by

$$H_c = H_o[1-(T/T_c)^2]$$

By applying magnetic field greater than H_o, the material can never become superconductor whatever may be the low temperature. The critical field need not be external but large current flowing in superconducting ring produce critical field and destroys superconductivity.



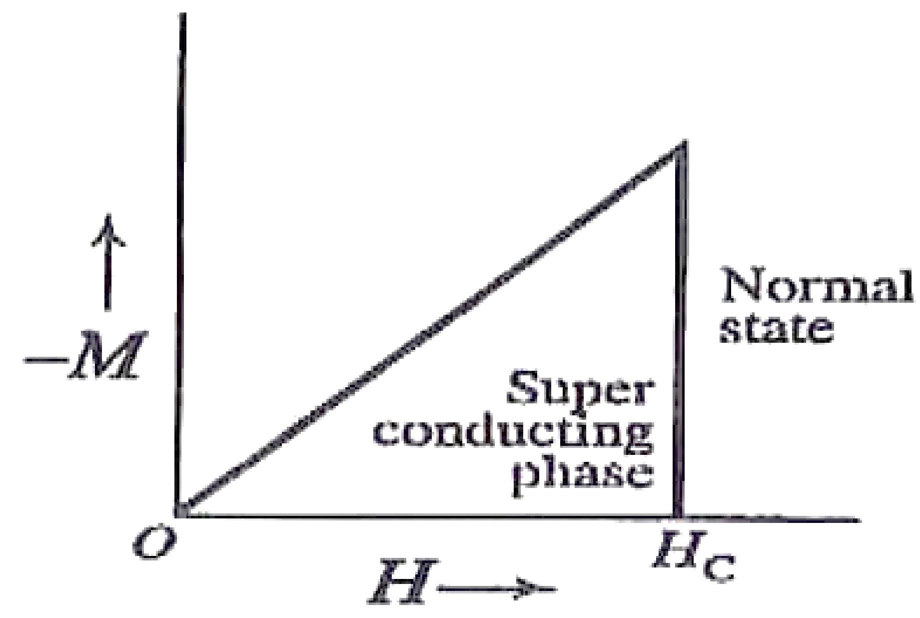
DEPENDENCE OF H_c ON T

Types of superconductors:

There are two types of superconductors. They are type-I superconductors and type-II superconductors.

i)Type-I superconductors:

Type-I superconductors exhibit complete Meissner effect. Below the critical field it behaves as perfect diamagnetic. If the external magnetic field increases beyond H_c the superconducting specimen gets converted to normal state. The magnetic flux penetrates and resistance increase from zero to some value. As the critical field is very low for type-I superconductors, they are not used in construction of solenoids and superconducting magnets.



DEPENDENCE OF MAGNETIC MOMENT ON *H*

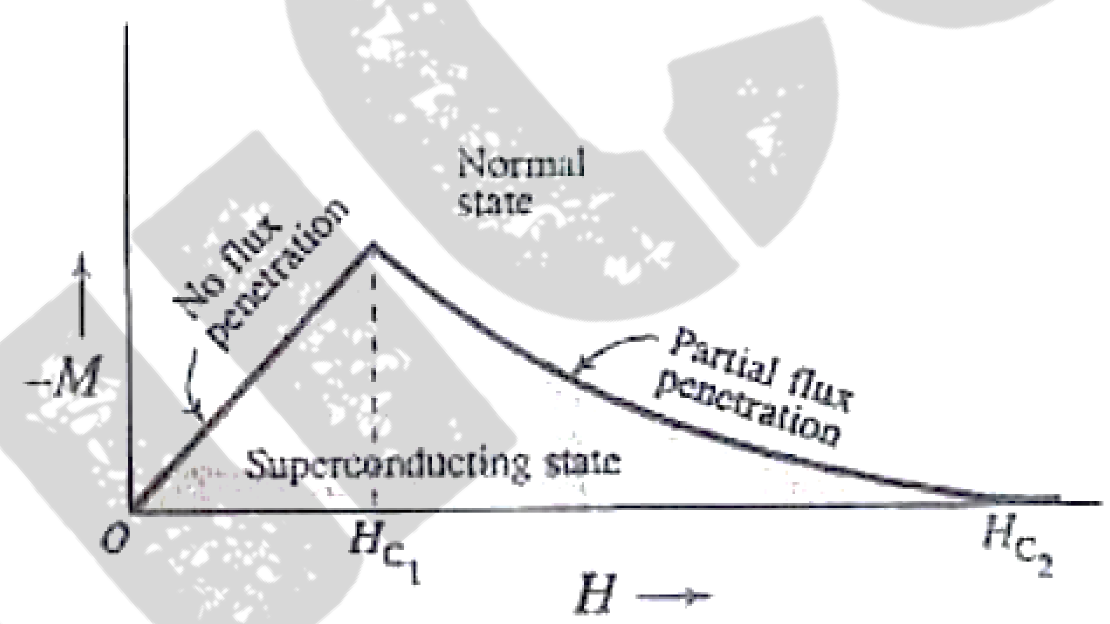
ii) Type-II superconductors

Type-II superconductors are hard superconductors. They exist in three states

i) Superconducting state

ii) Mixed state

iii) Normal state



DEPENDENCE OF MAGNETIC MOMENT ON *H*

They are having two critical fields H_{c1} and H_{c2}. For the field less then Hc_1, it expels the magnetic field completely and becomes a perfect diamagnetic. Between Hc_1 and Hc_2 the flux starts penetrating throughout the specimen. This state is called vortex state. Hc_2 is 100 times higher than Hc_1. At Hc_2 the flux penetrates completely and becomes normal conductor. Type-II superconductors are used in the manufacturing of the superconducting magnets of high magnetic fields above 10 Tesla.

BCS theory superconductivity:

- Bardeen, Cooper and Schrieffer (BCS) in 1957 explained the phenomenon of superconductivity based on the formation of cooper pairs. It is called BCS theory. It is a quantum mechanical concept.

- When a current flow in a superconductor, electrons come near a positive ion core of lattice, due to attractive force. The ion core also gets displaced from its position, which is called lattice distortion. The lattice vibrations are quantized in a term called Phonons.
- Consider elctron-1 at the center of the metal lattice as shown in fig. The electron exerts an attractive force on the neighboring ions and distorts the lattice. This distortion in the lattice is the phonon field. Now an electron-1 is surrounded by positive ions and positive charge density around it is high compared to other regions of the lattice. As the electron-1 moves through the lattice, the distortion shifts along with it.
- Consider another electron, say electron-2, near to electron-1 as shown in fig. As the positive field density around electron-1 is high, it eclipses the -ve field of electron-1, as a result, electron-2 is attracted towards electron-1via phono field.

- This attraction tends to reduce the net energy of the two electrons and a bond is formed between the two electrons. Therefore, these two electrons become pair called cooper pair and move together in the lattice as a single particle.
- "Cooper pairs are a bound pair of electrons formed by the interaction between the electrons with opposite spin and momentum in a phonon field".
- Below critical temperature the wave functions of all the cooper pairs overlap to form a huge wave packet and move as single wave packet. Within this packet, if one pair losses energy due to the collision with the lattice, the energy will be gained by another pair within the same wave packet and the resistance factor vanishes or in other words conductivity becomes infinity which is called as superconductivity.

High temperature superconductors:

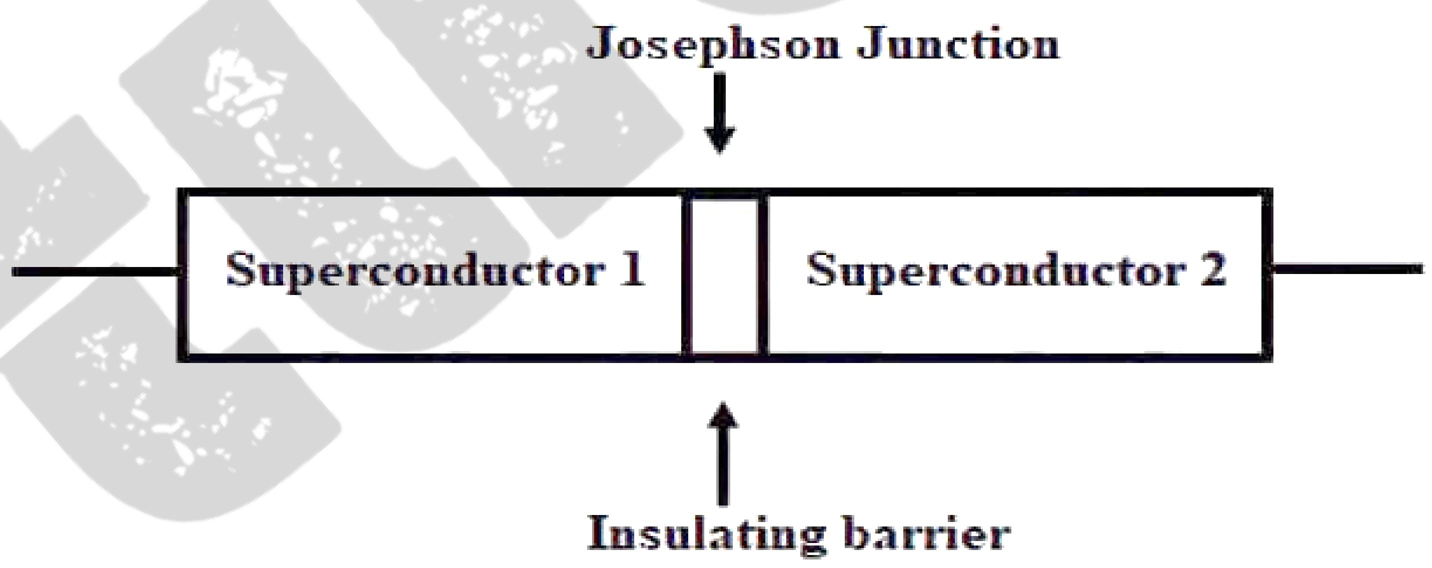
The term high-temperature superconductor was first used to designate the new family of cuprate-perovskite ceramic materials discovered by Bednorz and Müller in 1986. The first high-temperature superconductor, LaBaCuO, with a transition temperature of 30 K and in the same year LSCO ($La_{2-x}Sr_xCuO_4$) discovered with T_C of 40K. In 1987 it was shown that superconductors with T_c greater than 77K could be prepared, this temperature is greater than the liquid helium temperature. $YBa_2Cu_3O_7$ was discovered to have a Tc of 92 K. Bismuth/lead strontium Calcium Copper $(Bi\ Pb)_2Sr_2\ Ca_2Cu_3O_x$ (x<0.1) with T_c=105K. Thallium barium Calcium copper oxide ($Tl\ Ba_2Ca_2Cu_3O_4$) of T_c=115K. Mercury barium calcium copper oxide ($Hg\ Ba_2Ca_2Cu_3O_4$) with T_c=135K.

All high temperature superconductors are different types of oxides of copper, and bear a particular type of crystal structure called Perovskite crystal structure. The number of copper layers increases the T_c value increases. The current in the high T_C materials is direction dependent. It is strong in parallel to copper-oxygen planes and weak in perpendicular to copper-oxygen planes.

High T_c materials are Type-II superconductors and they are brittle and don't carry enough current. The formation of electron pairs is not due to interaction of electron lattice as in the BCS theory. Still it is not clear what does cause the formation of pairs. Research is being conducting in this direction. The high temperature superconductors are useful in high field applications. It can carry high currents of 10^5 to 10^6 amps in moderate magnetic fields. They are used in military applications, Josephson junction in SQUIDS, under sea communication, submarines.

Quantum Tunneling:

Consider two superconductors separated by insulating barrier of thickness less than 10-20 Å, then the cooper pairs tunneling through the insulating barrier is known as **Josephson superconducting quantum tunneling**. The junction between the two superconductors with insulating barrier is known as **Josephson junction**.



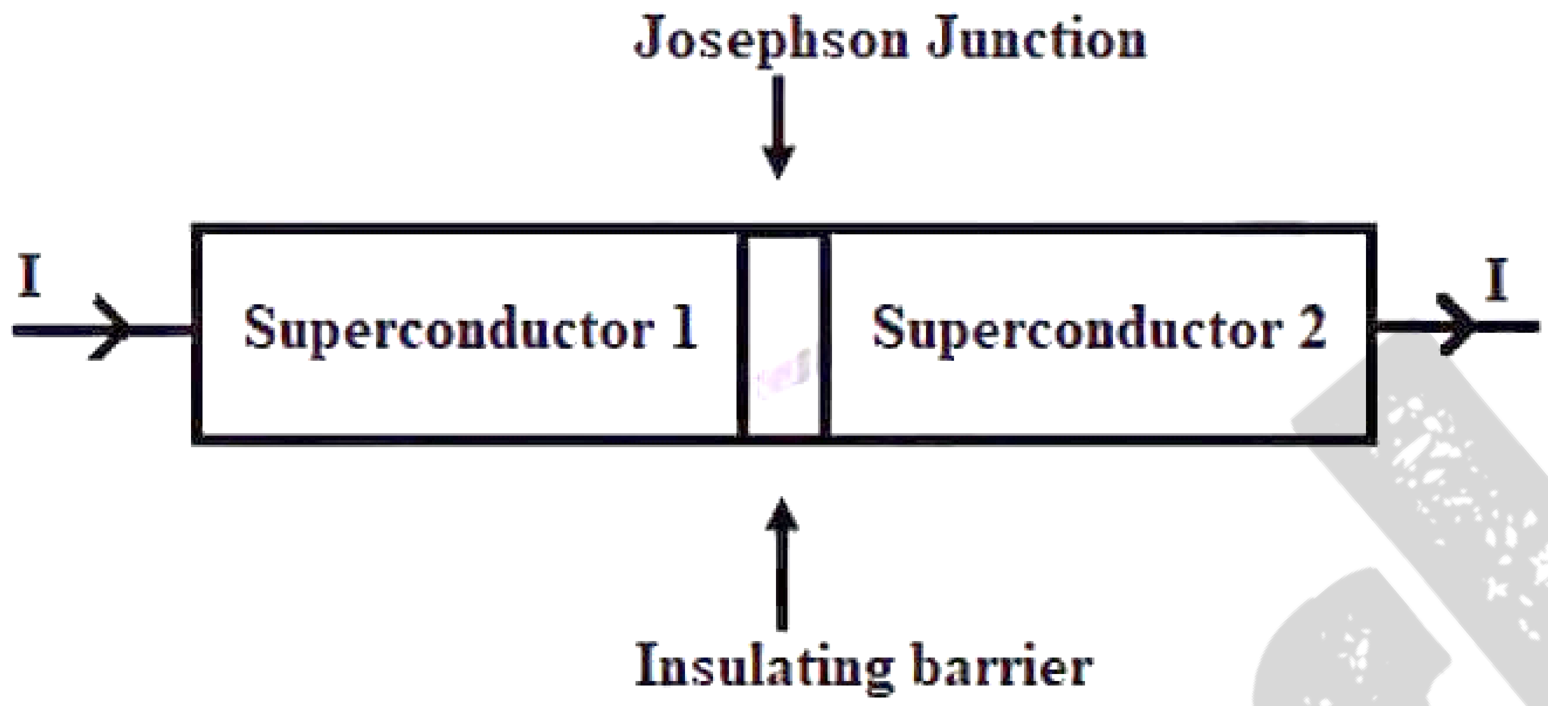
Josephson junction is an arrangement of two superconductors separated by an insulating barrier. When the barrier is thin enough, cooper pairs from one superconductor can tunnel through the barrier and reach the other superconductor.

Josephson proposed that this kind of tunneling leads to three kinds of effect, namely

1. D.C Josephson effect
2. A.C Josephson effect
3. quantum interference

1. **D C Josephson effect**

As per dc Josephson Effect, the tunneling of cooper pairs through the junction occurs without any resistance, which results in a steady dc current without any application of voltage between the two superconductors.



The super current through the junction is given by,

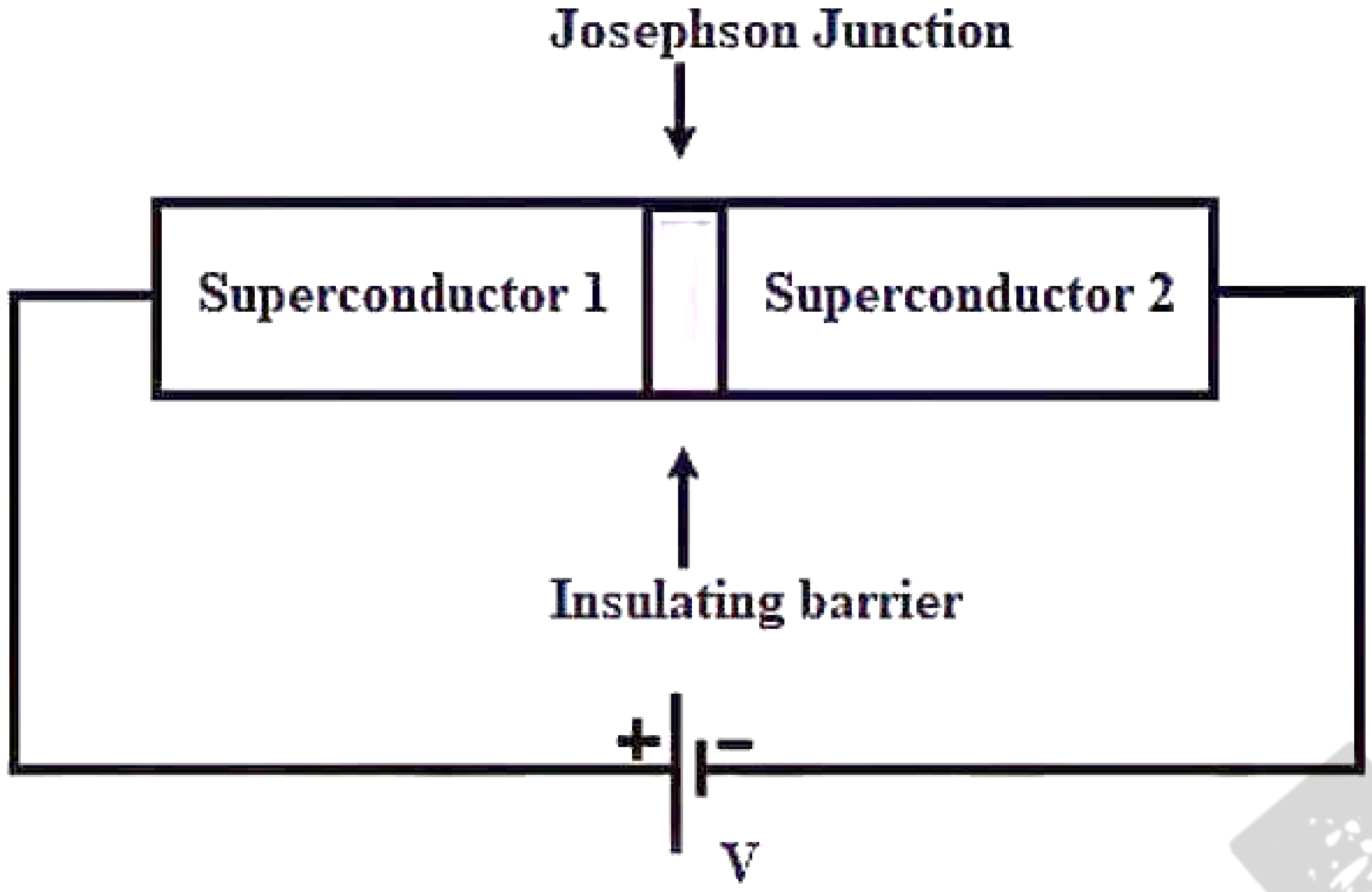
$$I_S = I_C \ sin\, sin\, \theta$$

Where, θ= phase difference between the wave functions describing the cooper pairs on both the sides of the junction.

I_C = critical current at zero voltage which depends on the thickness and width of the insulating layer.

2. **A C Josephson effect**

When a dc voltage is applied across the junction, the tunneling of cooper pairs occur in such a way than an ac current would develop in the junction and this effect is called as ac Josephson Effect. When a potential difference of 'V' is applied between the two sides of the junction then a radio frequency (RF) current oscillations across the junction is generated. This is known as ac Josephson Effect.



$$I = I_C \sin \sin (\Phi_0 + \Delta\Phi)$$

The energies of the cooper pairs on both sides of the barrier difference is E = hν = 2eV (Calculated using quantum mechanical concept).

Therefore it can be shown that, $\Delta\Phi = \omega t = 2\pi t \left(\frac{2\,eV}{h}\right)$

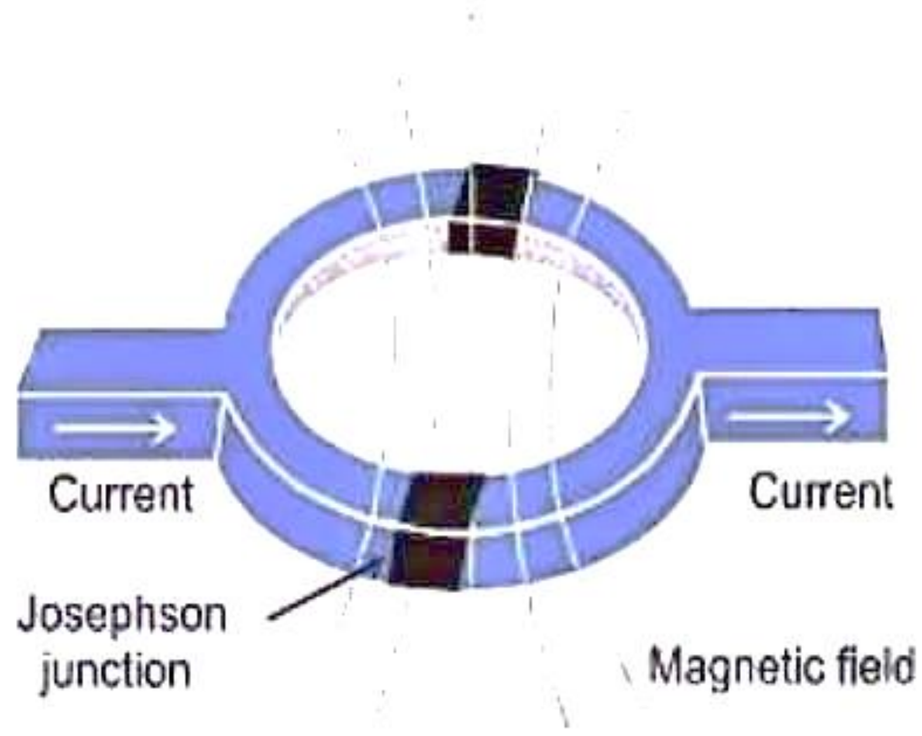
Hence, $I = I_C \sin \sin \left(\Phi_0 + 2\pi t \left(\frac{2\,eV}{h}\right)\right)$

I = Alternating current of frequency $\nu = \frac{2\,eV}{h}$

The above equation is an equation for alternate current. Thus when a voltage is applied across the junction, an ac current gets generated.

SQUID

- SQUID is an acronym for Superconducting Quantum Interference Device.
- It is an Ultra-sensitive measuring instrument used for the detection of very weak magnetic fields of the order of 10^{-14} T.
- A SQUID is formed by incorporating two Josephson's junction in the loop of a superconducting material.
- When a magnetic field is applied to this superconducting circuit, it induces a circulating current which produces just that much opposing magnetic field so as to exclude the flux from the loop.



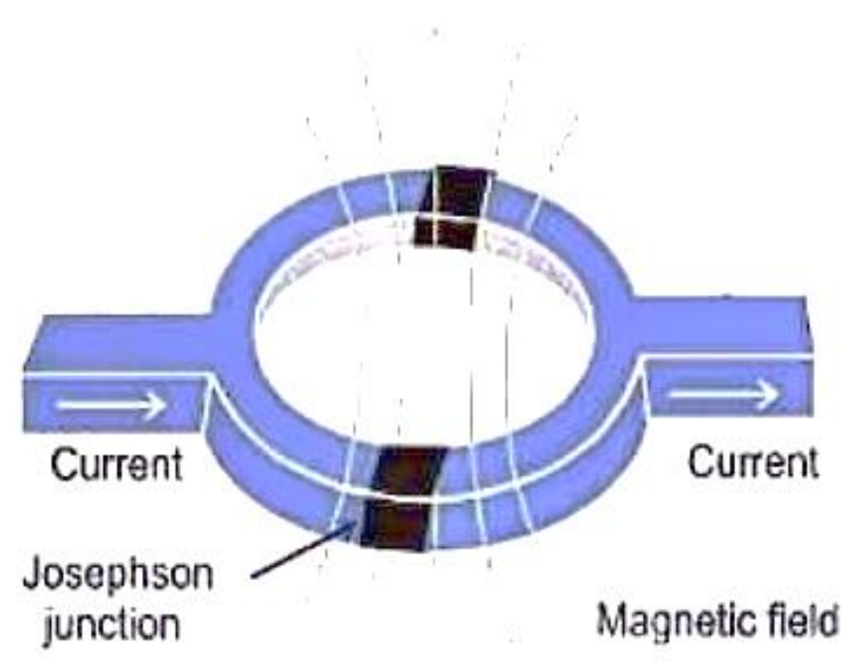
[Note: The flux remains excluded as long as the junction current do not exceed a critical value. But the circuit switches to resistive phase and thereby the flux passes into the loop once the current in either of the junction or in both the junction exceed the critical value. Thus the loop acts like a gate to allow or exclude the flux]

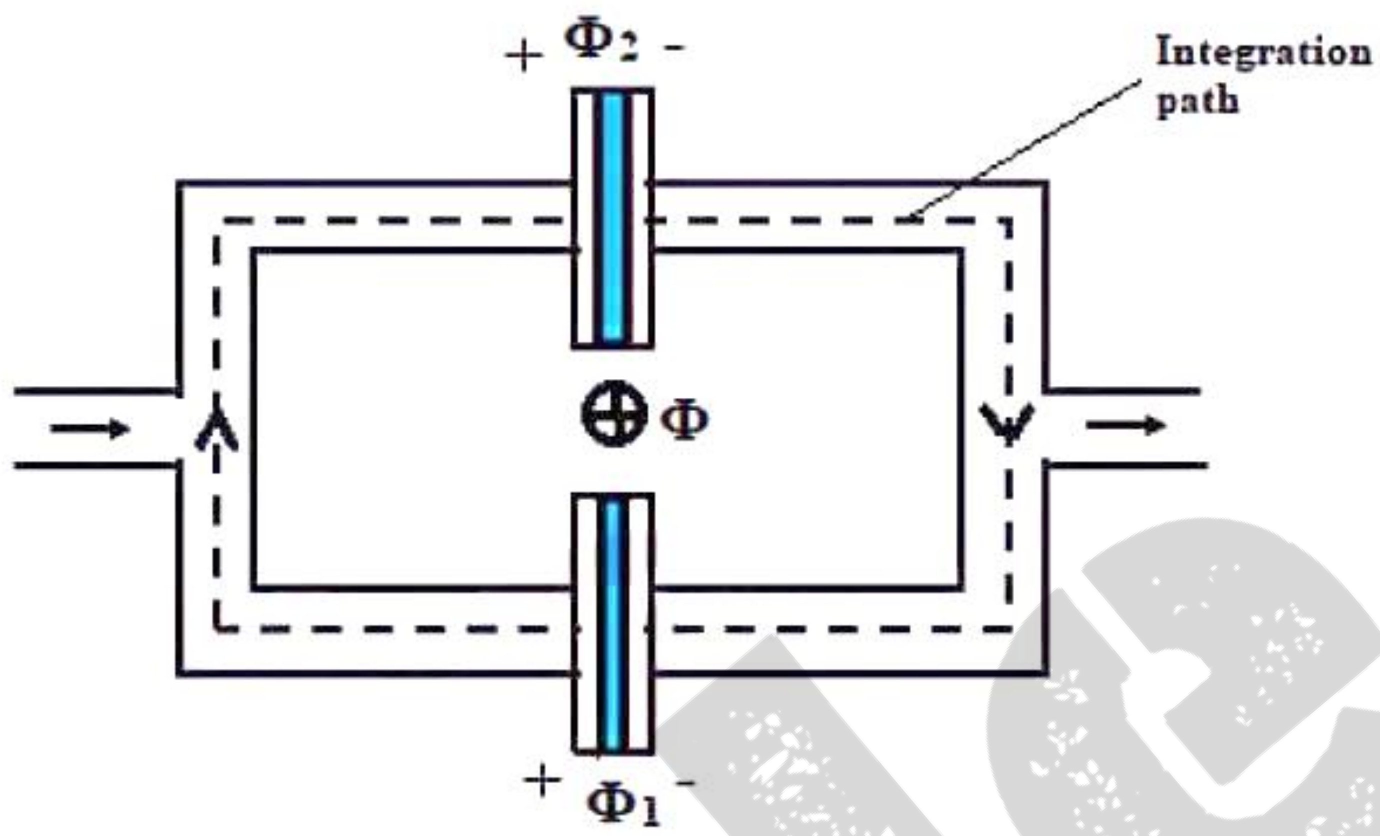
DC Squids

- A DC SQUID consists of two Josephson junctions connected in parallel on a closed superconducting loop as shown in the Fig.
- In this arrangement , a super current flows through the loop, whose magnitude is given by

$$I_S = I_C \ sin \, sin \, \theta$$

- This when magnetic field is applied normal to the plane of the loop, the value of I_C depends on the magnetic flux.
- If magnetic applied magnetic field is = nπ= nΦ_o, then I_c=0, I =0 i.e, the super current through the loop zero. If the super current is not flowing, the squid is not a superconductor and applied magnetic flux can pass through the loop.
- If magnetic applied magnetic field is ≠ nπ ≠Φ_o, then $I_c \neq 0$, I ≠ 0 i.e, the super current through the loop is non zero and the squid is a super conductor. Hence , magnetic flux cannot pass through the loop.
- Thus the squid acts loke a gate for magnetic field. It allows magnetic field is integral of Φ_o [where Φ_o =(h/2π) = 2.07 x 10^{-7} T, is the smallest unit of magnetic field called Fluxon]. SQUID can detect very small magnetic fields. Hence they are used in highly sensitive magnetometers.





RF SQUIDS

- The single junction SQUIDS are also known as RF SQUIDS.
- The junction is shorted by superconductor path; therefore the voltage response is obtained by coupling the loop to a RF bias tank circuit.
- The RF (Radio Frequency) SQUID is **a one-junction SQUID loop that can be used as a magnetic field detector.**
- In this configuration, **the RF SQUID is inductively coupled to the inductance L_T of an L_C tank circuit**. The tank circuit is driven by an rf current, and the resultant rf voltage is periodic in the flux applied to the SQUID loop with period Φ_0.